Ringing of black holes in higher-derivative gravity

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w/ Kwinten Fransen, Thomas Hertog and Simon Maenaut

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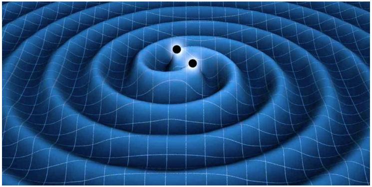
Gravitational waves: new opportunities

- test GR
- search for alternatives to GR



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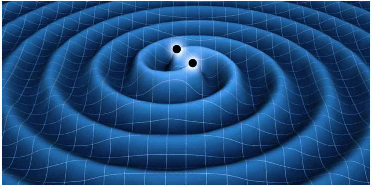


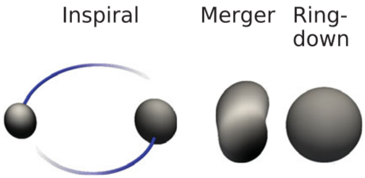
In this talk → **higher-derivative corrections**

$$\mathcal{L} = R + \ell^2 \mathcal{O}(Riem^2) + \ell^4 \mathcal{O}(Riem^3) + \ldots$$

- Motivated by high energy physics/EFT arguments
- Short-distance modifications. New length scale ℓ.
- Most sensitive test: **quasinormal ringing** of black holes

Introduction

After the merger, the remnant BH is in a vibrating stage → ringdown



- GWs emitted during the ringdown are determined by the **quasinormal modes**
- QNMs have complex frequencies → characteristic signature of a BH
- The QNM spectrum of Kerr black hole is determined by the mass and angular momentum → the first frequency determines the rest.
- Depends on the photon-sphere physics → small length scale → sensitive to short-distance modifications of GR

Introduction

QNMs of BHs in higher-derivative gravity

- Spherically symmetric BHs: solvable but unrealistic Cardoso, Gualtieri '09; Blázquez-Salcedo, Macedo, Cardoso, Ferrari, Gualtieri, Khoo, Kunz, Pani '16; Blázquez-Salcedo, Khoo, Kunz '17; Cardoso, Kimura, Maselli, Senatore '18; Konoplya, Zinhailo '20; Matyjasek '20
- Rotating black holes: real challenge

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- Rotating black holes: real challenge

Two miracles of Kerr black holes:

1. Decoupled equations for master scalar variables (Teukolsky)
2. Separability

In HDG we lose these properties

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In this talk: solution of 2 for a test scalar field. Solution of 1 for slowly-rotating BHs.

Higher-derivative extensions of GR:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\Big\{R + \alpha_1\ell^2\mathcal{X}_4 + \alpha_2\ell^2 R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} + \lambda_{\rm ev}\ell^4 R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}R_{\delta\gamma}{}^{\mu\nu} + \lambda_{\rm odd}\ell^4 R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}\tilde{R}_{\delta\gamma}{}^{\mu\nu}\Big\}$$

where $\mathcal{X}_4 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the Gauss-Bonnet density

EFT for gravity and rotating black holes

Higher-derivative extensions of GR: **scalar couplings**

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\Big\{R + \alpha_1\phi_1\ell^2\mathcal{X}_4 + \alpha_2\left(\phi_2\cos\theta_m + \phi_1\sin\theta_m\right)\ell^2 R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$$
$$+\lambda_{\rm ev}\ell^4 R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}R_{\delta\gamma}{}^{\mu\nu} + \lambda_{\rm odd}\ell^4 R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}\tilde{R}_{\delta\gamma}{}^{\mu\nu} - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2}(\partial\phi_2)^2\Big\}$$

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If these terms vanish, the leading corrections are quartic

$$S_{(4)} = \frac{\ell^6}{16\pi G}\int d^4x\sqrt{|g|}\left\{\epsilon_1\mathcal{C}^2 + \epsilon_2\tilde{\mathcal{C}}^2 + \epsilon_3\mathcal{C}\tilde{\mathcal{C}}\right\}$$

$$\mathcal{C} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\,, \quad \tilde{\mathcal{C}} = R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$$

