Freely Falling bodies in Standing Wave spacetime*

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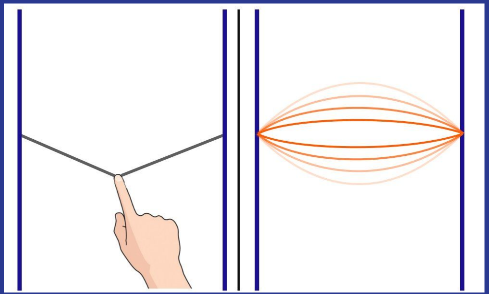
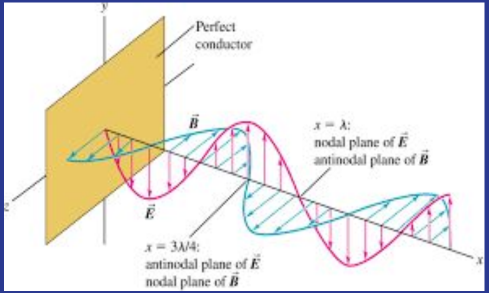
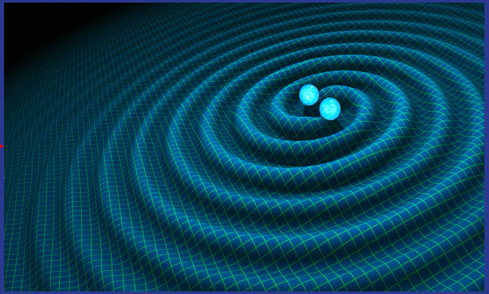
*PhysRevD.103.024011 Sebastian Szybka, Syed Naqvi.



Standing Mechanical Wave



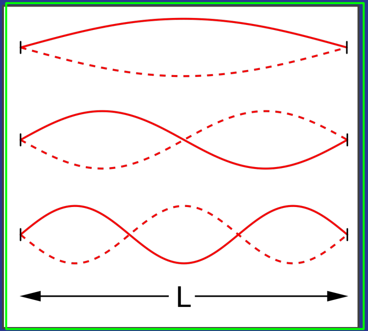
Standing Electromagnetic Wave



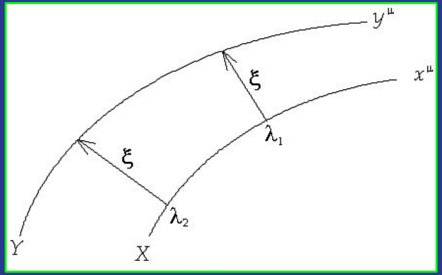
Standing Gravitational Wave ?

Outline:

- ➔ Linearised Gravity and GW Memory
- ➔ Gravitational Waves as Exact Solutions
- ➔ What are Standing Gravitational Waves?
- ➔ How do freely-falling observers behave in a standing wave spacetime?



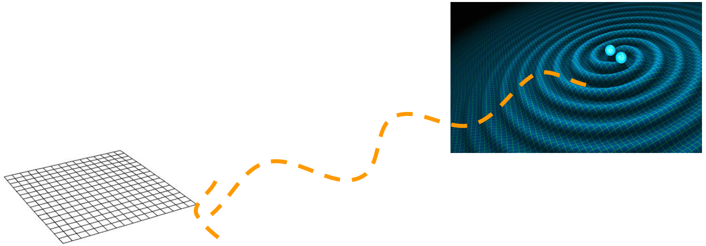
Standing GWs ???



Basic Review of GWs - Linearized Regime

- To describe geometry we need four coordinates $\boxed{x^\alpha}$

Metric : $\boxed{ds^2 = g_{\alpha\beta}(x)dx^\alpha dx^\beta}$



- **Metric perturbations** $\boxed{g_{\alpha\beta}(x) = \eta_{\alpha\beta} + h_{\alpha\beta}(x)}$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

- Linearised EFE far from source: $\boxed{\Box h_{\alpha\beta}(x) = 0,}$

- **Plane Wave soln :**

$$\tilde{h}^{\mu\nu} = A^{\mu\nu}e^{ik_\alpha x^\alpha} \qquad A^{\mu\nu} = h_+\epsilon_+^{\mu\nu} + h_\times\epsilon_\times^{\mu\nu}$$

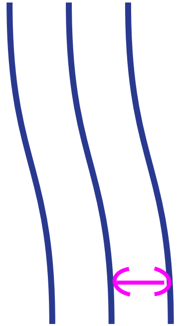
Basic Review of GWs – Linearized Regime

- To describe geometry we need four coordinates $\boxed{x^\alpha}$

Metric : $$\boxed{ds^2 = g_{\alpha\beta}(x)dx^\alpha dx^\beta}$$

- **Metric perturbations** $$\boxed{g_{\alpha\beta}(x) = \eta_{\alpha\beta} + h_{\alpha\beta}(x)}$$

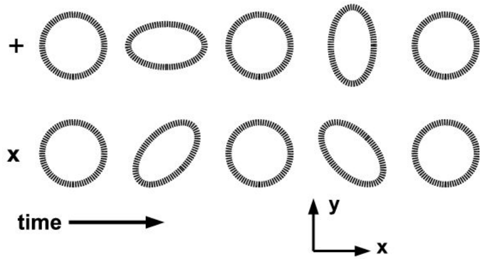
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



