Assessing the detectability of the secondary spin in EMRIs with fully-relativistic numerical waveforms*

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*Based on arXiv: 2105.07083

Extreme Mass-Ratio Inspirals (EMRIs)

Stellar-mass compact objects plunges into a supermassive black hole

Primary mass M: $10^6 - 10^9 M_\odot$ Secondary mass μ: $1 - 100 M_\odot$

mass ratio $q = \mu/\mathrm{M} \sim 10^{-4} - 10^{-7}$

$\implies$ we can expand Einstein field equations in q

Some features:

Modeled as point-particle moving in background spacetime (Kerr)

GWs in the mHz range $\implies$ targets for space observatories (LISA)

$\sim 10^5$ orbits in 1 year before inspiral in strong gravity regime

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We consider EMRI binaries in Kerr with a spinning secondary

Why should we consider a spinning secondary in EMRIs?

- $\sim$ 1mrad precision in the GW phase needs 1-post-adiabatic order

$$\Phi_{\mathrm{GW}} = \underbrace{q^{-1}\mathcal{C}^{(0)}}_{\text{adiabatic}} \quad + \quad \underbrace{q^{-1/2}\mathcal{C}^{(1/2)}}_{\text{resonances}} \quad + \quad \underbrace{q^{0}\mathcal{C}^{(1)}}_{\text{post-1-adiabatic}} \quad + \quad \mathcal{O}(q)$$

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- Parameter estimation done with PN fluxes and kludge waveforms[2]
- **What happens with relativistic fluxes and waveforms?**
- We estimated errors on χ with a Fisher matrix approach

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Mathisson-Papapetrou-Dixon (MPD) equations of motion

- MPD equations for a spinning body

$$\boxed{\nabla_{\vec{v}} p^{\mu} = -\frac{1}{2} R^{\mu}{}_{\nu\alpha\beta} v^{\nu} S^{\alpha\beta} \qquad \nabla_{\vec{v}} S^{\mu\nu} = 2 p^{[\mu} v^{\nu]} \qquad v^{\mu} = \frac{\mathrm{d}z^{\mu}}{\mathrm{d}\lambda}}$$

spin vector $\quad S_{\mu} \equiv \dfrac{1}{2\mu} \epsilon_{\mu\nu\alpha\beta} p^{\nu} S^{\alpha\beta}$

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$$p_{\mu} S^{\mu\nu} = 0 \implies \mu^2 = \text{cost} \;\; \text{and } S = \text{cost}$$

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- Circular equatorial orbits (CEO) with $S^\mu = \delta^\mu_\theta S^\theta$

$\implies$ S^μ (anti)-aligned to primary spin a

$\implies$ dynamics fixed by $E = E(r, \sigma), J_z = J_z(r, \sigma), \Omega = \Omega(r, \sigma)$

Radiation reaction effects and balance laws

GW fluxes and waveforms computed with the Teukolsky formalism

$$\sigma := \frac{S}{\mu M} = \frac{S}{\mu^2} q = \chi q \qquad \text{with } q \ll \sigma \ll 1 \text{ in EMRIs}$$

At first order in the secondary spin[3]

$$\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\mathrm{GW}} = -\frac{\mathrm{d}E}{\mathrm{d}t} \qquad \left(\frac{\mathrm{d}J_z}{\mathrm{d}t}\right)_{\mathrm{GW}} = -\frac{\mathrm{d}J_z}{\mathrm{d}t}$$

Technical assumptions:

- no radiation reaction on $S^{\mu\nu}$ and assumed $S = cost$
- orbits remain circular and equatorial

We employed the numerical routines of the BHPToolkit [4]

[3] Akcay+, 2019

[4] Black Hole Perturbation Toolkit https://bhptoolkit.org

Linear spin corrections to the fluxes

Piovano, Maselli, Pani Phys. Rev. D 102 (2020) 024041

Linear spin corrections $\delta\mathcal{F}^\sigma$ obtained by interpolating $(\mathcal{F}^0 \equiv \mathcal{F}(\sigma = 0))$

$$\mathcal{F}(r,\Omega) - \mathcal{F}^0(r,\Omega) = \sigma\delta\mathcal{F}^\sigma(r,\Omega) + \mathcal{O}(\sigma^2) \qquad \mathcal{F}(r,\Omega) := \frac{1}{q^2}\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\mathrm{GW}}$$

