Amplitude and Polarisation of Light in Gravitational Wave Detectors

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September 13, 2021

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- TM, P. T. Chruściel and S. Palenta. The electromagnetic field in gravitational wave interferometers. *Classical and Quantum Gravity* pre-published (2021)
  DOI: 10.1088/1361-6382/ac2270
  arXiv: 2107.07727 [gr-qc]

How to understand theoretical models of laser-interferometric gravitational wave detectors?

Previous Work

- Jacobi equation for geodesic deviation (Forward 1978; Schutz, Tinto 1987)
- Round-trip time of null curves (Weiss 1972, Estabrook, Wahlquist 1975; Popławski 2006; Rakhmanov, Romano, Whelan 2008; Rakhmanov 2009; Finn 2009; Melissinos, Das 2010; Saulson 2017; Koop, Finn 2014; Błaut 2019)
- Maxwell's equations (Cooperstock 1968; Lobo 1992; Cooperstock, Faraoni 1993; Montanari 1998; Calura, Montanari 1999)

Aims

- Describe phase, amplitude and polarisation perturbations of light in laser interferomters for arbitrary GW incidence angles and polarisations (Lobato et al. 2021: normal incidence).
- Clarify conflicting geometrical optics predictions e.g. one-way phase shifts for parallell emission (Angélil, Saha 2015; Lobo 1992).
- Analyse the quality of geometrical optics predictions by comparison with the full Maxwell equations (Park, Kim 2021)

Geometrical Optics

Geometrical optics (at leading order) is based on the eikonal ψ, scalar amplitude $\mathscr{A}$ and polarisation two-form $f_{\mu\nu}$, satisfying

$$g^{\mu\nu}(\nabla_\mu\psi)(\nabla_\nu\psi) = 0\,,$$

$$\nabla^\mu\psi\nabla_\mu\mathscr{A} + \tfrac{1}{2}\mathscr{A}\Box\psi = 0\,,$$

$$\nabla^\mu\psi\nabla_\mu f_{\rho\sigma} = 0\,, \qquad f_{\mu\nu}\nabla^\nu\psi = 0\,, \qquad f_{[\mu\nu}\nabla_{\rho]}\psi = 0\,.$$

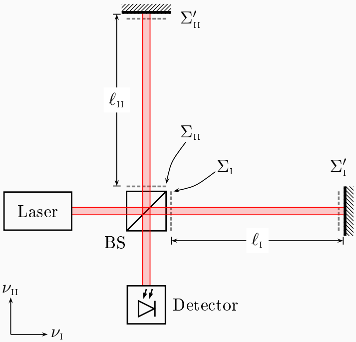
The electromagnetic field is then taken to be $F_{\mu\nu} \approx \mathscr{A} f_{\mu\nu} e^{i\psi}$.

The equations for $f_{\mu\nu}$ are simplified by writing $f_{\mu\nu} = E_\mu\nabla_\nu\psi - E_\nu\nabla_\mu\psi$ and imposing

$$\nabla^\mu\psi\nabla_\mu E_\nu = 0\,, \qquad E_\mu\nabla^\mu\psi = 0\,,$$

the freedom $E_\mu \to E_\mu + \alpha\nabla_\mu\psi$ corresponds to the freedom of choosing the observer with respect to which the EM field is decomposed into electric and magnetic fields (Dehnen 1972; Christie 2007; Santana et al. 2020).

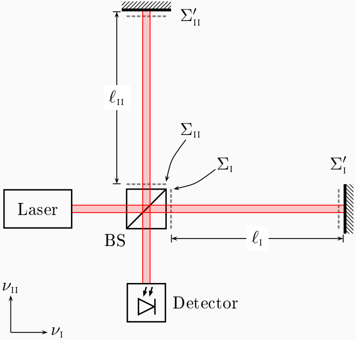
Application to Laser Interferometers



- We assume

$$g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon h_{\mu\nu}(\kappa_\sigma x^\sigma) + O(\varepsilon^2)\,,$$

  with the GW wave vector κ and the GW polarisation $h_{\mu\nu}$ satisfying $\kappa^\mu \kappa_\mu = 0$, $h_{\mu 0} = 0$, $h_{\mu\nu}\kappa^\nu = 0$ and $h^\mu{}_\mu = 0$.
- Error terms $O(\varepsilon^2)$ are not written explicitly.
- Assume the geometry of the GW interferomter to be well described in TT coordinates. In particular, mirrors are held at a fixed TT-coordinate distance from the beam splitter at the origin.

