Gravitational-wave imprints of non-integrable extreme-mass-ratio inspirals

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Extreme-mass-ratio inspirals

- Extreme-mass-ratio inspirals (EMRIs) consist of a primary supermassive black hole and a secondary small-mass compact object
- Originate in galactic cores
- The mass ratio of EMRIs span in the range $\sim 10^{-7} - 10^{-4}$
- Generate gravitational waves (GWs) in the frequency range $\sim 10^{-4} - 10^{-1}$ Hz
- EMRIs are prime targets for space-borne detectors (LISA, Taiji)
- Perform $\sim 10^5$ revolutions in the strong-field regime
- Rich waveform phenomenology

Short-timescale EMRI modeling: geodesic motion

Stationary/axisymmetric spacetime: $ds^2 = g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2$

Stationarity: $E = -\mu\left(g_{tt}\dot{t} + g_{t\phi}\dot{\phi}\right)$

Axisymmetry: $L_z = \mu\left(g_{t\phi}\dot{t} + g_{\phi\phi}\dot{\phi}\right)$

Conservation of rest mass: $\dot{r}^2 + \frac{g_{\theta\theta}}{g_{rr}}\dot{\theta}^2 + V_{\text{eff}} = 0, \quad V_{\text{eff}} \equiv \frac{1}{g_{rr}}\left(1 + \frac{g_{\phi\phi}E^2 + g_{tt}L_z^2 + 2g_{t\phi}EL_z}{\mu^2\left(g_{tt}g_{\phi\phi} - g_{t\phi}^2\right)}\right)$

$V_{\text{eff}} = 0 \rightarrow$ Curve of zero velocity

Carter constant $\leftrightarrow$ separation of radial and polar motion $\leftrightarrow$ integrability

Long-timescale EMRI modeling: radiation reaction

Adiabatic approximation: The change in the momenta is sufficiently small over a single orbit $\rightarrow$ 'averaging' the self-force [Mino, PRD (2003)]

- Short timescales: orbit neglects radiative backreaction
- Long timescales: inspiral behaves like a 'flow' through successive geodesics

Hybrid kludge scheme: approximate the flux of momenta with post-Newtonian (PN) formulae [Barack, Cutler, PRD (2004), Gair, Glampedakis, PRD (2006)]

- augmentation: include an additional PN term which represents the effect of anomalous quadrupole moments [Barack, Cutler, PRD (2007), Gair, Li, Mandel PRD (2008), Apostolatos, Lukes-Gerakopoulos, Contopoulos, PRL (2009)]

"Shaken, not stirred"

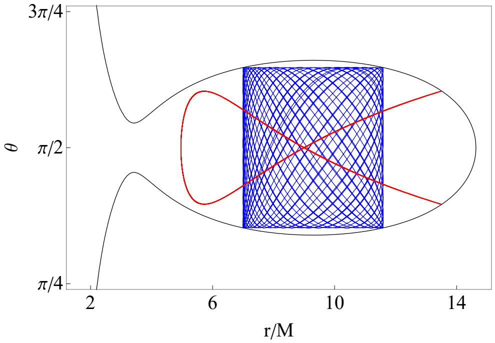
Theory-agnostic spacetime: [inspired by Johannsen, PRD (2013)]

$$
\begin{aligned}
ds^2 = &- \frac{\Sigma[(\alpha_Q/r)\,M^3 + \Delta - a^2 A(r)^2 \sin^2\theta]}{[(r^2+a^2) - a^2 A(r)\sin^2\theta]^2} dt^2 \\
&- \frac{2a[(r^2+a^2)A(r) - \Delta]\Sigma \sin^2\theta}{[(r^2+a^2) - a^2 A(r)\sin^2\theta]^2} dt d\phi \\
&+ \frac{(\alpha_Q/r)\,M^3 + \Sigma}{\Delta} dr^2 + \Sigma d\theta^2 \\
&+ \frac{\Sigma \sin^2\theta\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]}{[(r^2+a^2) - a^2 A(r)\sin^2\theta]^2} d\phi^2,
\end{aligned}
$$

$$\Sigma = r^2 + a^2\cos^2\theta,\; \Delta = r^2 - 2Mr + a^2,\; A(r) = 1 + r^{-2}\alpha_{22}M^2.$$