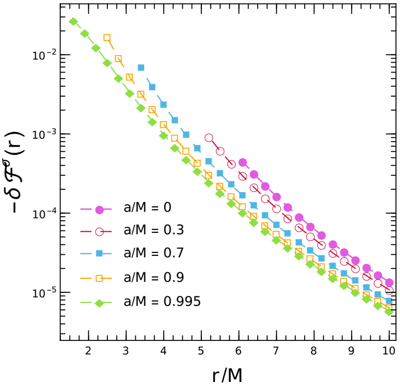


Figure: fixed r

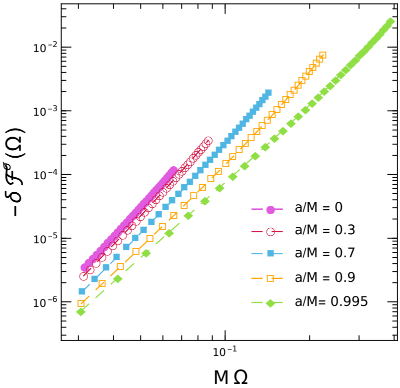
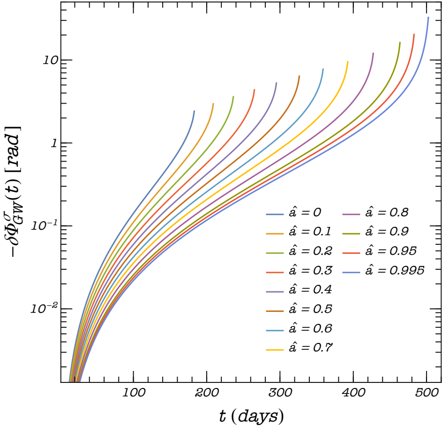


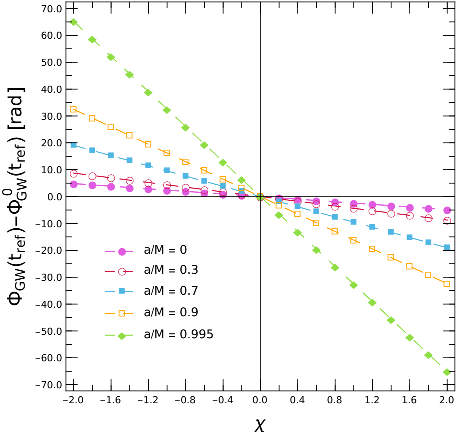
Figure: fixed Ω

Linear spin corrections to the GW phase

Piovano, Maselli, Pani Phys. Rev. D 102 (2020) 024041
GW phase is $\Phi_{\rm GW} = 2\Phi$ for dominant mode ($\Phi^0_{\rm GW} \equiv \Phi^0_{\rm GW}(\sigma = 0)$)

$$\Phi_{\rm GW}(t) - \Phi^0_{\rm GW}(t) = (\sigma/q)\delta\Phi^\sigma_{\rm GW}(t) + \mathcal{O}(\sigma^2/q)$$





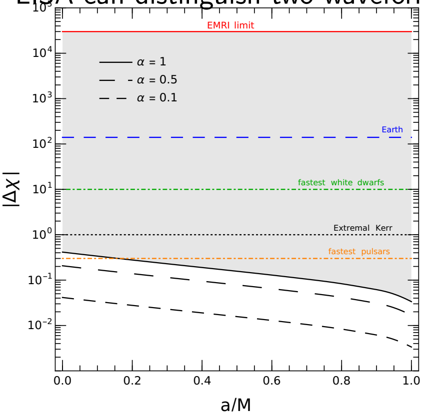
Spin resolution and Kerr bound test

Piovano, Maselli, Pani PLB 811 135860 (2020)

Minimum $\Delta\chi = \chi_B - \chi_A$ leading to a phase difference of $\alpha\,\mathrm{rad}$ is:

$$|\Delta\chi| > \alpha|\delta\Phi^{\sigma}_{\mathrm{GW}}(a)|^{-1}$$

LISA can distinguish two waveforms out-of-phase of $\simeq 1\ \mathrm{rad}$



- With $\alpha = 1\ \mathrm{rad}$, for

$$a/M = 0.7 \qquad |\Delta\chi| > 0.1$$

$$a/M = 0.9 \qquad |\Delta\chi| > 0.05$$

$$\chi \approx a/M \approx 0.7, |\Delta\chi/\chi| \sim 15\%$$

- For Kerr BHs $|\chi| \leq 1$ (Kerr bound)

We may ask:

1. Can we measure χ?
2. Can we test the Kerr bound?

Parameter estimation with Fisher matrices

In the high SNR limit

$$\Sigma_{ij} = (\Gamma^{-1})_{ij} \qquad \text{covariance matrix}$$

$$\Gamma_{ij} = \sum_{\alpha=I,II} \left(\frac{d\tilde{h}_\alpha}{dx^i} \,\middle|\, \frac{d\tilde{h}_\alpha}{dx^j} \right)_{\vec{x}=\vec{x}_0} \qquad \text{Fisher matrix}$$

$$(p_\alpha|q_\alpha) = 4\mathrm{Re}\int_{f_{\min}}^{f_{\max}} \frac{df}{S_n(f)} \tilde{p}^*_\alpha(f)\tilde{q}_\alpha(f) \qquad \text{scalar product}$$

11 parameters for circular equatorial orbits with spins (anti)aligned

- intrinsic: masses $(\ln\mu, \ln M)$, spins (a, χ), r_0
- extrinsic: ϕ_0, angles $(\vartheta_S, \varphi_S, \vartheta_K, \varphi_K)$, distance $\ln D$

Γ_{ij} is ill-conditioned: small error in Γ_{ij} $\implies$ large error in Σ_{ij}

Some cases required waveforms with >90 digits for stable Γ_{ij} and Σ_{ij} !

Fisher-matrix errors neglecting the secondary spin

Piovano, Brito, Maselli, Pani arXiv: 2105.07083

Masses: $M = 10^6 M_\odot, \mu = 10 M_\odot$. Spins: $a = 0.9, \chi = 0$. Sky location fixed. 1 year of observation. Errors are all normalized to $\mathrm{SNR} = 30$.

ℓ	$\ln M$	$\ln \mu$	a/M	r_0/M	ϕ_0
2	-3.24	-3.53	-4.15	-4.45	0.48
2+3	-3.25	-3.54	-4.16	-4.46	-0.52
2+3+4	-3.25	-3.55	-4.16	-4.46	-0.53

Table: $\log_{10}$ of the errors

ℓ	$\ln D$	$\Delta\Omega_S$	$\Delta\Omega_K$
2	4.7×10^{-1}	7.9×10^{-4}	2.5
2+3	4.7×10^{-2}	7.3×10^{-4}	1.3×10^{-2}
2+3+4	4.6×10^{-2}	7.2×10^{-4}	1.1×10^{-2}

Table: errors on $\ln D$ and sky location of source ($\Delta\Omega_S$) and primary spin ($\Delta\Omega_K$).

Fisher-matrix errors including the secondary spin

Piovano, Brito, Maselli, Pani arXiv: 2105.07083

$M = 10^6 M_\odot, \chi = 1$. Sky location fixed. 1 year observation

Errors are normalized to $\mathrm{SNR} = 30$ ($\mathrm{SNR} = 150$) for $\mu = 10 M_\odot (100 M_\odot)$.

a/M	$\mu/M_\odot$	prior	$\ln M$	$\ln \mu$	a/M	χ	r_0/M	ϕ_0
0.9	10	no	-2.26	-2.41	-2.66	2.85	-3.88	0.48
		yes	-3.24	-3.53	-4.14	0.48	-4.45	0.48
	100	no	-2.20	-2.39	-2.78	1.66	-4.14	-0.015
		yes	-3.30	-3.52	-4.32	0.064	-4.93	-0.024
0.99	10	no	-2.81	-2.96	-4.55	1.98	-3.89	0.47
		yes	-3.51	-3.76	-4.67	0.52	-4.32	0.47
	100	no	-2.14	-2.33	-3.39	1.21	-3.75	-0.12
		yes	-3.01	-3.22	-4.03	0.11	-4.50	-0.12

Table: $\log_{10}$ of the errors with and without imposing a prior on χ. Only $\ell = 2$

- Secondary spin is not measurable
- With no prior on χ, correlations spoil errors on intrinsic parameters

Conclusions

We estimated errors on binary parameters for fully relativistic waveforms with a Fisher matrix approach

Summary:

- Our analysis confirm results obtained with kludge waveforms
- Secondary spin might not be measurable due to correlations...
- ...but we considered circular orbits without self-force or spin evolution

Future work

- Consider general orbits (eccentric and inclined)
- Include secondary spin in fast-waveform? (Challenging)[5]
- More sophisticated statistical analysis? (MonteCarlo)[6]
- Include conservative first order in q self force
- Investigate spin evolution and radiation back-reaction

[5] Chua+,2020

[6] Katz+,2021

Final notes and acknowledgments

- code and data are available at the GitHub repository https://web.uniroma1.it/gmunu/resources
- this work makes use the BHPToolkit https://bhptoolkit.org/
- all tensors computation performed with "xAct" www.xact.es
- Feel free to contact me at gabriel.piovano@uniroma1.it

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Thank you for you attention!

