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Computing positioning errors in Relativity Positioning Systems

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Abstract

General Relativity Theory (GRT) provides a framework to compute the most precise orbits of Earth satellites. Four satellites are needed to locate a user in Relativistic Positioning Systems (RPS). In 2014, Puchades and Sáez (Astrophys. Space Sci. 352, 307, 2014) computed the difference in positioning taking satellites world lines with Schwarzschild metric and with a statistical perturbation of such world lines. Such differences are named the U-errors. To compute the photons null geodesics of the satellites signals they used the solution given by Coll, Ferrando and Morales Lladosa (Class. Quantum Grav. 27, 065013, 2010).

Abstract

Our team (Puchades, Arnau and Fullana, Astrophys. Space Sci., Volume(366):66 (19pp), 2021) has taken more accurate satellites trajectories as perturbations of Schwarzschild world lines. These more accurate trajectories consider the gravitational effects of the Earth, the Moon and the Sun, and the Earth oblateness.

Abstract

A robust algorithm has been built to compute the U-errors with this more accurate description of satellites orbits. We are now incorporating more relativistic perturbations in the metric to describe the satellites world lines. Our method is applied to the ESA Galileo Satellites Constellation ***(h = 23222 Km)***. However, our algorithm is also applied to other satellites at different heights. In this presentation a summary of this research is given.

Introduction

1) Determine the orbits of 4 satellites

2) Compute the proper times at some user's position

3) Then one has the user's position

4) Do it with different descriptions of the 4 orbits

5) Compute the difference of positioning

6) Such differences are the positioning errors, U-errors

7) Plot the HEALPIx mollweide representation of such errors in a spherical surface

Positioning a satellite in RPS: Metric:

(GRT from the beginning)

$$g_{00} = -\left[1 - 2\left(\omega_0(t,\mathbf{x}) + \omega_L(t,\mathbf{x})\right)\right] = -(1 - 2\,\Phi)$$

$$g_{0i} = 0,$$

$$g_{ij} = \delta_{ij}\left[1 + 2\left(\omega_0\,(t,\mathbf{x}) + \omega_L\,(t,\mathbf{x})\right)\right] = \delta_{ij}(1 + 2\Phi)$$

$$\omega_0 = \Phi = \textstyle\sum_B \Phi_{\mathrm{B}} = G \sum_B \frac{M_B}{r_B}\,;\ \textit{B=Earth, Moon and Sun}$$

$$2\omega_L = \Phi^{J_2}\ \textit{Earth quadrupole potential}$$

RPS: Timelike Geodesic Equations of Satellites

$$\frac{du^{\alpha}}{d\tau} = -\,\Gamma_{\mu\nu}{}^{\alpha} u^{\mu} u^{\nu}$$

$$\frac{du^{0}}{d\tau} = \frac{1}{1-\Phi} \Phi_{,i} u^{0} u^{i}$$

$$\frac{dx^{0}}{d\tau} = u^{0}$$

$$\frac{du^{k}}{d\tau} = \frac{1}{1+\Phi} \left[\frac{1}{2} \Phi_{,k} \left(u^{0} u^{0} + u^{i} u^{i} \right) - u^{k} (\Phi_{,i} u^{i}) \right]$$

$$\frac{dx^{k}}{d\tau} = u^{k}$$

Constraint: $g(u,u) = -1$

RPS: Timelike Geodesic Equations of Satellites

- Those equations are in pseudo-cartesian isotropic GCRS (Geo-Centre)
- Numerical integration of the ODE
- Runge-Kutta method: Accuracy 10^{-18}
- Using 40 significant digits

