Generalizing the close-limit approximation

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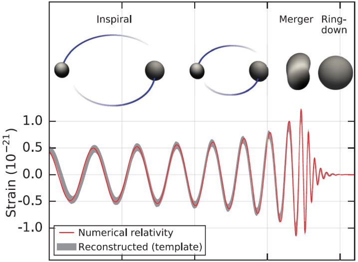
Modelling GW generation



Credit: LIGO/Virgo

Models which do not assume GR

Models which do not assume BHs

What is the close limit approximation (CLAP)?

CLAP as an alternative approach [Price&Pullin, 1994]

(i) Initial data to model last stages of the merger

(ii) Perturbative expansion in the initial separation

(iii) Small deformation of the final object

(iv) Perturbation equations to obtain GWs

Agreement with simulations?

[Anninos+,1993]

Spherically symmetric compact body with mass M and a surface at $r = r_0$,

$$r_0 = 2M(1+\epsilon), \ \ \epsilon \ll 1$$

Binary ECOs

Brill-Lindquist initial data: $${}^3ds^2_{\mathrm{BL-ECO}} = \left(1 + \frac{4}{1+\frac{M}{2R}} \sum_{\ell=2,4,\ldots}^{\infty} \xi_\ell P_\ell\left(\cos\theta\right)\right)\left(f^{-1}dr^2 + r^2 d\Omega^2\right)$$

Decomposing the metric $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$ the only non-vanishing perturbations are h_{rr}, $h_{\theta\theta}$, $h_{\phi\phi}$

[Zerilli, 1971] [Moncrief,1974] [Cunningham+, 1979]

$$-\frac{\partial^2\psi}{\partial t^2} + \frac{\partial^2\psi}{\partial r_*^2} - \left(1 - \frac{2M}{r}\right)\frac{6\left(3M^3 + 6M^2 r + 4Mr^2 + 4r^3\right)}{r^3(3M+2r)^2}\psi = 0$$

with $\psi(t_0, r) \propto \xi_2 = \dfrac{1}{2}\left(\dfrac{Z_0}{M}\right)^2$

ECOs head-on collision

Collision to a single BH: same as original CLAP

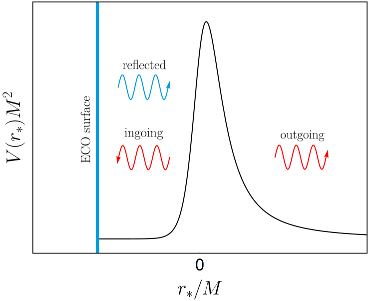
Collision to a single ECO

- Exterior spacetime excitations
  [Vishveshwara, 1970] [Berti+, 2009]
- Contributions from interior: lapse very small = large delays
  [Mark+, 2017] [Ferrari&Kokkotas, 2000] [Cardoso&Pani, 2018]

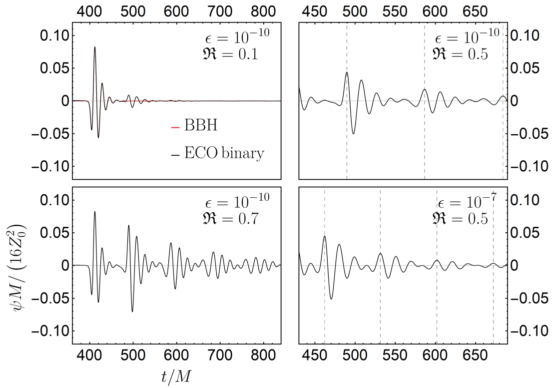
"Freezing" the inner region:
incoming waves partially reflected

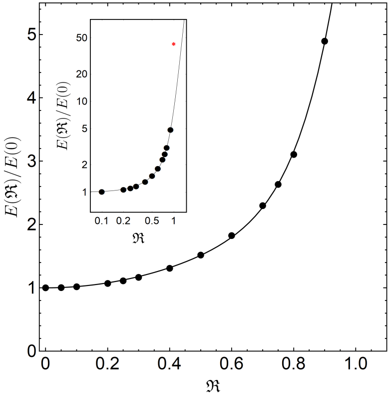
$$\tilde{\phi}(r_*) \sim e^{-i\omega r_*} + \mathfrak{R} e^{i\omega r_*}$$

$\mathfrak{R}$ is the *reflectivity coefficient*.