Displacement vector $\delta x^\mu(\tau)$

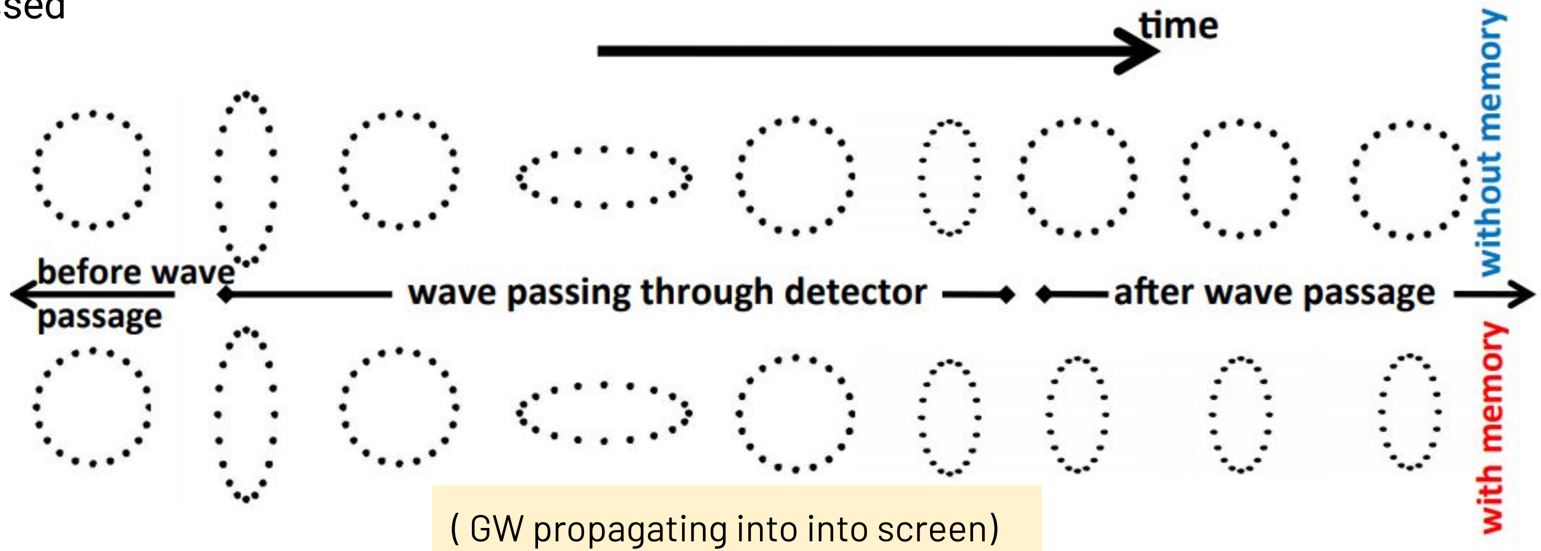
- Linearised EFE far from source: $$\boxed{\Box h_{\alpha\beta}(x) = 0,}$$

$$h \approx \frac{\Delta L}{L}$$



Ripples of spacetime leave behind ‘Memories’

GW Memory Effect : permanent change in the configuration of spacetime after a GW has passed

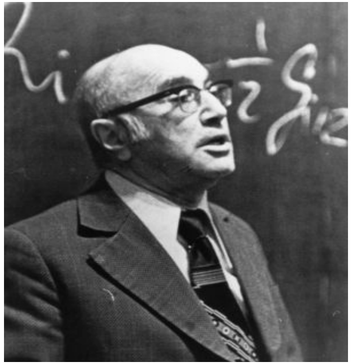


(GW propagating into into screen)

- Permanent change in displacement or velocity of particles
- size of the memory is one order of magnitude smaller than that of the oscillatory waveform*

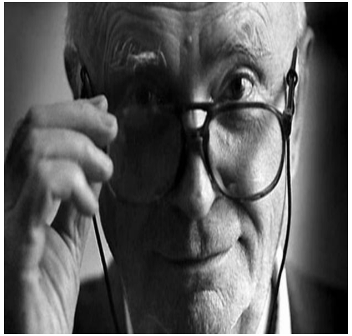
*A Study of Gravitational Wave Memory and Its Detectability With LIGO Using Bayesian Inference, Jillian Doane,Alan W einstein

Ripples leave behind :

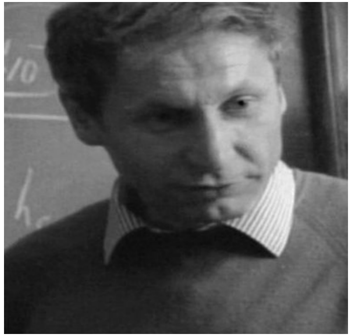


Zeldovich

Polnarev



Braginsky



Grishchuk

Linear Memory

- ★ Two Types: Displacement and velocity

- ★ change in the value of quadrupole moment

$$h_{jk}^{\mathbf{TT}}(t,r) = \frac{2G}{c^4 r} \cdot \left[\frac{d^2}{dt^2} Q_{jk}^{\mathbf{TT}}(t - \frac{r}{c})\right]$$

Reduced quadrupole moment

How they occur ?