Motion of Celestial Bodies

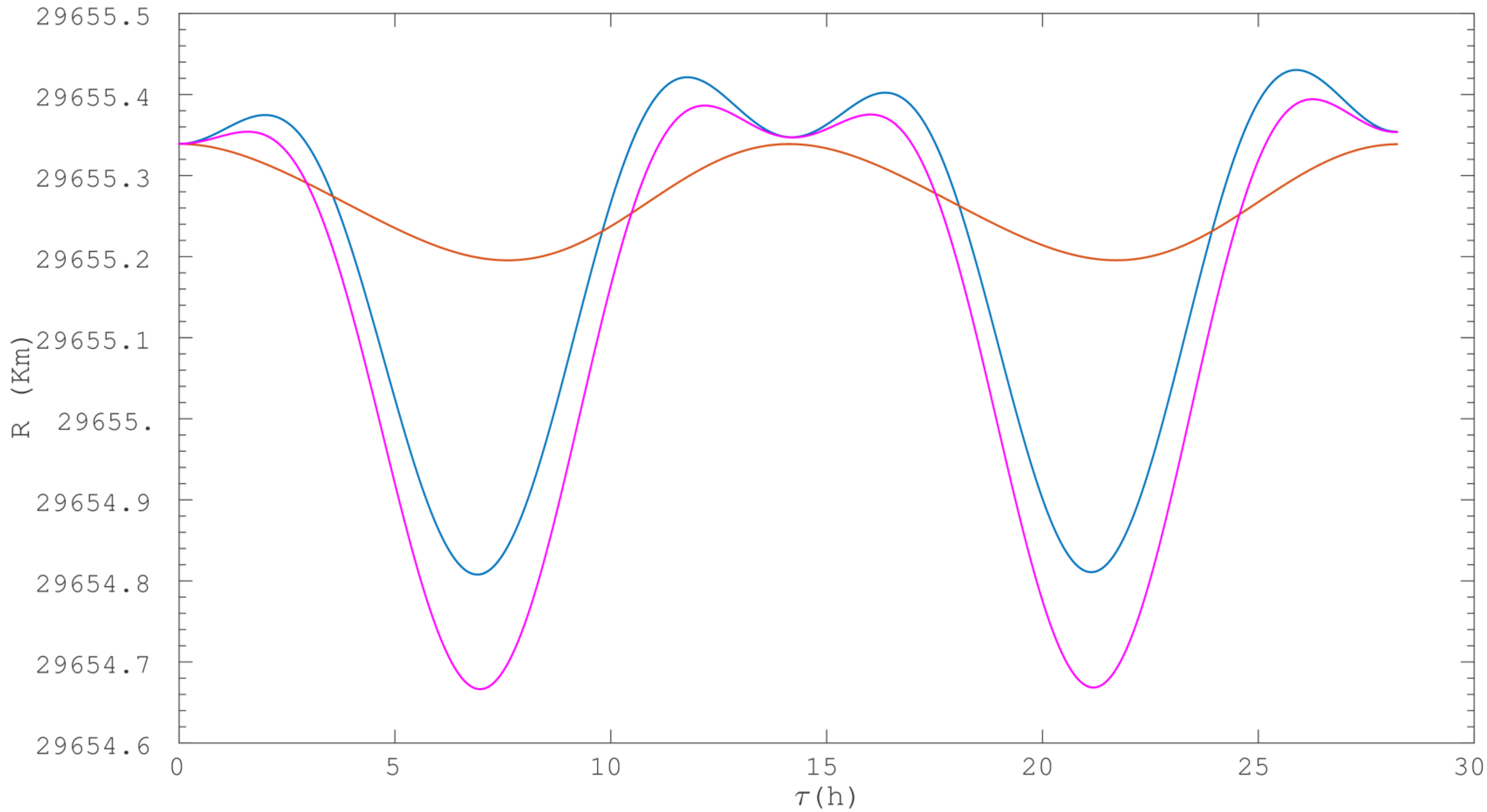
Sufficient considering Newtonian movement

$$\frac{dx^k{}_E}{d\tau} = v^k{}_E u^0$$

$$\frac{dv^k{}_E}{d\tau} = -G\,\frac{M_S\left(x^k{}_E - x^k{}_S\right)}{|\mathbf{x}_E - \mathbf{x}_S|^3}u^0 - G\,\frac{M_M(x^k{}_E - x^k{}_M)}{|\mathbf{x}_E - \mathbf{x}_M|^3}u^0$$