Waveforms and echoes





$$\Delta t_{\mathrm{ECO}} \sim 4.3\, |\log \epsilon|\, M \qquad \frac{E}{M} \approx 10^{-6} \left(6.14 + \mathfrak{R}^2 (1.29 + 3.26 \mathfrak{R}^6)\right) \frac{256 Z_0^4}{M^4}$$

Follow-up

Inspiral of equal-mass, compact, horizonless objects

Binary BH waveforms in alternative theories, e.g. Einstein-scalar-Gauss-Bonnet

BHs collision at the speed of light

...

Appendix

Initial data

Set of $3D$ spatial hypersurfaces Σ_t

$$ds^2 = -(\alpha^2 - \beta_i\beta^i)dt^2 + 2\gamma_{ij}\beta^i dt dx^j + \gamma_{ij}dx^i dx^j$$

$$\mathcal{H}^{\mathrm{GR}} \equiv {}^3R + K^2 - K_{ij}K^{ij} = 16\pi\rho \text{ (Hamiltonian constraint)}$$

Not unique solution for BH binaries

[Brill&Lindquist, 1963] [Bowen&York, 1980] [...]

Brill Lindquist

$${}^3ds^2_{\mathrm{BL}} = \varphi^4_{\mathrm{BL}}\left(dR^2 + R^2 d\Omega^2\right) \implies \nabla^2\varphi_{\mathrm{BL}} = 0$$

$$\varphi_{\mathrm{BL}} = 1 + \frac{m_1}{2|\boldsymbol{R} - \boldsymbol{R}_1|} + \frac{m_2}{2|\boldsymbol{R} - \boldsymbol{R}_2|} = 1 + \frac{M}{2R} + \sum_{\ell=1}^{\infty}\xi_\ell\left(\frac{M}{R}\right)^{\ell+1} P_\ell(\cos\theta)$$

$$\xi_\ell = \left\{\left(\frac{R_1}{M}\right)^\ell \frac{m_1}{2M} + \left(\frac{R_2}{M}\right)^\ell \frac{m_2}{2M}\right\}$$

Equal mass head-on collision

Defining the $4D$ metric with BHs on Z-axis: $\xi_\ell = \frac{1}{2}\left(\frac{Z_0}{M}\right)^\ell$, for $\ell = 2, 4, 6, \ldots$

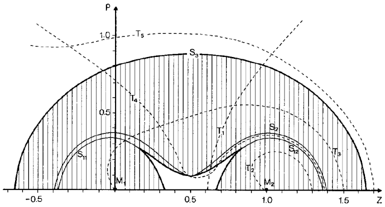
$$ds^2 = -f dt^2 + \left(f^{-1} dr^2 + r^2 d\Omega^2\right) \left[1 + \frac{4}{1 + \frac{M}{2R}} \sum_{\ell=2,4,\ldots}^{\infty} \xi_\ell \left(\frac{M}{R}\right)^{\ell+1} P_\ell\left(\cos\theta\right)\right]$$

$$R = \frac{1}{4}\left(\sqrt{r} + \sqrt{r - 2M}\right)^2, \quad f = 1 - \frac{2M}{r}$$

Initial separation

$$L = \int_{Z_1}^{Z_2} \left[1 + \frac{M}{4}\left(\frac{1}{Z_0 + Z} + \frac{1}{Z_0 - Z}\right)\right]^2 dZ$$

[Andrade, 1996]



[Bishop, 1982]