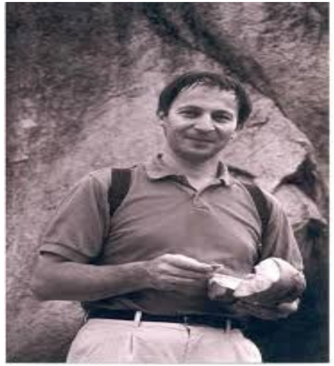
➔ Hyperbolic orbits

➔ Asymmetric supernovae explosion

➔ GRB

Ripples leave behind :

Christodoulou



Blanchet

Damour

Nonlinear Memory

★ due to change in the mass of a binary caused by the emission of GWs

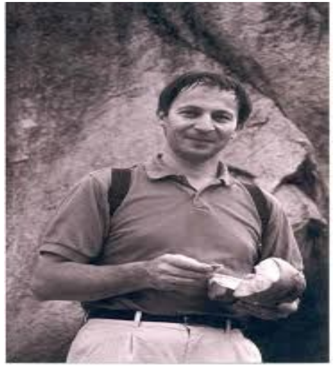
★ GWs produced by GWs

Full nonlinear theory :

-> time-integral has the entire past history of the source

Ripples leave behind :

Christodoulou



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Nonlinear Memory

★ due to change in the mass of a binary caused by the emission of GWs

★ GWs produced by GWs

PN approximation : nonlinear hereditary effects in the generation of gravitational radiation

-> stress-energy distribution of gravitational waves -> ‘memory effect’

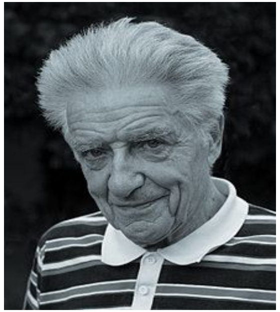
Exact plane wave spacetimes

pp-wave spacetimes

exact solutions of Einstein's field equation

model radiation moving at the speed of light

Ehlers



Kundt

Brinkmann Coordinates : 1925

$$ds^2 = H(u,x,y)du^2 + 2dudv + dx^2 + dy^2$$

A special case : Plane Wave spacetimes

Easy to interpret : (i) coordinate free definition,
(ii) NP tetrad -> null dust solutions

Memory in Exact plane wave spacetimes

Zhang , Duval, Gibbons : 'The Memory Effect for Plane Gravitational Waves'

Plane Gravitational waves
Brinkmann coordinates

$$\boxed{g = \delta_{ij}\, dX^i dX^j + 2dU\,dV + K_{ij}(U) X^i X^j\, dU^2}$$

Profile of wave

$$\boxed{K_{ij}(U)X^iX^j = \frac{1}{2}\mathcal{A}(U)\Big((X^1)^2 - (X^2)^2\Big)}$$

Wave produced by
gravitational collapse
modelled as
sandwich wave,

$$\boxed{\mathcal{A}(U) = \frac{1}{2}\,\frac{d^3(e^{-U^2})}{dU^3}}$$

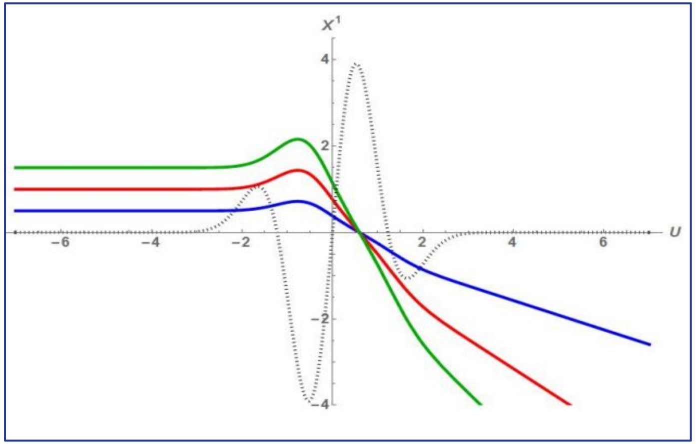
$$\frac{d^2\mathbf{X}}{dU^2} - \frac{1}{2}\mathrm{diag}(\mathcal{A}, -\mathcal{A})\,\mathbf{X} = 0$$