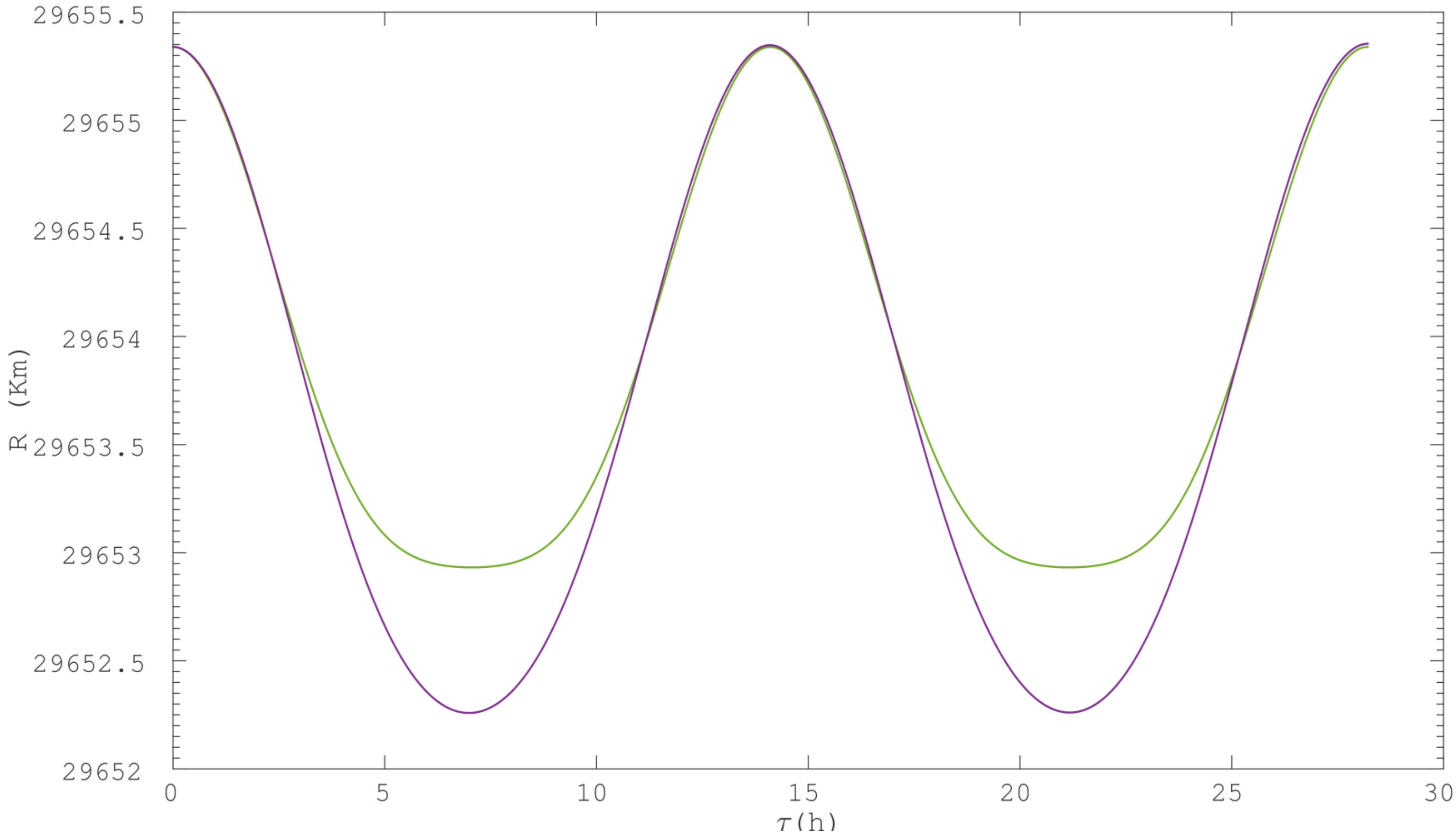
Corresponding Eqs. for Sun and Moon

RESULTS: Sun + Moon (at Gallileo Sat. distance)



~ 600m Moon (Blue, shifts the satellite position after one period)
~ 200m Sun (Orange, recover the radial distance in 1 or 2 periods)
~ 700m Sun + Moon (Magenta)
Coincide with Teunissen & Montenbruck (2015)

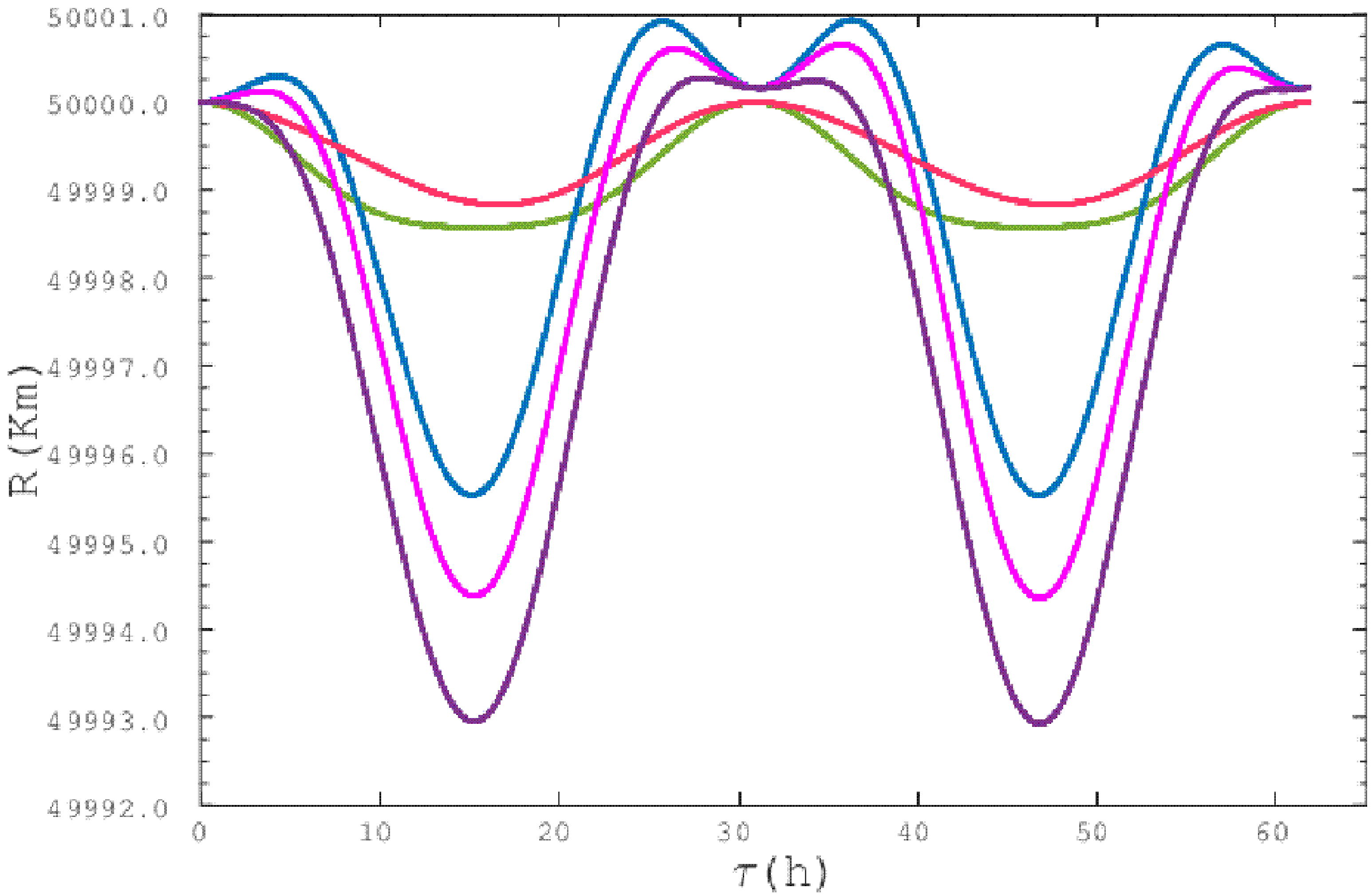
RESULTS: Earth oblateness (at Gallileo Sat. distance)