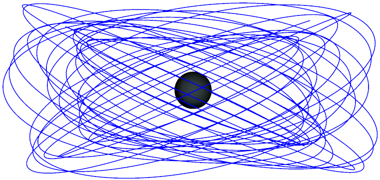
- $\alpha_Q = \alpha_{22} = 0 \to$ Kerr
- α_{22} deforms frame-dragging, $\alpha_Q = 0$ and $\alpha_{22} \neq 0 \to$ Carter constant ('deformed Kerr')
- α_Q controls integrability, $\alpha_Q \neq 0 \to$ no Carter constant ('non-Kerr')

(r, θ)-bound motion

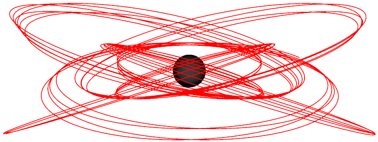


$3D$ Cartesian coordinates

Generic orbit: $\omega_r/\omega_\theta =$ irrational



Resonant orbit: $\omega_r/\omega_\theta =$ rational

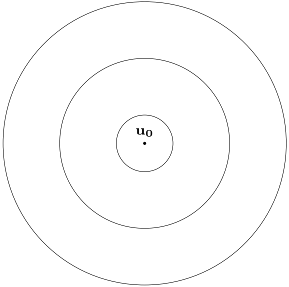


“Two theorems to rule them all”

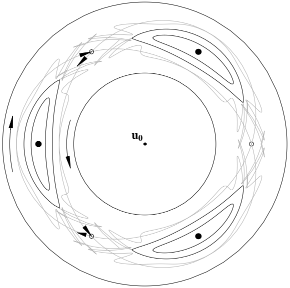
Kolmogorov-Arnold-Moser (KAM) theorem: If a system is non-integrable $\rightarrow$ dynamics smoothly depart from the integrable one, provided that ω_r/ω_θ is sufficiently irrational

Poincaré-Birkhoff theorem: If ω_r/ω_θ is rational $\rightarrow$ formation of ‘Birkhoff’ chain

integrable

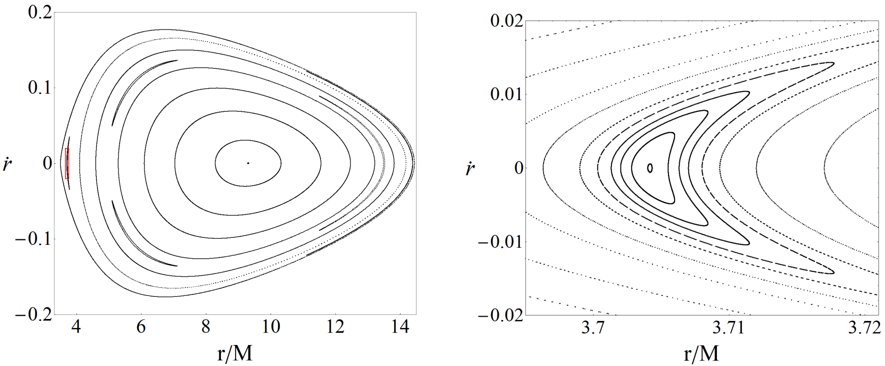


non-integrable



[Lukes-Gerakopoulos et al. PRD (2010)]

Poincaré map $(\alpha_Q \neq 0)$



[KD, Suvorov, Kokkotas, PRD (2020)]

- Away from resonances, KAM curves surround the central point
- Resonant points are surrounded by nested KAM curves (resonant islands)
- Orbits share the same rational ratio ω_r/ω_θ across the island

Impact of non-integrability: orbital level

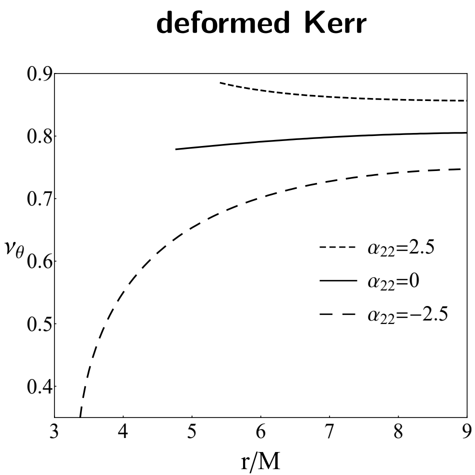
$\boldsymbol{\vartheta_n}$ **:** angle between two successive intersections of a Poincaré map