Initial data for ECOs

$$^3ds^2_{\mathrm{ECO}} = \varphi^4\, {}^3d\eta^2 = \varphi^4\left(dR^2 + R^2 d\Omega^2\right)$$

Hamiltonian constraint becomes

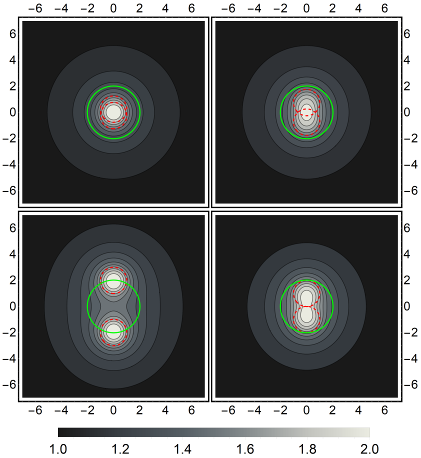
$$\rho = \frac{\varphi^{-5}}{2\pi}\left[\frac{3M}{2R_0^3}\Theta\left(R_0 - R\right)\right]$$

$$\nabla^2_\eta\varphi = \frac{3M}{2R_0^3}\Theta\left(R_0 - R\right)$$

Conformal factor for ECO

$$\varphi = 1 + \frac{M}{2}\left(\frac{3}{2R_0} - \frac{R^2}{2R_0^3}\right) + \frac{M}{2}\left(\frac{1}{R} + \frac{R^2}{2R_0^3} - \frac{3}{2R_0}\right)\Theta\left(R - R_0\right)$$

Toy model



Theories with a scalar field

Einstein-scalar-Gauss-Bonnet

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\nabla\Phi)^2 + \frac{\eta}{4}\mathfrak{f}(\Phi)\mathcal{R}_{\rm GB}\right] \quad \left\{\mathcal{R}_{\rm GB} = R^2 - 4R_{ij}R^{ij} + R_{ijkl}R^{ijkl}\right\}$$

$$G_{\mu\nu} = \frac{1}{2}T_{\mu\nu} - \frac{1}{8}\eta\,\mathcal{G}_{\mu\nu}, \quad \Box\Phi = -\frac{\eta}{4}\frac{\partial\mathfrak{f}(\Phi)}{\partial\Phi}\mathcal{R}_{\rm GB}$$

Scalar instabilities

$$\mathfrak{f}(\Phi) = \frac{\Phi^2}{2}$$

KG equation admits (depending on η): $\begin{cases} \text{Constant scalar around GR solutions} \\ \text{Unstable GR solutions} \Longrightarrow \end{cases}$

Scalarized (or hairy) BHs or stars [Silva+, 2018] [Doneva&Yazadjiev, 2018] [Cunha+, 2019] ...

Ansätze

$$\text{Spacetime: } \Box\Phi = -\frac{\eta}{4}\Phi\mathcal{R}_{\rm GB} \Longrightarrow \begin{cases} \Box = \Box^{(0)} + Z_0^2\,\Box^{(1)} + \mathcal{O}(Z_0^3) \\ \mathcal{R}_{\rm GB} = \mathcal{R}_{\rm GB}^{(0)} + Z_0^2\,\mathcal{R}_{\rm GB}^{(1)} + \mathcal{O}(Z_0^3) \end{cases}$$

$$\text{Scalar: } \Phi = \frac{\psi_{\ell\,m}\,(t,r)\,Y^{\ell\,m}\,(\theta,\phi)}{r} + Z_0^2 \sum_{\ell'\neq\ell} \frac{\psi_{\ell'\,m}\,(t,r)\,Y^{\ell'\,m}\,(\theta,\phi)}{r}$$