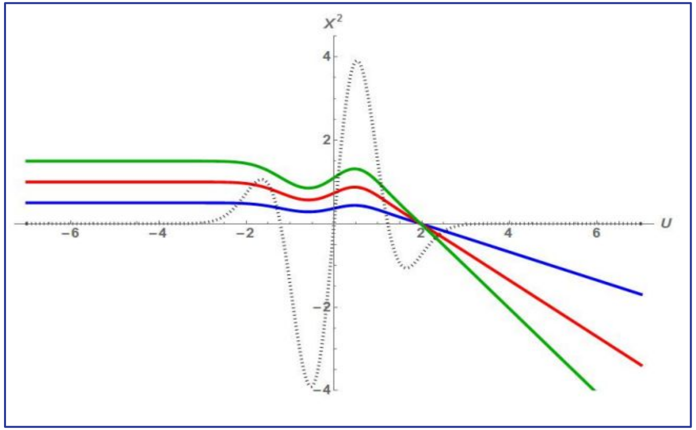
Geodesics in Brinkmann coordinates for particles initially at rest

$$\frac{d^2\mathbf{X}}{dU^2} - \frac{1}{2}\mathrm{diag}(\mathcal{A}, -\mathcal{A})\,\mathbf{X} = 0$$

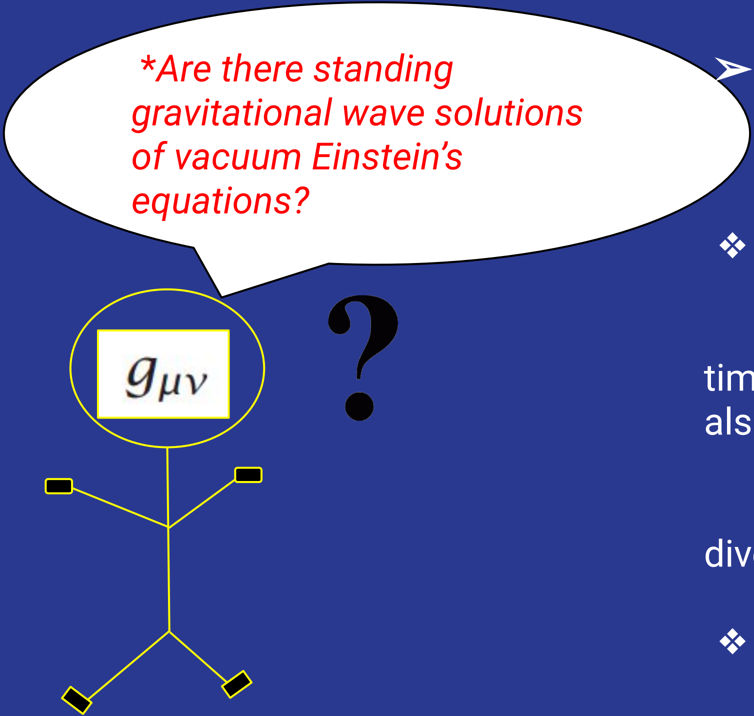
They found that test particles initially at rest move, after the wave has passed, with constant but non-zero relative velocity

=>velocity memory effect

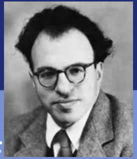




Exact solutions having SGWs...



- ➢ **#Bondi** - Studied GWs of unsymmetric body rotating about z-axis



- ➢ **Issue** - If nonlinearities are taken into account, the lack of superposition principle complicates studies.

- ❖ ***Stephani** - Look for exact solutions with :

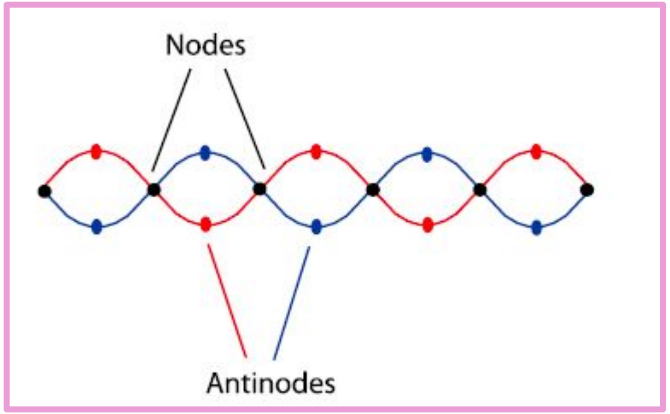
(i)constitutive parts of the metric functions to depend on the timelike coordinate only through a periodic factor, and they should also depend on spacelike coordinates.

(ii) analogue of the Poynting vector (if there is any) should be divergence-free

- ❖ **Issue** - Method is not covariant

#H. Bondi. Gravitational waves in general relativity XVI. Standing waves.
*H. Stephani. Some remarks on standing gravitational waves.

How to define Standing GWs?



- Standing Wave may imply => alternating energy flow through the nodes which averages to zero
- ! Energy of GWs cannot be localised, need a covariant averaging procedure
- It can work* in the high frequency limit which captures the dominant contribution to the average energy flow

A spacetime (*M*,*g*) contains standing GWs if *

i)it belongs to a one-parameter family of spacetimes (M, g(λ)) satisfying the Green-Wald assumptions

ii)the Ricci tensor of the background metric is of a Serge type .

*Standing waves in general relativity : Sebastian J. Szybka and Adam Cieślik.

