The modified gravity landscape

looking through the lens of Quasinormal modes.

*A talk at **The Spanish-Portuguese Relativity Meeting, EREP2021**.*

$\{ \quad + \quad \} \longrightarrow$

Outline for the talk

1. Classical physics and gravitation
2. Various gravitational models
3. The *unary* problem
4. Consequences to GW physics

Classical theories of physics:

Models of classical physics are derived from an action. The action is a functional of one or many field variables, and their derivatives with respect to a parameter that is (or can be) regarded as the ’time’.

Classical equations of motion are obtained by varying the action with respect to the field variables and their derivatives with respect to the parameter that is regarded as time.

Elementary gravitational processes:
it takes two

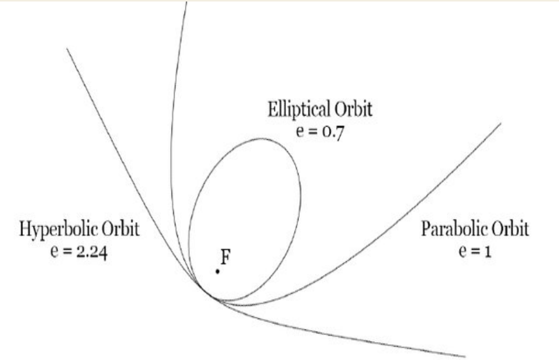

Figure: The three different kinds of elementary two-body gravitational interactions based on the total mechanical energy of the two bodies.
Source: Engelhardt, Toni. (2014). Setting an Observational Upper Limit to the Number Density of Interstellar Objects.

Elliptical orbits, the apparent picture

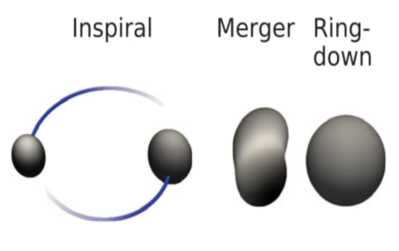

Figure: BREAKING NEWS!!! Nature says no to stable two-body orbits with a negative total mechanical energy, contrary to the predictions of Newtonian gravity!

Source: LIGO Scientific Collaboration, Testing General Relativity with Gravitational Waves from the first half of the LIGO-Virgo 3rd Observing Run.

Elliptical orbits, with some new "ears"

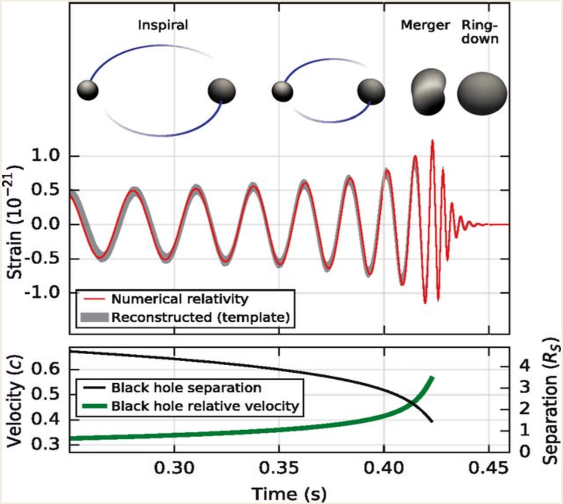

Figure: Source: LSC 2016, Observation of Gravitational Waves from a Binary Black Hole Merger.

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General relativity and its various modifications

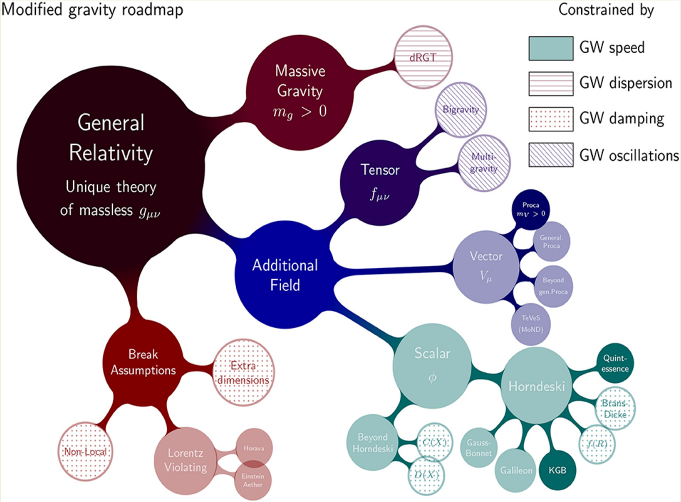

Figure: Source: Ezquiaga Jose María, Zumalacárregui Miguel, 2018; Dark Energy in Light of Multi-Messenger Gravitational-Wave Astronomy