Master equation

$$\frac{\partial^2\psi_{\ell m}}{\partial t^2} + \frac{\partial^2\psi_{\ell m}}{\partial r^2}\left(U_0 + Z_0^2\tilde{U}_0\right) + \frac{\partial\psi_{\ell m}}{\partial r}\left(U_1 + Z_0^2\tilde{U}_1\right) + \psi_{\ell m}\left(\left(W_0 + \frac{\eta}{4}\tilde{W}_0\right) + Z_0^2\left(W_1 + \frac{\eta}{4}\tilde{W}_1\right)\right) = 0$$

Single BH bound states

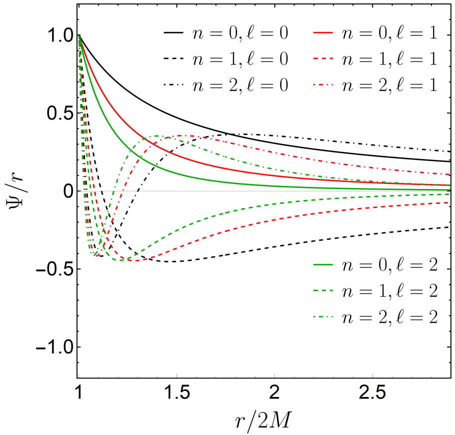
Onset of the instability

$$\psi_{\ell m}(t,r) = \Psi(\omega,r)\, e^{-i\omega t}$$

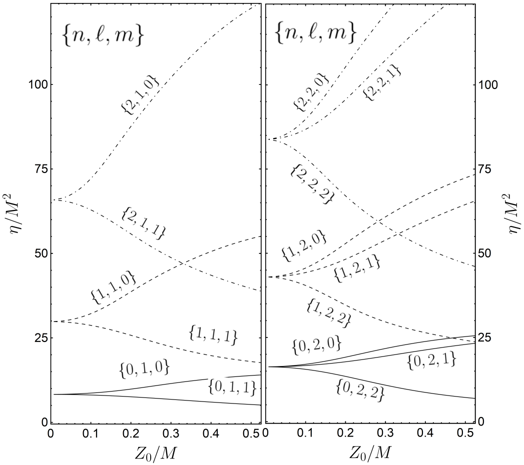
$$\omega = \omega_R + i\omega_I, \quad \text{with } \omega_I > 0$$

... plus regular asymptotic behaviours

	$\left(\eta/M^2\right)^{n\ell m}_{Z_0=0}$		
ℓ	$n=0$	$n=1$	$n=2$
0	2.902	19.50	50.93
1	8.282	29.82	65.84
2	16.30	42.97	83.82



Binary BH bound states

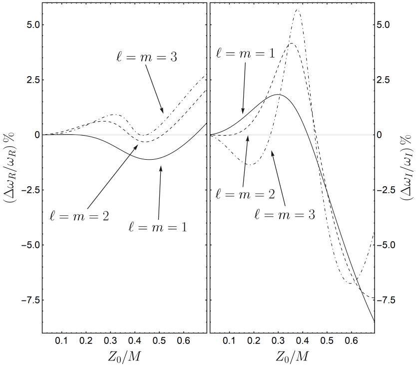


...and in GR? $(\eta = 0, Z_0 \neq 0)$

$$\frac{\partial^2 \psi_{lm}}{\partial t^2} + \frac{\partial^2 \psi_{lm}}{\partial r^2}\left(U_0 + Z_0^2 \tilde{U}_0\right) + \frac{\partial \psi_{lm}}{\partial r}\left(U_1 + Z_0^2 \tilde{U}_1\right) + \psi_{lm}\left(W_0 + Z_0^2 W_1\right) = 0$$

QNM deviations from static BHs

$$\frac{\Delta\omega_{R/I}}{\omega_{R/I}} = \frac{\omega_{R/I} - \omega_{R/I}^{(\mathrm{Schw})}}{\omega_{R/I}^{(\mathrm{Schw})}}$$



[i] Can a BH binary make vanishingly small scalar grow?

[ii] When the ringdown starts?