Corrections to the Kerr metric PAC, Ruipérez '19:

$$ds^2 = -\left(1 - \frac{2M\rho}{\Sigma} - H_1\right) dt^2 - (1 + H_2)\,\frac{4Ma\rho(1-x^2)}{\Sigma} dt d\phi$$
$$+ (1 + H_3)\,\Sigma\left(\frac{d\rho^2}{\Delta} + \frac{dx^2}{1-x^2}\right) + (1 + H_4)\left(\rho^2 + a^2 + \frac{2M\rho a^2(1-x^2)}{\Sigma}\right)(1-x^2)d\phi^2$$

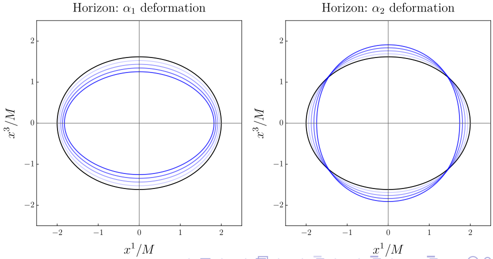
where Σ and Δ are given by

$$\Sigma = \rho^2 + a^2 x^2\,, \qquad \Delta = \rho^2 - 2M\rho + a^2$$

Power series in χ: **analytic solution**

$$H_i = \sum_{n=0}^{\infty} \chi^n \sum_{p=0}^{n} \sum_{k=0}^{k_{\max}} H_i^{(n,p,k)} x^p \rho^{-k}$$

$n = 14$ accurate for $\chi \sim 0.7$



Scalar QNMs

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Test scalar field in the background of a rotating BH

$$\nabla^2\psi = 0$$

Separation of t and ϕ variables

$$\psi = e^{i(m\phi-\omega t)}\psi_{m,\omega}(\rho, x) \qquad \Rightarrow \qquad \mathcal{D}^2_{m,\omega}\psi_{m,\omega} = 0$$

The operator $\mathcal{D}^2_{m,\omega}$ is **non-separable.**

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We can expand ψ in spheroidal harmonics

$$\psi_{m,\omega} = \sum_{l=|m|}^{\infty} S_{l,m}(x; a\omega)R_{l,m}(\rho) = S_{l,m}(x; a\omega)R_{l,m}(\rho) + \lambda\sum_{l\neq l'} S_{l',m}(x; a\omega)R_{l',m}(\rho)$$

Projecting on $S_{l,m}$ we get a decoupled equation for $R_{l,m}$

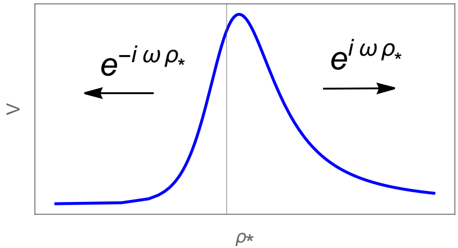
$$\mathcal{D}^2_{l,m,\omega}R_{l,m} = 0\,, \qquad \mathcal{D}^2_{l,m,\omega} = \langle S_{l,m}|(\rho^2 + a^2x^2)\mathcal{D}^2_{m,\omega}|S_{l,m}\rangle$$

Scalar QNMs

Radial equation → 1-dimensional Schrödinger equation

$$\frac{d^2\varphi_{l,m}}{d\rho_*^2} + \left(\omega^2 - V(\rho_*,\omega)\right)\varphi_{l,m} = 0\,, \qquad \varphi_{l,m} \propto R_{l,m}$$

BC for QNMs: outgoing waves at horizon and infinity



Scalar QNMs

$$\omega_{l,m,n} = \omega^{(0)}_{l,m,n} + \frac{\ell^4}{M^5}\left(\alpha_1^2 \Delta\omega^{(1)}_{l,m,n} + \alpha_2^2 \Delta\omega^{(2)}_{l,m,n} + \lambda_{\rm ev} \Delta\omega^{(\rm ev)}_{l,m,n}\right)$$