~ 2km Earth oblateness (green, recover the radial distance in 1 or 2 periods)

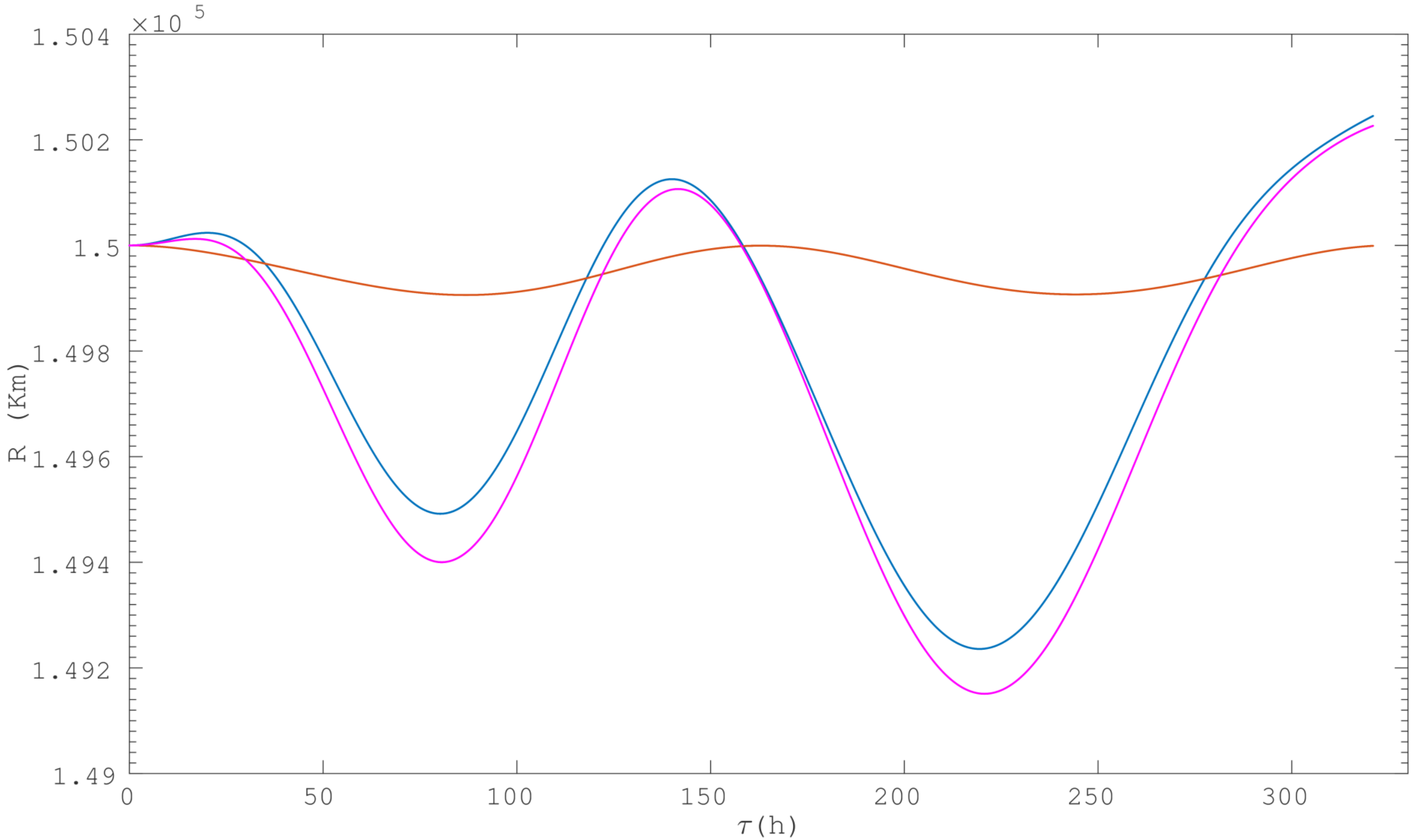
~ 3Km (purple) Earth oblateness + Sun + Moon