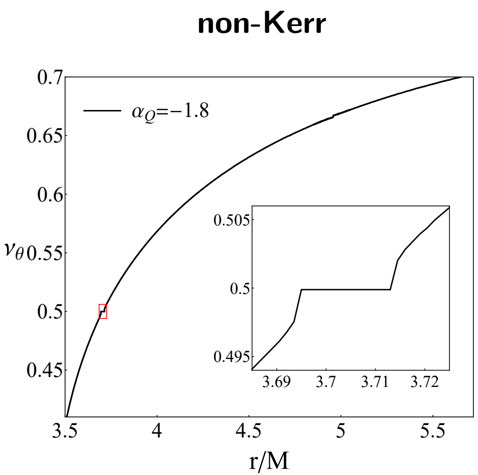
Rotation number:

$$\nu_{\theta,N} = \frac{1}{2\pi N}\sum_{i=1}^{N}\vartheta_i$$

- The limit $N \to \infty$ converges to $\nu_\theta = \omega_r/\omega_\theta$ [Contopoulos (2002)]
- Successive rotation numbers form a rotation curve
- The rotation curve of integrable systems is monotonous
- The rotation curve of non-integrable systems exhibits a plateau inside resonant islands [Lukes-Gerakopoulos et al. PRD (2010)]

Rotation curves





[KD, Suvorov, Kokkotas, PRD (2020)]

Waveform modeling and LISA response

Numerical kludge waveforms: Combine exact particle trajectories with approximate expressions for GW emission [Babak et al., PRD (2007)]

- 'quick and dirty' EMRI waveforms, perfectly-suited for phenomenology
- remarkable agreement with Teukolsky-based waveforms ($\sim 95\%$)

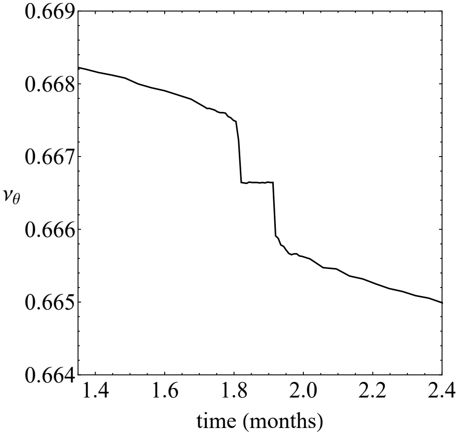
Quadrupole formula: $h_{ij}^{\mathsf{TT}} = \frac{2}{d}\frac{d^2 I_{ij}(Z^i(t))}{dt^2}$, $\mathbf{Z}(t)$: inspiral trajectory

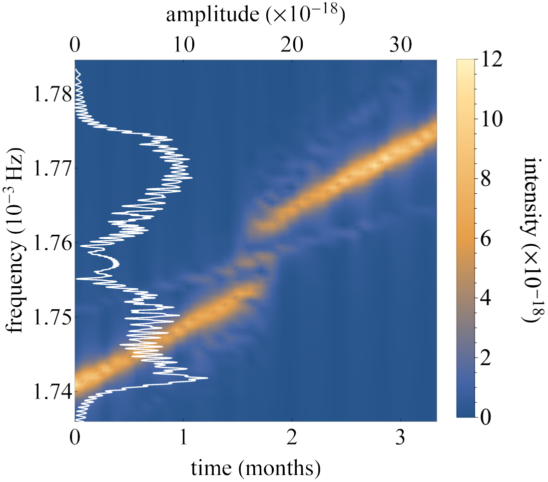
Incoming GW: $h_{+,\times}(t) = \frac{2\mu}{d}\epsilon_{ij}^{+,\times}\left[\frac{Z^i(t)}{dt^2}Z^j(t) + \frac{Z^i(t)}{dt}\frac{Z^j(t)}{dt}\right]$

LISA response: $h_\alpha(t) \sim [F_\alpha^+(t)h_+(t) + F_\alpha^\times(t)h_\times(t)]$ [Barack, Cutler, PRD (2004)]

We use a **single-channel approach**, for simplicity, and **omit any noise** in the data stream.