- **The action of GR:** The action of GR is the Ricci scalar R times the square root of the negative of the metric determinant, or

$$\mathscr{S} \;=\; \int_{\mathscr{M}} d^4x \sqrt{-g}\,\left(\frac{R}{16\,\pi} + \mathscr{L}_m\right)$$

- **Basic statement of GR:** The *trace reversed* Ricci tensor(Einstein tensor), itself being obtained by contracting the Riemann tensor, is directly proportional to the energy-momentum-stress density tensor that is modeling the behavior of classical matter.

$$G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \;=\; 8\,\pi\,G\,T_m^{\mu\nu}$$

Alternative gravity, seen with the lens of GR

or the classical picture using the metric formalism.

Any modified theory of gravity with/without extra fields (other than the metric tensor) can be written in the GR form, which (historically) already has some inbuilt tools to solve such system of DEs, given by

$$G^{\mu\nu} = 8\pi \left(T^{\mu\nu}_{matter} + T^{\mu\nu}_{effective} \right)$$

$T^{\mu\nu}_{effective}$: Tensor (real or effective) comprising of terms (/effects?) from "modifications" to GR. Modifications to GR also lead to other DEs (followed by scalar and/or vector **D**egrees **o**f **F**reedom for example) which must be solved simultaneously along with the metric DEs above.

$f\,(R)$ action and equations of motion

- **The action:**

$$\mathscr{S} \;=\; \int_{\mathscr{M}} d^4x \sqrt{-g}\left[\frac{f\,(R)}{16\,\pi} + \mathscr{L}_m\right]$$

- **The field equations:**

$$G^{\mu\nu} = \frac{8\,\pi}{f'}\left(T^{\mu\nu}_m + T^{\mu\nu}_{eff}\right)$$

$$T^{\mu\nu}_{eff} \equiv \nabla^\mu \nabla^\nu f' + \frac{g^{\mu\nu}}{2}\left(f - R f'\right) - g^{\mu\nu} g^{\alpha\beta} \nabla_\alpha \nabla_\beta f'$$

$$3\,g^{\mu\nu} \nabla_\mu \nabla_\nu f' + f' R - 2 f = 8\,\pi\, T_m$$

Chern-Simons modified gravity

- General relativity can be extended in an explicitly parity violating manner by adding a dynamical pseudoscalar field Θ, and making it couple non-trivially with the curvature

$$\mathscr{S} = \int d^4x\sqrt{-g}\left[\frac{R}{16\,\pi} + \frac{\alpha}{4}\Theta\,{}^*RR - \frac{\beta}{2}(\nabla\Theta)^2\right]$$

$${}^*RR = \frac{1}{2}R_{\mu\nu\rho\sigma}\epsilon^{\mu\nu\alpha\beta}R_{\alpha\beta}{}^{\rho\sigma}$$

- ${}^*RR$ is known as the Pontryagin density in literature.
- Equations of motion become

$$\begin{aligned} G_{\mu\nu} &= 8\,\pi\,\alpha\,\nabla_\sigma\nabla_\tau\Theta\left({}^*R^{\tau}{}_{\mu}{}^{\sigma}{}_{\nu} + {}^*R^{\tau}{}_{\nu}{}^{\sigma}{}_{\mu}\right) \\ \Box\Theta &= -\frac{\alpha}{4\beta}\,{}^*RR \end{aligned}$$