RESULTS: Different orbital radius from GCRS



At 5×10^4 Km. Compatible with figure of Montenbruck & Gill (2005)
Earth oblateness (green), Moon (blue), Sun (red), Moon + Sun (magenta) and Earth oblateness + Moon + Sun (violet).

RESULTS: Different orbital radius from GCRS



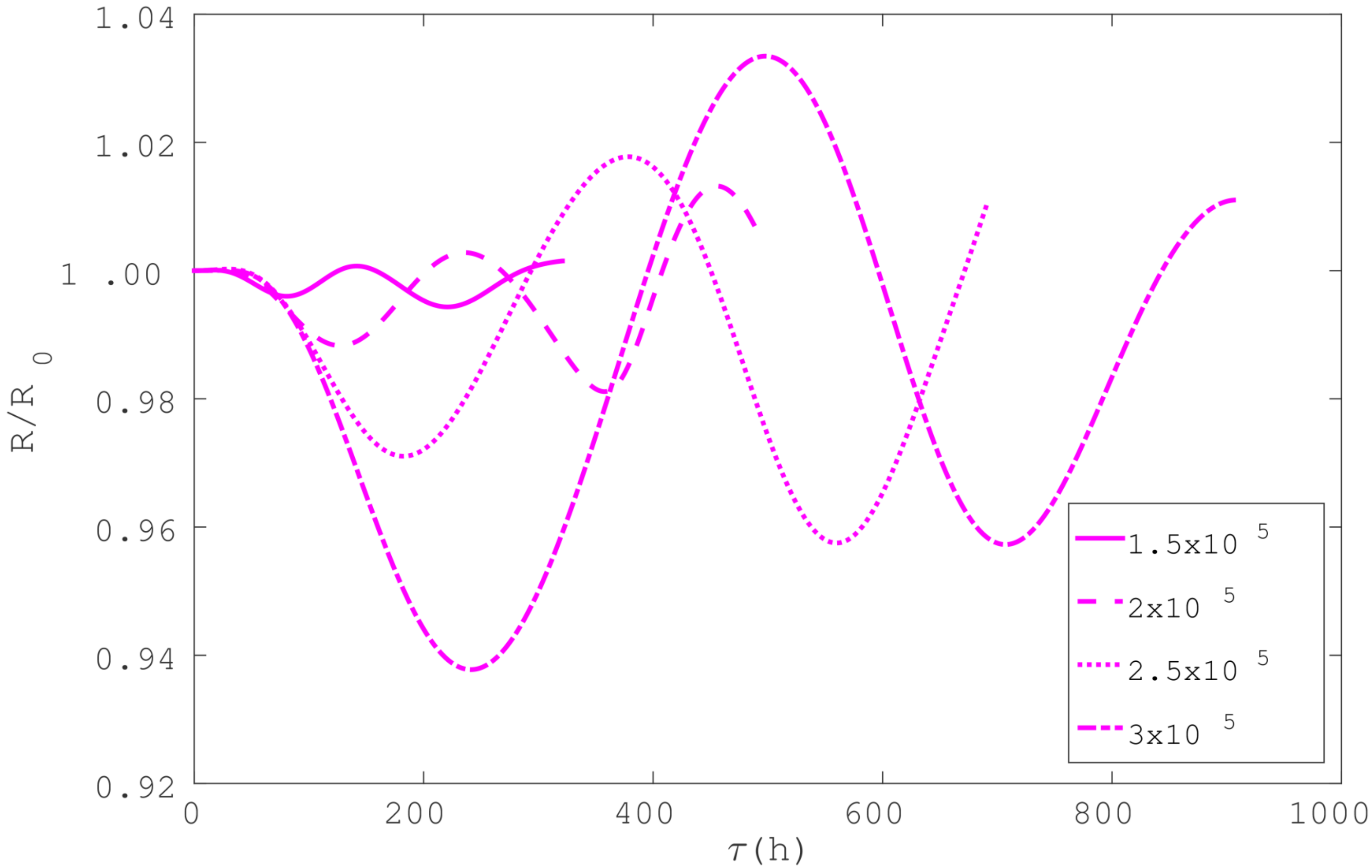
At $1.5 \times \ 10^5$ Km

As orbital radius increases:

Earth oblateness dicreases (orange) and Sun (red) & Moon (blue) increases

As nearer the Moon is, greater Moon effect than Sun one is

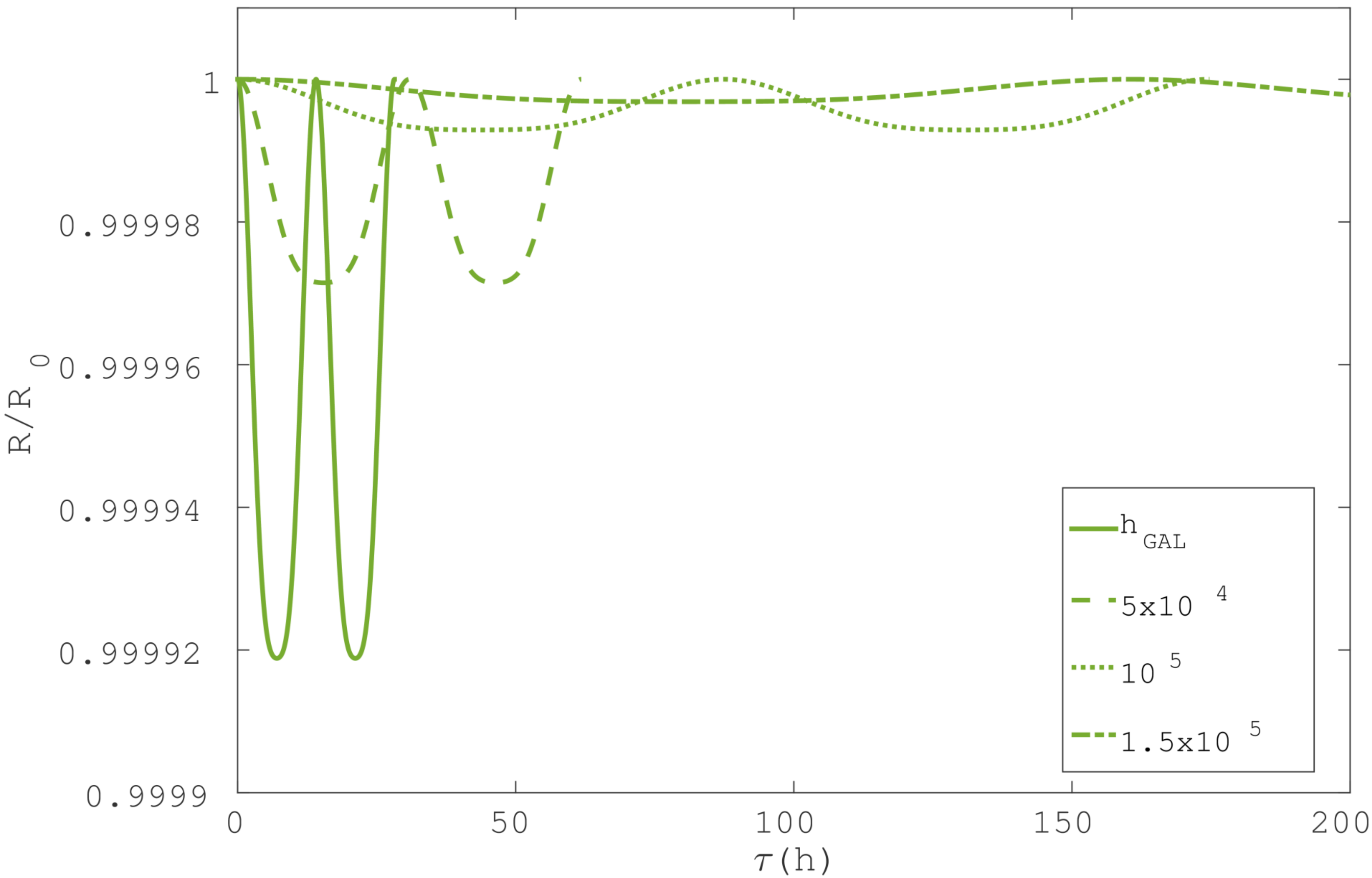
RESULTS: Different orbital radius from GCRS