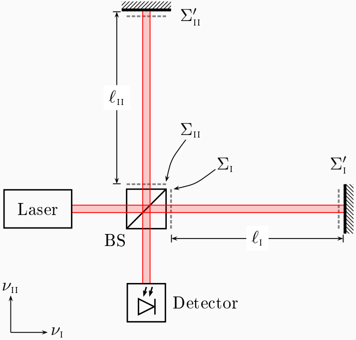


For emission, we impose

$$
\begin{aligned}
\psi_{\mathsf{I}}|_{\Sigma_{\mathsf{I}}} &= \psi_{\mathsf{II}}|_{\Sigma_{\mathsf{II}}} = -\omega t\,, \\
\mathscr{A}_{\mathsf{I}}|_{\Sigma_{\mathsf{I}}} &= \mathscr{A}_{\mathsf{II}}|_{\Sigma_{\mathsf{II}}} = 1\,, \\
E_{\mathsf{I}}|_{\Sigma_{\mathsf{I}}} &= E^{(0)} + \varepsilon\nu_{\mathsf{I}} h(\nu_{\mathsf{I}}, E^{(0)}) + \varepsilon\tilde{E}_{\mathsf{I}}\,, \\
E_{\mathsf{II}}|_{\Sigma_{\mathsf{II}}} &= -\hat{R}_{\mathsf{BS}} E^{(0)} - \varepsilon\nu_{\mathsf{II}} h(\nu_{\mathsf{II}}, \hat{R}_{\mathsf{BS}} E^{(0)}) - \varepsilon\tilde{E}_{\mathsf{II}}\,,
\end{aligned}
$$

where $\hat{R}_{\mathsf{BS}}$ is the orthogonal reflection interchanging ν_{I} and ν_{II}, while keeping their orthogonal complement unchanged.

Application to Laser Interferometers



For reflection at the mirror surfaces Σ'_I, we impose

$$
\begin{aligned}
\check{\psi}_I|_{\Sigma'_I} &= \psi_I|_{\Sigma'_I} \,, \\
\check{\mathscr{A}}_I|_{\Sigma'_I} &= \mathscr{A}_I|_{\Sigma'_I} \,, \\
\check{E}_I|_{\Sigma'_I} &= -\hat{R}_I E_I|_{\Sigma'_I} \,,
\end{aligned}
$$

where $\hat{R}_I$ is the orthogonal reflection along the surface normal ν_I. Similar conditions imposed at the second mirror Σ'_{II}.

Detector Output

From this, one can compute explicitly the field reaching the detector and the time-averaged field energy density

$$\langle T_{00} \rangle = \langle T_{00} \rangle^{(0)} + \varepsilon\delta\langle T_{00} \rangle_{\psi} + \varepsilon\delta\langle T_{00} \rangle_{\mathscr{A}} + \varepsilon\delta\langle T_{00} \rangle_{K} + \varepsilon\delta\langle T_{00} \rangle_{E} + O(\varepsilon^2)\,,$$

with

$$\langle T_{00} \rangle^{(0)} = \sin^2(\omega\Delta\ell)\,.$$

In the low-frequency regime, one assumes $\omega_g \ell \ll 1$, so that the gravitational is almost constant over the timespan of a ray round-trip. In the DC readout scheme:

$$\begin{aligned}
\delta\langle T_{00} \rangle_{\psi} &\approx -2\omega\ell\sin(2\omega\Delta\ell)[h(\nu_{\mathsf{I}}, \nu_{\mathsf{I}}) - h(\nu_{\mathsf{II}}, \nu_{\mathsf{II}})]\,,\\
\delta\langle T_{00} \rangle_{\mathscr{A}} &\approx \tfrac{1}{2}\omega_g\ell\cos^2(\omega\Delta\ell)[h'(\nu_{\mathsf{I}}, \nu_{\mathsf{I}}) + h'(\nu_{\mathsf{II}}, \nu_{\mathsf{II}})]\,,\\
\delta\langle T_{00} \rangle_{K} &\approx -\omega_g\ell\cos^2(\omega\Delta\ell)[h'(\nu_{\mathsf{I}}, \nu_{\mathsf{I}}) + h'(\nu_{\mathsf{II}}, \nu_{\mathsf{II}})]\,,\\
\delta\langle T_{00} \rangle_{E} &= 0\,.
\end{aligned}$$

In general, the quality of the geometrical optics approximation ($F_{\mu\nu} \approx \mathscr{A} f_{\mu\nu} e^{i\psi}$) is difficult to assess.

In a separate analysis we were able to show that, for boundary values as considered here, the electromagnetic field has the structure

$$F_{\mu\nu} = f_{\mu\nu}(\kappa_\rho x^\rho, \nu_\sigma x^\sigma, \omega_g/\omega, \varepsilon) e^{i\omega\psi} + O(\varepsilon^2)\,,$$

where ψ satisfies the eikonal equation and $f_{\mu\nu}$ is analytic in the frequency ratio ω_g/ω.

This puts the geometrical optics method on a firm basis. In particular, geometrical optics remains valid for all GW incidence angles.

Conclusion

- Determined amplitude and polarisation perturbations in laser interferomters for arbitrary GW waveforms and incidence angles.
- Polarisation perturbations do not contribute to the observed signal.
- Implementing plausible boundary conditions, previously found ambiguities are resolved.
- The use of geometrical optics is justified in the context considered.