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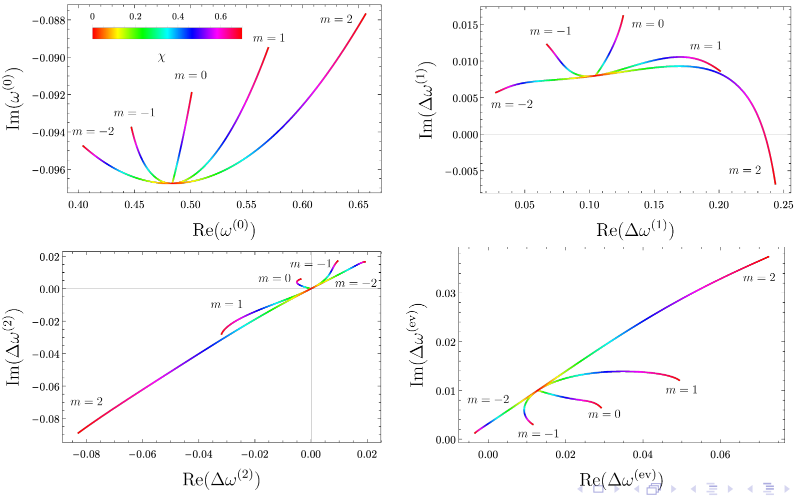

Gravitational QNMs

Gravitational perturbations: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$. Problem: decouple the linearized EOMs

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$$h_{\mu\nu} = e^{-i\omega t} S^{lm,A}_{\mu\nu}(\theta, \phi) h^{lm}_A(r)$$

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$$h_{\mu\nu} = e^{-i\omega t} S^{lm,A}_{\mu\nu}(\theta, \phi) h^{lm}_A(r) + \chi e^{-i\omega t} \sum_{(l',m') \neq (l,m)} S^{l'm',A}_{\mu\nu}(\theta, \phi) h^{l'm'}_A(r) + \mathcal{O}(\chi^2)$$

Applying the same logic as for the scalar field:

$$\langle S^{lm,A,\mu\nu} | G^L_{\mu\nu} + \lambda \mathcal{E}^L_{\mu\nu} \rangle = 0$$

→ **these equations only involve the variables h^{lm}_A at $\mathcal{O}(\chi)$**

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This idea has been applied to EdGB and dCS gravities Pierini, Gualtieri '21; Wagle, Yunes, Silva '21; Srivastava, Chen, Shankaranarayanan '21

Gravitational QNMs

Parity-preserving corrections: decoupled equations

$$\frac{d^2\Psi^\pm}{dr_*^2} + \left(\omega^2 - V_\pm - \delta V_\pm\right)\Psi^\pm = 0$$

Parity-breaking corrections: coupled equations for the master variables

$$\frac{d^2\Psi^\pm}{dr_*^2} + \left(\omega^2 - V_\pm\right)\Psi^\pm = \delta W_\pm \Psi^\mp$$

V_+ and V_- are isospectral $\Rightarrow$ linear shift in the QNM frequencies

In general, we have

- Mode mixing
- Loss of isospectrality

Results coming soon for cubic and quartic terms! PAC, Hertog, Fransen, Maenaut

Conclusions

- Calculation of QNMs of highly-rotating BHs in HDG is still an open problem
- Progress: scalar QNMs for high rotation, gravitational QNMs for slow rotation
- Going beyond this: χ^n expansion of $h_{\mu\nu}$? NP formalism? Geodesic approximation for the eikonal limit ($l \to \infty$)?
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Thank you for your attention

Bonus

Can we really see higher-derivative corrections? Depends on the scale ℓ

The first corrections are $\sim \ell^4 Riem^3$. The relative deviation Δ with respect to GR is of order

$$\Delta \sim \frac{\ell^4 (GM)^2}{r^6}$$

$$\Delta_{Surf.Sun} \sim \left(\frac{\ell}{5\times 10^8\text{km}}\right)^4, \quad \Delta_{Surf.Earth} \sim \left(\frac{\ell}{2\times 10^8\text{km}}\right)^4, \quad \Delta_{BH}(10M_{\odot}) \sim \left(\frac{\ell}{40\text{km}}\right)^4$$

We may measure this if $\ell \sim$ km.

Constraints from cosmology? NO

BHs$\rightarrow$ Weyl curvature. Cosmology $\rightarrow$ Ricci curvature