Radial distance / orbital distance

As orbital radius increases: Sun & Moon effect increases

RESULTS: Different orbital radius from GCRS



Radial distance / orbital distance

As orbital radius increases: Earth oblateness effect dicreases

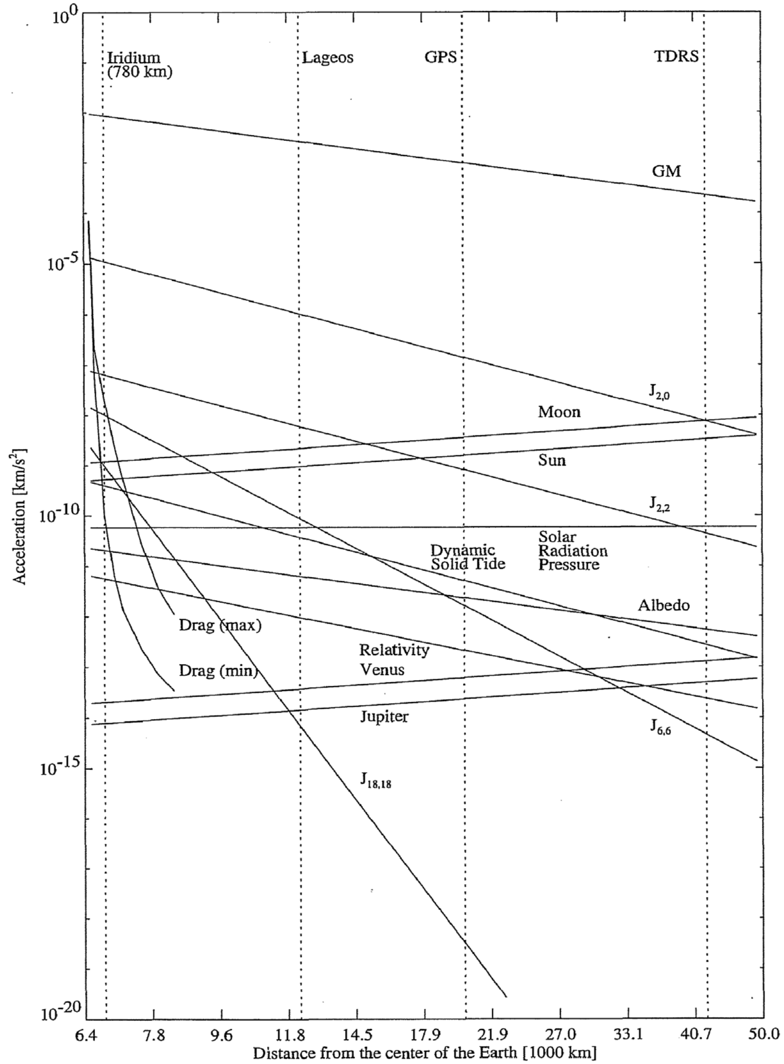


Fig. 3.1. Order of magnitude of various perturbations of a satellite orbit. See text for further explanations.

Montenbruck, O. Gill, E., 2005. Satellite Orbits: Models, Methods and Applications, Springer-Verlag, Heldeberg, Germany. Springer, ISBN-13: 978-3540672807

U-errors

Emission coordinates: τ^A

which are not to be varied since they are broadcasted by the satellites and received by the user without ambiguity

Nominal coordinates, described in Schwarzschild ST: $x^{\alpha}(\tau^A)$

Perturbed satellite world lines in the space-time:

TX-code gives **new inertial coordinates:** $[x^{\alpha}+\Delta(x^{\alpha})](\tau^A)$

Both coordinates are to be compared:

U-errors: $\Delta_d=[\Delta^2(x^1)+\Delta^2(x^2)+\Delta^2(x^3)]^{1/2}$

Same emission coordinates, which are received from the satellites, but from different satellite world lines.

Improvement: most accurate description of satellite perturbations using a metric which better accounts of a more accurate trajectory of the satellites.

The perturbations computed here using metrics improve our previous works based on statistical methods as:

1) A better description of the real satellite world lines is achieved.

2) The effect of each perturbing contribution in the satellite world lines is studied.

3) Also, the combination of two of the three terms in the metric is studied and the three of them together. So, the orbits of the satellites are described depending on the terms considered.

4) Therefore, the contribution of each effect on the user's positioning can also be studied.

5) The value of the U-errors is now smaller.

6) That means a more precise computation of the user's positioning.

Present work

- Consider more perturbations in the metric and see the effects in the computations of the orbits of satellites. Then do the RPS computations, U-errors.

- GRT is used from the beginning. No Newtonian computation with GRT perturbations.

Perspectives

- The use of our method in space navigation is being planned. The Barycentric Celestial Reference System is more appropriate as reference system to locate the emitters (four satellites…) in the solar system.

Perspectives

- For example, in the vicinity of the Moon, two emitters fixed on the Moon surface (North and South poles) and two emitters from Galileo satellites.

- The positioning of a spacecraft that navigates in the solar system could be determined considering emitters in other appropriate locations to be studied.