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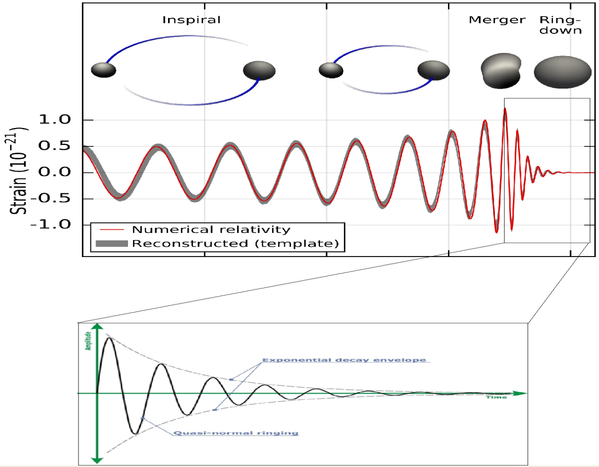
Epoch of interest

Source: Berti et al. (2009), Quasinormal modes of black holes and black branes



The semi-final state of a binary gravitational process with negative total mechanical energy is a single deformed object, which must also be an unstable solution of the same theory, eventually damping down to a stable solution.

Can be treated in the realm of linear perturbation theory about a specific background solution.

Linear perturbations about a background solution

Field equations of GR:

$$R_{\mu\nu}\left(g_{\mu\nu}\right) \quad = \quad 0$$

Perturbation:

$$g_{\mu\nu} \quad = \quad g^{B}_{\mu\nu} + \epsilon\, h_{\mu\nu}$$

$$\epsilon < \frac{|h|}{|g^B|}$$

Characteristic length scale over which the background sufficiently changes.

Characteristic length scale over which the ripples sufficiently change.

Helmholtz decomposition

or the irreducible representation of a symmetric tensor under $SO(3)$ transformation in spherically symmetric space-times.

Parity operation in a spherically symmetric coordinate system:

$$\mathbf{P} \quad : \quad \{\theta \to \pi - \theta,\ \phi \to \pi + \phi\}$$

$$h_{\mu\nu} \equiv \sum_{l,m} \left(\boxed{\begin{matrix} 0 & 0 & (f^A)_t(t,r)\,V_\theta(\theta,\phi) & (f^A)_t(t,r)\,V_\phi(\theta,\phi) \\ 0 & 0 & (f^A)_r(t,r)\,V_\theta(\theta,\phi) & (f^A)_r(t,r)\,V_\phi(\theta,\phi) \\ \mathrm{Sym} & \mathrm{Sym} & (H^A)_{TT}(t,r)\,V_{\theta\theta}(\theta,\phi) & (H^A)_{TT}(t,r)\,V_{\theta\phi}(\theta,\phi) \\ \mathrm{Sym} & \mathrm{Sym} & \mathrm{Sym} & (H^A)_{TT}(t,r)\,V_{\phi\phi}(\theta,\phi) \end{matrix}} \right)$$

$$+$$

$$\left(\boxed{\begin{matrix} (f^P)_{tt}\,S(\theta,\phi) & (f^P)_{tr}\,S(\theta,\phi) & (f^P)_t(t,r)\,S_\theta(\theta,\phi) & (f^P)_t(t,r)\,S_\phi(\theta,\phi) \\ \mathrm{Sym} & (f^P)_{rr}\,S(\theta,\phi) & (f^P)_r(t,r)\,S_\theta(\theta,\phi) & (f^P)_r(t,r)\,S_\phi(\theta,\phi) \\ \mathrm{Sym} & \mathrm{Sym} & r^2[(H^P)_L(t,r)\,\gamma_{\theta\theta}\,S(\theta,\phi) + (H^P)_{TT}(t,r)\,V_{\theta\theta}(\theta,\phi)] & r^2[(H^P)_L(t,r)\,\gamma_{\theta\phi}\,S(\theta,\phi) + (H^P)_{TT}(t,r)\,S_{\theta\phi}(\theta,\phi)] \\ \mathrm{Sym} & \mathrm{Sym} & \mathrm{Sym} & r^2[(H^P)_L(t,r)\,\gamma_{\phi\phi}\,S(\theta,\phi) + (H^P)_{TT}(t,r)\,S_{\phi\phi}(\theta,\phi)] \end{matrix}} \right)$$

A : Axial, 3 scalar functions of (t, r) : $(f^A)_t$, $(f^A)_r$, $(H^A)_{TT}$

P : Polar, 7 scalar functions of (t, r) : $(f^P)_{tt}$, $(f^P)_{tr}$, $(f^P)_{rr}$, $(f^P)_t$, $(f^P)_r$, $(H^P)_L$, $(H^P)_{TT}$

Total 10 unknown scalars.However, only 2 independent.