Resonant islands and GW frequency evolution ($\mu/\mathrm{M} = 10^{-6}$)

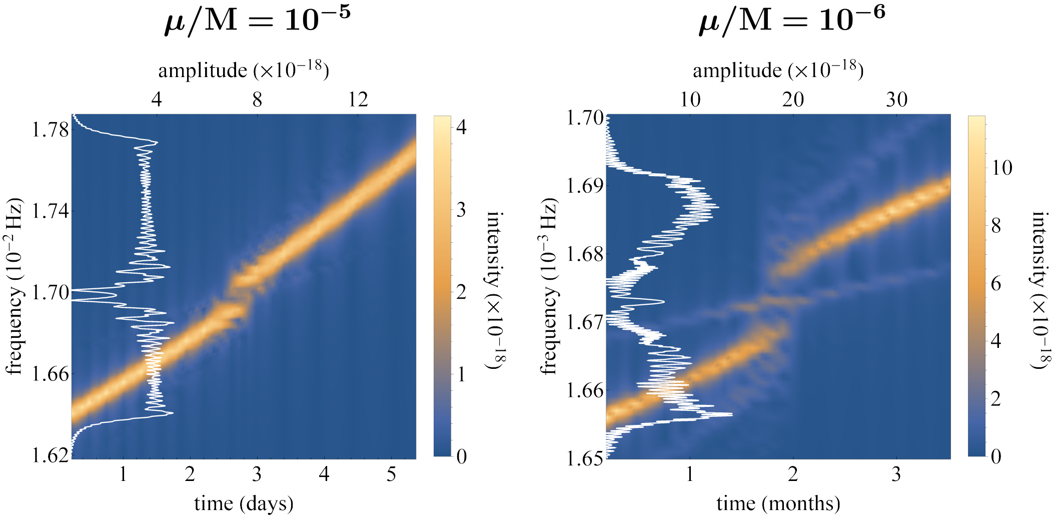




Average crossing time: ~ 250 cycles

[KD, Suvorov, Kokkotas, PRL (2021)]

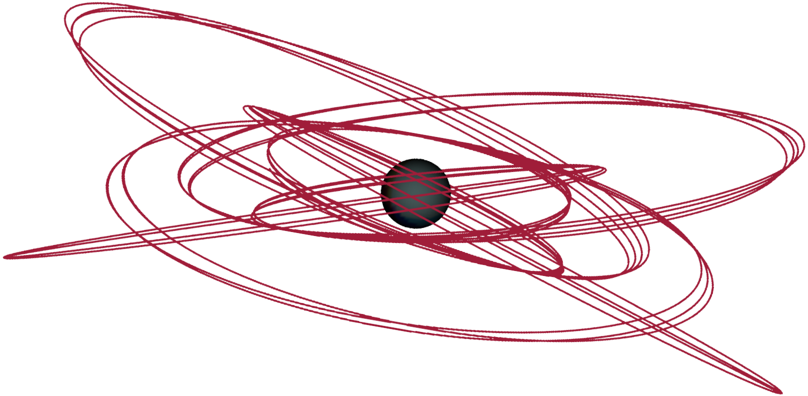
Manko-Novikov glitches: a generic feature



[KD, Kokkotas, PRD (2021)]