Standing Grav. Wave Solution*

- **analyze and admit arbitrary wavelength gravitational waves in expanding universe**

*#Metric

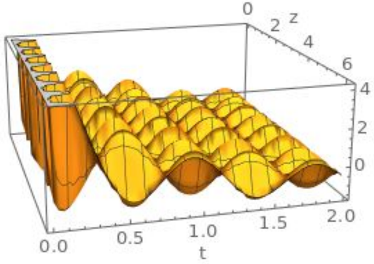
$$\hat{g} = e^f(-dt^2 + dz^2) + t(e^p dx^2 + e^{-p} dy^2)$$

where $0 \leq z < 2\pi$, $t > 0$ (t is a cosmic time function), $0 \leq x, y < 2\pi$, $f = f(t,z)$ and $p = p(t,z)$. The particular solution we are interested in is given by

After solving Einstein Field Eqn.

$$\begin{aligned} p &= -\ln t + 2\beta\sqrt{\lambda}J_0(\frac{t}{\lambda})\sin(\frac{z}{\lambda});\,, \\ f &= \frac{\beta^2}{\lambda}t^2\left[J_0^2(\frac{t}{\lambda}) + J_1^2(\frac{t}{\lambda}) - 2\frac{\lambda}{t}J_0(\frac{t}{\lambda})J_1(\frac{t}{\lambda})\sin^2(\frac{z}{\lambda})\right] \\ &\quad -2\beta\sqrt{\lambda}J_0(\frac{t}{\lambda})\sin(\frac{z}{\lambda})\,, \end{aligned}$$

Parameters ~ amplitude of Grav. Waves
~no. of waves



*Standing waves in general relativity, Sebastian J. Szybka and Adam Cieślik(2019).
*#Backreaction for Einstein-Rosen waves coupled to a massless scalar field, Sebastian J. Szybka,Michał J. Wyrębowski(2016).

Standing Grav. Wave Solution*

Metric

$$\hat{g} = e^f(-dt^2 + dz^2) + t(e^p dx^2 + e^{-p} dy^2)$$

Parameters

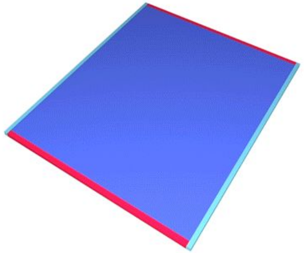
$$p = -\ln t + 2\beta\sqrt{\lambda}J_0(\frac{t}{\lambda})\sin(\frac{z}{\lambda});\,,$$

$$f = \frac{\beta^2}{\lambda}t^2\left[J_0^2(\frac{t}{\lambda}) + J_1^2(\frac{t}{\lambda}) - 2\frac{\lambda}{t}J_0(\frac{t}{\lambda})J_1(\frac{t}{\lambda})\sin^2(\frac{z}{\lambda})\right]$$
$$-2\beta\sqrt{\lambda}J_0(\frac{t}{\lambda})\sin(\frac{z}{\lambda})\,,$$

Topology of our spacetime : "Donut"

This solution is T^3 Gowdy Model(polarised 3-torus)*

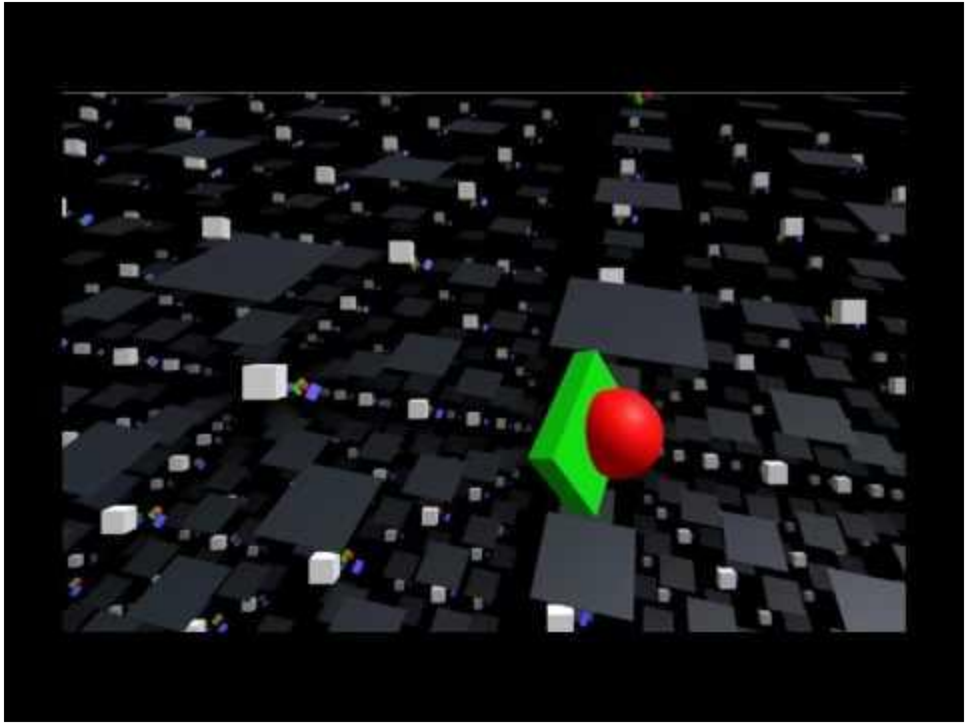
2-torus



- Amplitude of wave decreases as the universe expands
- Geodesic Eqn has **stationary solutions to antinodes** + at antinodes we study behavior of freely-falling bodies