Backup slides

In the high SNR limit, the posterior distribution is approximated by a Gaussian distribution

$$\log p(\vec{\theta}|s) \propto \log p_0(\vec{\theta}) - \frac{1}{2}(\vec{\theta} - \vec{\theta}_{\rm true})^t \Gamma (\vec{\theta} - \vec{\theta}_{\rm true}) \equiv f(\vec{\theta})$$

Gaussian prior on χ, with $\sigma_{0,\chi} = 1$ and centered in $\chi = 0$

$$\log p_0(\vec{\theta}) = -\frac{1}{2}\chi^2 \delta_{\chi i} \quad \text{with } \delta_{\chi i} \text{ delta Kronecker}$$

We can then rewrite $f(\vec{\theta})$ as single quadratic form plus a constant term

$$f(\vec{\theta}) = -\frac{1}{2}\,(\vec{\theta} - \Gamma_{\rm pos}^{-1}\vec{b})^t \Gamma_{\rm pos} (\vec{\theta} - \Gamma_{\rm pos}^{-1}\vec{b}) + R$$

where

$$\Gamma_{\rm pos} = \Gamma + \delta_{\chi i} \qquad \vec{b} = \vec{\theta}_{\rm true}\Gamma \qquad R = \frac{1}{2}\left(\vec{b}^t \Gamma_{\rm pos}^{-1}\vec{b} - \vec{\theta}_{\rm true}\Gamma\vec{\theta}_{\rm true}\right)$$

Linearization in σ: angular Teukolsky equation

Piovano, Brito, Maselli, Pani   arXiv: 2105.07083

Angular Teukolsky equation defines an operator $\mathcal{H}$ ($x = cos\theta$, $c = a\omega$)

$$\mathcal{H}|S\rangle = -\lambda_{\ell m\omega}|S\rangle \qquad |S\rangle \equiv S^{c}_{\ell m}(x) \qquad \mathcal{H} = \mathcal{K} + \mathcal{V}$$

$\mathcal{K}$ "kinetic" operator $\mathcal{V}$ "potential" operator

Expansion in σ, $c = c^0 + \sigma c^1 + \mathcal{O}(\sigma^2)$

$$\lambda_{\ell m\omega} = \lambda^0_{\ell m} + \sigma\lambda^1_{\ell m} \qquad |S\rangle = |S^0\rangle + \sigma|S^1\rangle$$

$$\lambda^1_{\ell m} = \langle S^0|\mathcal{V}^1|S^0\rangle \equiv \int_{-1}^{1} S^0_{\ell m}\mathcal{V}^1 S^0_{\ell m}\mathrm{d}x$$

$|S^1\rangle$ obtained by expanding Leaver method in σ

$\lambda^0_{\ell m}$ and $|S^0\rangle$ calculated with the BHPToolkit package "SWSH" [7]

[7] Black Hole Perturbation Toolkit https://bhptoolkit.org

Linearization in σ: radial Teukolsky equation

Piovano, Brito, Maselli, Pani arXiv: 2105.07083

Homogeneous solutions computed in hyperboloidal-slicing coordinates (HSC) [8]

$$R^{\text{in,up}}_{\ell m\omega}(r) = \frac{\Delta^s}{r} e^{iH\omega r^*} e^{im\tilde{\phi}} \psi(r) \qquad \tilde{\phi} = \frac{a}{r_+ - r_-} \ln\left(\frac{r - r_+}{r - r_-}\right)$$

$R^{\text{in}}_{\ell m\omega}$ ($R^{\text{up}}_{\ell m\omega}$) for $H = -1$ ($H = +1$). $\psi(r)$ satisfies

$$\Delta^2 \frac{\mathrm{d}^2\psi}{\mathrm{d}r^2} + \Delta \tilde{F}(r;H) \frac{\mathrm{d}\psi}{\mathrm{d}r} + \tilde{U}(r;H)\psi = 0 \quad (1)$$

HSC method is fast and accurate:

- $\tilde{U}(r;-1)/\Delta^2$ is short ranged
- oscillating behavior is factored out

Found exact boundary conditions for Eq. (1) $\implies$ easy to expand $R^{\text{in,up}}_{\ell m\omega}(r)$:

$$R^{\alpha}_{\ell m\omega}(r) = R^{\alpha,0}_{\ell m}(r,\omega^0) + \sigma R^{\alpha,1}_{\ell m}(r,\omega^0,\omega^1) \qquad \alpha = \text{in, up}$$

[8] Zenginoglu 2011