Isospectrality in Schwarzschild black holes

- DEs followed by $\mathbf{h}^{P}_{\ell m}$ and $\mathbf{h}^{A}_{\ell m}$ decouple from each other (only at the linear order).
- Gauge invariant combinations of the components of $\mathbf{h}^{P}_{\ell m}$ and $\mathbf{h}^{A}_{\ell m}$ obey two wave equations.

Expansion in plane wave fronts ⟶ $e^{i\omega t}$ time dependence.

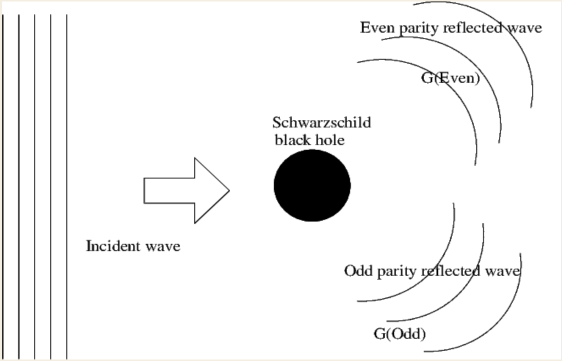
$$\frac{d^2\Phi^{P/A}_{\ell m}}{d{r_*}^2} + \left(\omega^2 - V_{P/A}\right) = 0$$

Eddington-Finkelstein/Tortoise coordinates.

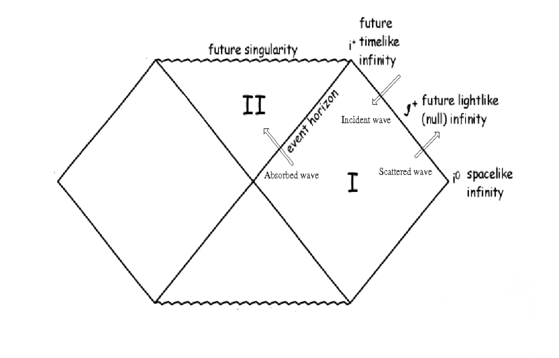
Background curvature induced scattering potential, same physical profile for both parities.

- Problem of ring-down modelled in the wave scattering picture.

The wave scattering picture



Wave scattering boundary conditions



Irreducible representation of a four-vector under $SO(3)$

$$A^{\mu} \equiv \begin{pmatrix} 0 \\ 0 \\ \mathcal{A}_T{}^A\, V_\theta(\theta,\phi) \\ \mathcal{A}_T{}^A\, V_\phi(\theta,\phi) \end{pmatrix} + \begin{pmatrix} \mathcal{A}^P{}_t\, S(\theta,\phi) \\ \mathcal{A}^P{}_r\, S(\theta,\phi) \\ \mathcal{A}_L{}^P\, S_\theta(\theta,\phi) \\ \mathcal{A}_L{}^P\, S_\phi(\theta,\phi) \end{pmatrix}$$

A: Axial, 1 scalar function : $\mathcal{A}_T{}^A$

P: Polar, 3 scalar functions : $\mathcal{A}^P{}_t$, $\mathcal{A}^P{}_r$, $\mathcal{A}_L{}^P$

Total 4 unknown scalars. However, two independent independent degress of freedom.