*Standing waves in general relativity Sebastian J. Szybka and Adam Cieślik.
*Robert H. Gowdy, "Of gravitational waves and spherical chickens" in: Einstein Online Band 03 (2007), 03-1008

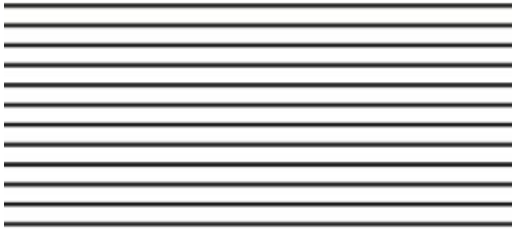
Visualizing 3-torus



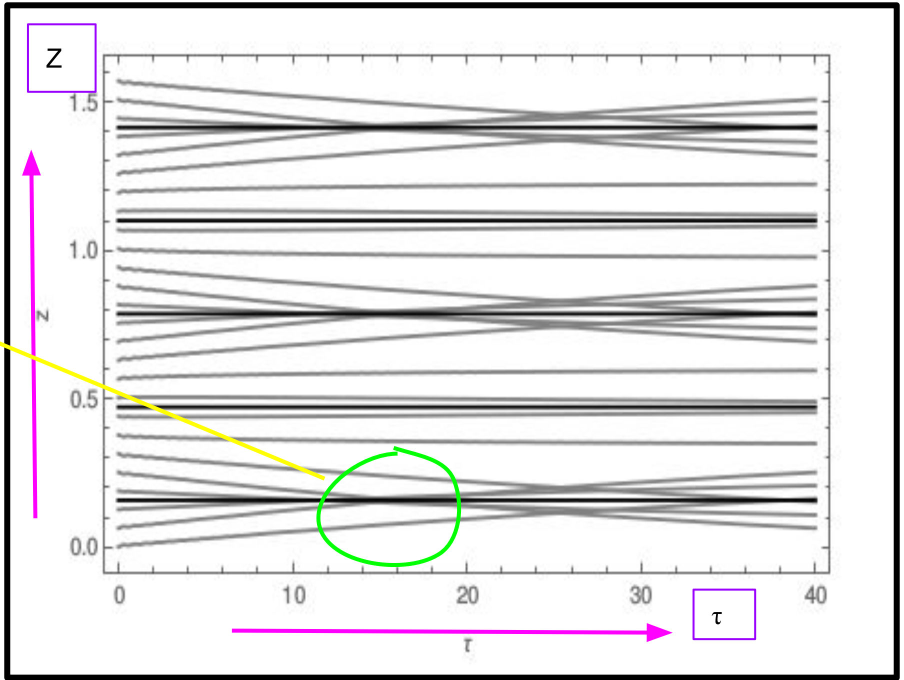
Mathematical Computing Laboratory

Prototype of a VR experience under development which allows the user to play a game of "catch" in various multiply-connected spaces. In this case, the player is in the 3-dimensional torus. Created with Unity 3D; running on Oculus Rift HMD with Touch controllers. :Horalia Armas , Brandon Reichman ,Hai Tran,David Dumas

Trajectories of Test Particles - Geodesic Eqn.



Particles attracted at antinodes



Wave is 'z' direction from bottom to up

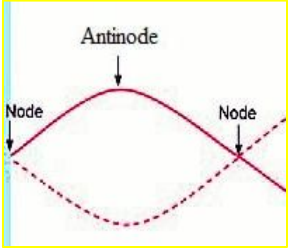
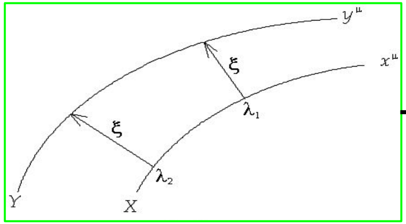


Figure : Geodesic equation for 'z' vs proper time τ, Here λ=1/10, β=0.2

Geo. Eqn has stationary soln. at antinodes & we **studied the motion of of freely-falling bodies at antinodes**

X,Y,Z components of Geodesic Deviation Eqn.



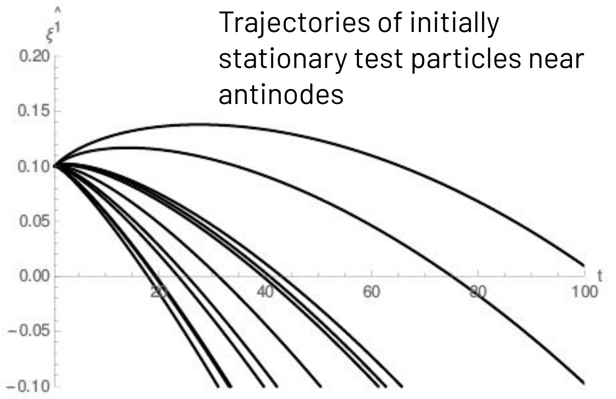
Freely-falling frame

$$\frac{d^2\xi^{\hat\alpha}}{d\tau^2} = -R^{\hat\alpha}{}_{\hat0\hat\beta\hat0}\,\xi^{\hat\beta}$$