Discussion

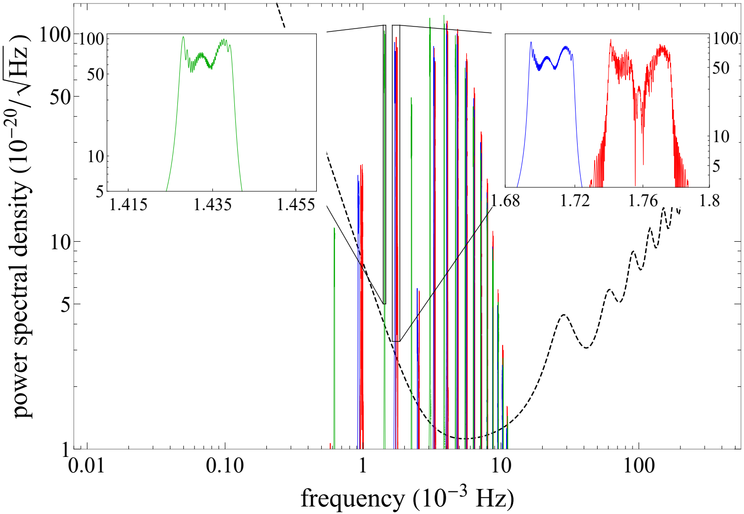
- Integrable deformations lead to EMRIs which smoothly deviate from those in Kerr
- Non-integrable perturbations lead to the formation of resonant islands
- Non-integrability manifests into the waveform in the form of a frequency glitch
- Thousands of EMRIs are expected to be detected by LISA [Gair et al. J. Phys. (2017)]
- The glitches presented here may serve as “smoking-guns” of chaotic phenomena in EMRIs



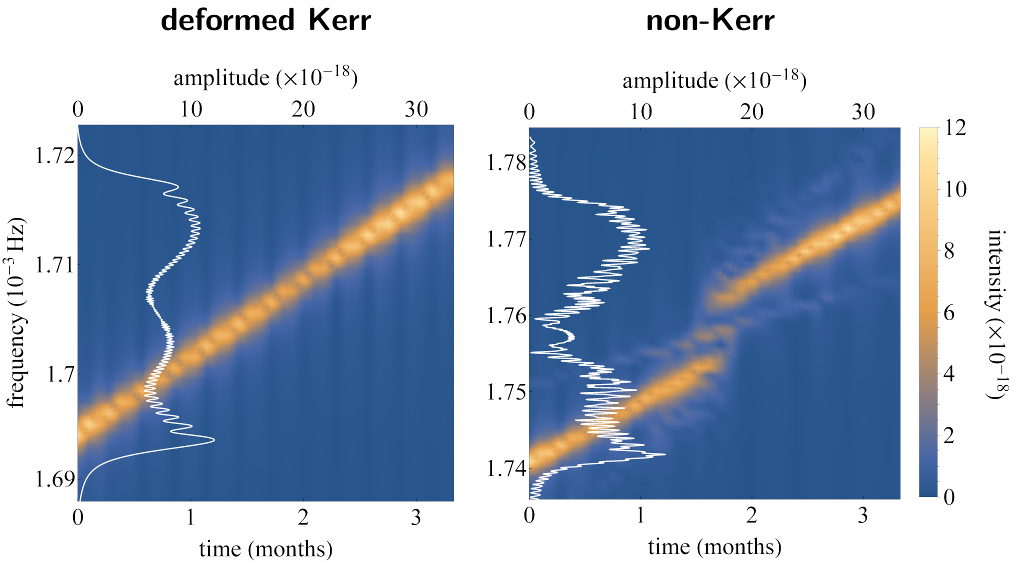
Thanks a lot for your attention!

Appendix

Detectability and frequency evolution $(\mu/M = 10^{-6})$



[KD, Suvorov, Kokkotas, PRL (2021)]



[KD, Suvorov, Kokkotas, PRL (2021)]

EMRI evolution scheme

The calculation proceeds via the following steps:

1. Define the EMRI initial parameters M, μ, a and α_Q, α_{22}
2. Initialize the trajectory with E_0, $L_{z,0}$ and $(r(0), \theta(0), \dot{r}(0), \dot{\theta}(0))$
3. Calculate the orbital elements e, p, ι
4. Approximate the fluxes

$$\left\langle \frac{dE}{dt} \right\rangle_{\text{2PN}} = \frac{\mu}{M^2} f_E(e, p, \iota, M_2), \ \left\langle \frac{dL_z}{dt} \right\rangle_{\text{2PN}} = \frac{\mu}{M} f_L(e, p, \iota, M_2)$$ [Gair, Glampedakis, PRD (2006)]

5. Evolve the coupled (r, θ) geodesic equations with E, L_z changing adiabatically as:

$$E(t) = E_0 + \langle dE/dt \rangle t, \quad L_z(t) = L_{z,0} + \langle dL_z/dt \rangle t$$

6. Update the fluxes every N_r cycles and repeat from step 2

Orbital elements