Master equations of perturbation

of Reissner-Nordström black holes

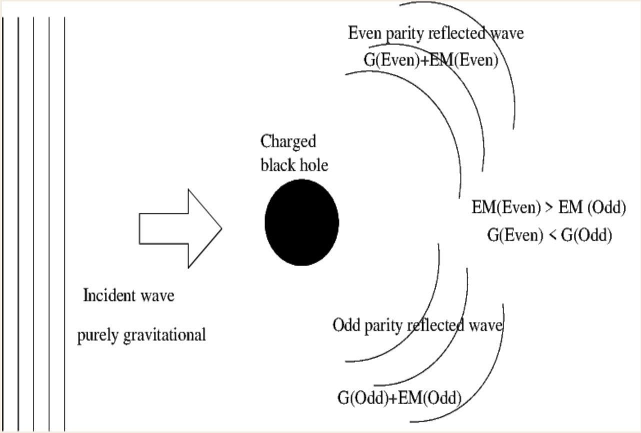
$$
\begin{aligned}
\frac{d^2\Phi^O_\pm}{dx^2} + \left(\tilde{\omega}^2 - V^O_\pm\right)\Phi^O_\pm &= 0 \\
\frac{d^2\Phi^E_\pm}{dx^2} + \left(\tilde{\omega}^2 - V^E_\pm\right)\Phi^E_\pm &= 0
\end{aligned}
$$

$$
\begin{aligned}
x &= y - \frac{\ln(y-1)}{q^2-1} + \frac{q^4\ln\left(y-q^2\right)}{q^2-1} \\
y &= \frac{r}{r_H}; \qquad q = \frac{Q}{r_H}; \qquad r_H = M + \sqrt{M^2 - Q^2}
\end{aligned}
$$

Isospectral relations: Charged black holes in GR

- Two kinds of perturbations per parity: gravitational and electromagnetic mode.
- Strength of coupling of electromagnetic perturbation to gravitational perturbation is parity dependent (Gunter, 1980).
- Although, net (gravitational+electromagnetic) energy equality holds for both modes, **even parity reflection coefficient is less than the reflection coefficient of odd parity**.
- **A change in the relative difference of radiated gravitational energies, compared to the corresponding GR value, between two modes is dependent on black hole charge and can be used to detect the same** (SB & SS, 2018).

The wave scattering picture



Linear perturbations in modified theories of gravity

in $f(R)$ and dynamical Chern-Simons modifications to GR

- Gravitational perturbations of a Schwarzschild BH in $f(R)$ / **d**ynamical **C**hern-**S**imons modification to GR lead to the extra massive / massless degree of freedom coupling only to the even / odd parity perturbations leaving the other gravitational parity untouched (SB & SS '17 / SB & SS '19).

$$\frac{d^2\Phi_E^{(\ell,m)}}{dr_*^2} + \left(\omega^2 - V_E^{(\ell,m)}\right)\Phi_E^{(\ell,m)} = S_{f(R)}^{(\ell,m)}$$

$$\frac{d^2\Phi_O^{(\ell,m)}}{dr_*^2} + \left(\omega^2 - V_O^{(\ell,m)}\right)\Phi_O^{(\ell,m)} = S_{dCS}^{(\ell,m)}$$

- Effective source term for $f(R)$ modifications.
- Effective source term for dCS modification to GR.

Broken/dynamic isospectrality

under modifications to the Einstein-Hilbert action

$$S = \int_{\mathscr{M}} d^4x \, (\mathscr{L}_{EH} + \mathscr{L}_{PC} + \mathscr{L}_{PV} + .?. + \mathscr{L}_{matter})$$

$$\mathscr{L}_{EH} \equiv \sqrt{-g}\, R \qquad ; \qquad \mathscr{L}_{matter} : \text{Lagrangian of classical matter.}$$

- So far only two classes based on the effect of extra degrees of freedom with the GR metric degrees of freedom, parity conserving generalization and parity violating modifications.
- If there can be a third independent classification remains to be seen.

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Amplitude ratios as a novel parameter to distinguish between GR and alternative models.

- Modifications to GR lead to extra degrees of freedom, which may couple preferentially to either parities of the massless spin-2 field, depending on the motivation for modifying the Einstein-Hilbert action.
- Preferential coupling leads to smoking gun effects on relative amplitude ratios of the spin-2 amplitudes. For example, see Shankaranarayanan in 2019 Essay Competition of the Gravity Research Foundation, arXiv:1905.03943.

Thank you for your attention...