Eqn.s decouple => the motion of test particles in **transverse** and **longitudinal directions**

$$\frac{d^2\xi^{\hat1}}{dt^2} - \frac{1}{2}f_{,t}\frac{d\xi^{\hat1}}{dt} = \frac{1}{2}(f_{,tt} - f_{,zz})\xi^{\hat1} = -\frac{\beta}{\lambda t}J_1(\frac{t}{\lambda})\left[\sqrt{\lambda} + t\beta J_1(\frac{t}{\lambda})\right]\xi^{\hat1}$$

Deviation in 'Z'



Trajectories of initially stationary test particles near antinodes

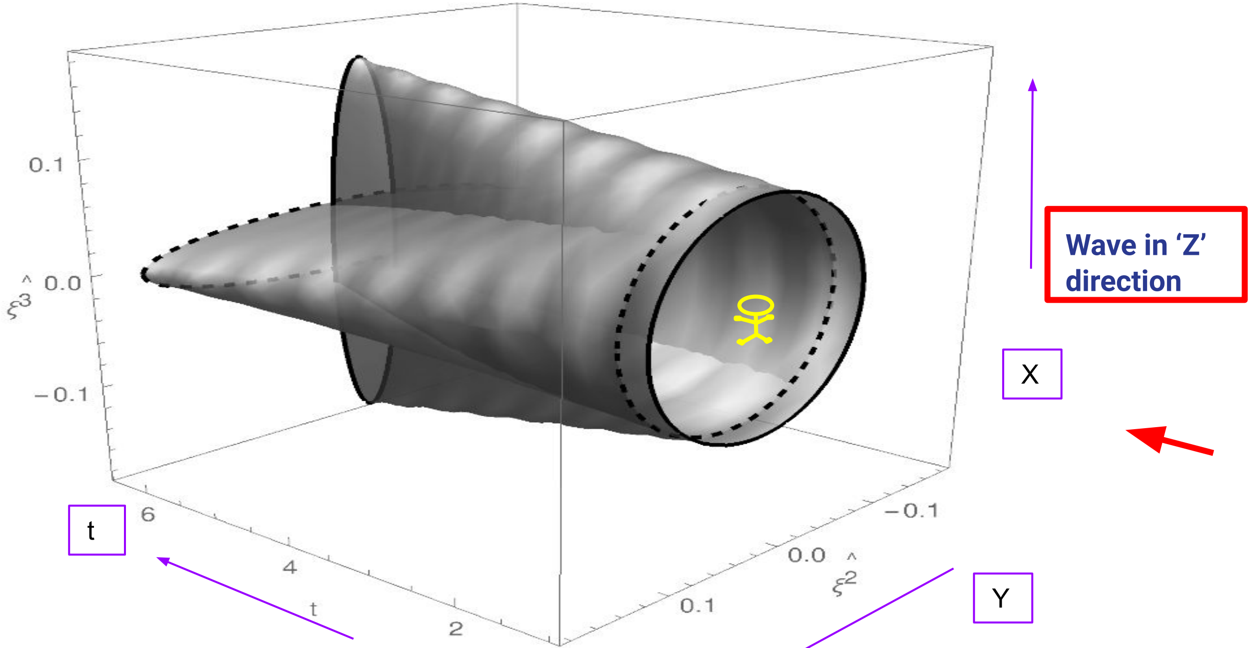
Deviation in 'X' and 'Y'

$$\begin{aligned}
\frac{d^2\xi^{\hat2}}{dt^2} - \frac{1}{2}f_{,t}\frac{d\xi^{\hat2}}{dt} &= \frac{1}{4t}\left[f_{,t} + (p_{,t}(2 - tf_{,t}) + 2t\,p_{,tt})\right]\xi^{\hat2} \\
&= -\frac{\beta}{\lambda^{3/2}}\left[J_0(\frac{t}{\lambda}) - \frac{\lambda^2}{4\beta^2 t}J_1^{-1}(\frac{t}{\lambda})f_{,t}^2\right]\xi^{\hat2} \\
\frac{d^2\xi^{\hat3}}{dt^2} - \frac{1}{2}f_{,t}\frac{d\xi^{\hat3}}{dt} &= \frac{1}{4t}\left[f_{,t} - (p_{,t}(2 - tf_{,t}) + 2t\,p_{,tt})\right]\xi^{\hat3} \\
&= \frac{\beta}{\lambda^{3/2}}\left[J_0(\frac{t}{\lambda}) - \frac{\lambda}{2}J_1(\frac{t}{\lambda})f_{,t}\right]\xi^{\hat3}\,,
\end{aligned}$$

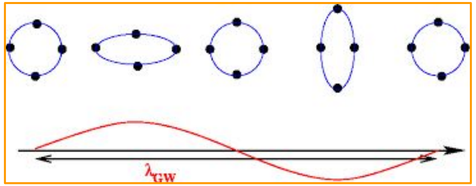
Freely-falling bodies – Geodesic Deviation Eqn in X and Y

-two rings (solid and dashed) of particles initially at rest ,**later permanently deformed into an ellipse**

-the shift in the initial conditions alters a long-range behavior of trajectories and **reveals the 'plus' polarization of gravitational waves.**



Usual GW Picture :

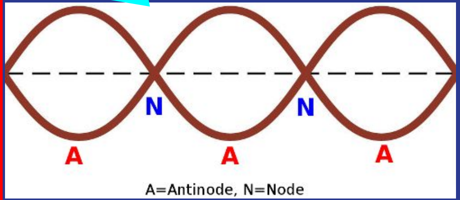


Permanent change : GW Memory

Figure : Tissot plot showing how a ring of test particles will behave for waves for slightly different initial conditions. Here λ=1/10, β=0.2

Frame Fields : Mathematical technique to investigate spacetimes

We evaluated the Geodesic Deviation Eqn in an **Orthonormal frame**



Correspond to a family of ideal observers immersed in the given spacetime

Newman-Penrose formalism

- **Work with Null-Tetrad,**
- **Formalism well suited for studying how radiation propagates in curved spacetime**

Weyl Scalars

$$\{\Psi_0, \Psi_1, \Psi_2, \Psi_3, \Psi_4\}$$

Physical Interpretation : Szekeres (1965)gave an interpretation of the different Weyl scalars at large distances:

Ψ_2 is a "Coulomb" term,

Ψ_1 & Ψ_3 are ingoing and outgoing "longitudinal" radiation terms;

Ψ_0 & Ψ_4 are ingoing and outgoing "transverse" radiation terms.

Newman-Penrose formalism for our spacetime

$$\hat{g} = e^{f}(-dt^2 + dz^2) + t(e^{p}dx^2 + e^{-p}dy^2)$$

Our standing gravitational waves studied in this paper may be thought of as a **non-trivial superposition of two waves moving in the opposite spatial directions.**

Hence to analyse our GWs we decompose the Weyl tensor into NP tetrad components.

Weyl Scalars

$$\{\Psi_0, \Psi_1, \Psi_2, \Psi_3, \Psi_4\}$$

$$\boxed{\psi_2 \neq 0}$$

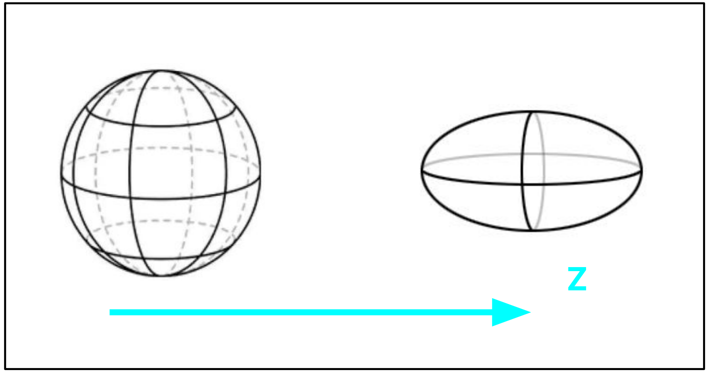
Coulomb part

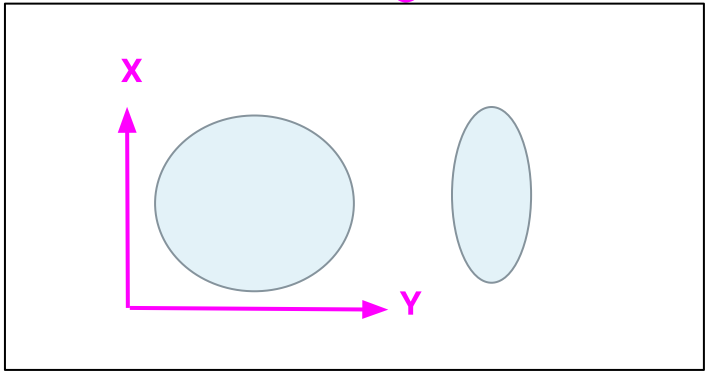
$$\boxed{\psi_1 = \psi_3 = 0}$$

Longitudinal effects

$$\boxed{\psi_0, \psi_4 \neq 0}$$

Transverse effects





Conclusion

PhysRevD.103.024011 / arXiv:2010.12549

Motivation

What are Standing Gravitational waves?

Behaviors of test particles in such a spacetime

- ❖ Analyzed the **Geodesic Equation** => **Particles are attracted to antinodes**
- ❖ Analyzed the **Geodesic Deviation Eqn** => permanent deformation of ring of test particles(**velocity memory effect**)
- ❖ Standing Grav. Waves induce additional effect **: Longitudinal effect** is due to Coulomb part of Weyl

Future Work : EM standing waves coupled to gravity???

★ Some Trivia - Grishchuk & Sazhin(1975)* : - toroidal electromagnetic resonator with alternating current
Interference of radiated GWs => Standing GW

*Excitation and detection of standing gravitational waves L. P. Grishchuk and M. V. Sazhin