Acknowledgements

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Acknowledgements

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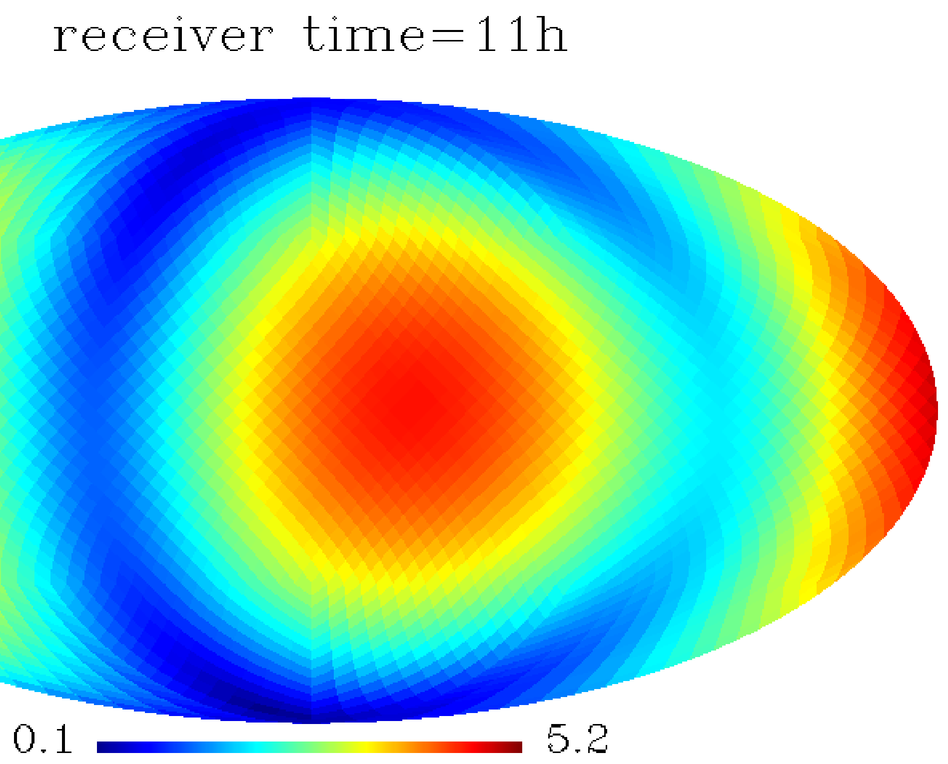
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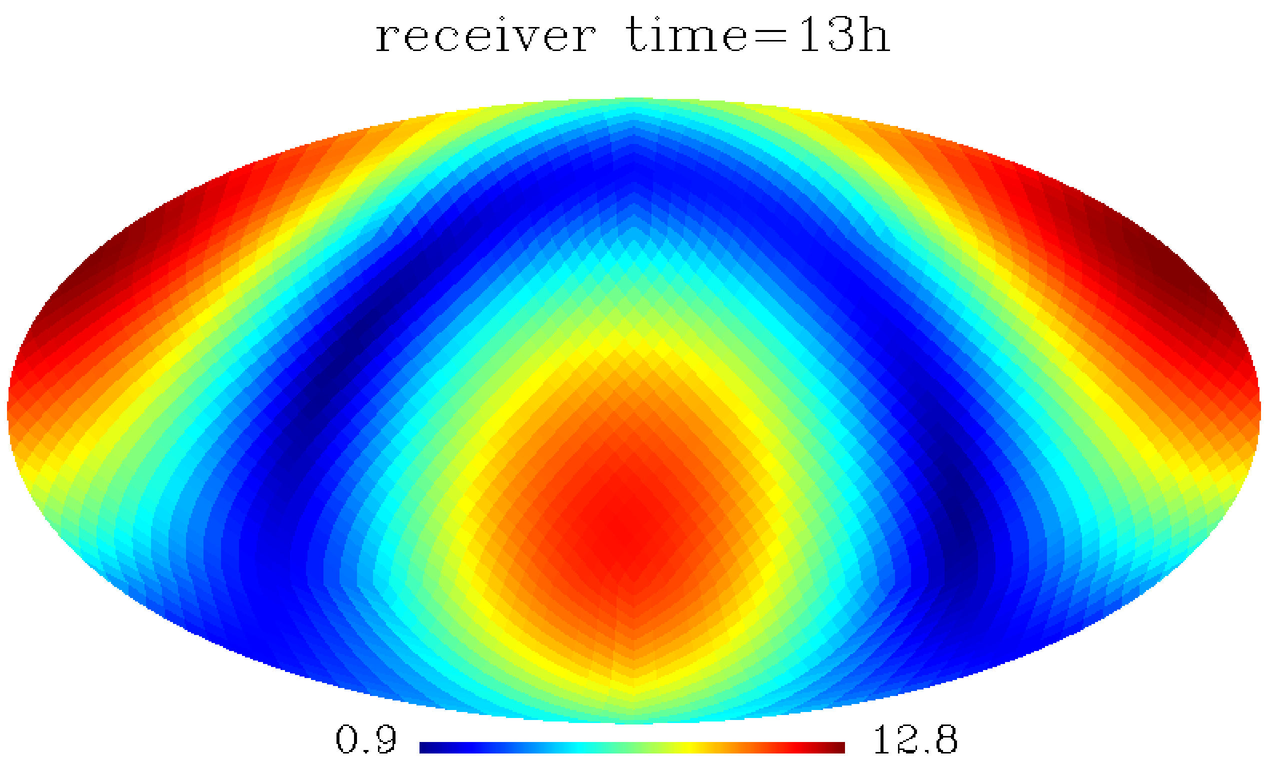
5.2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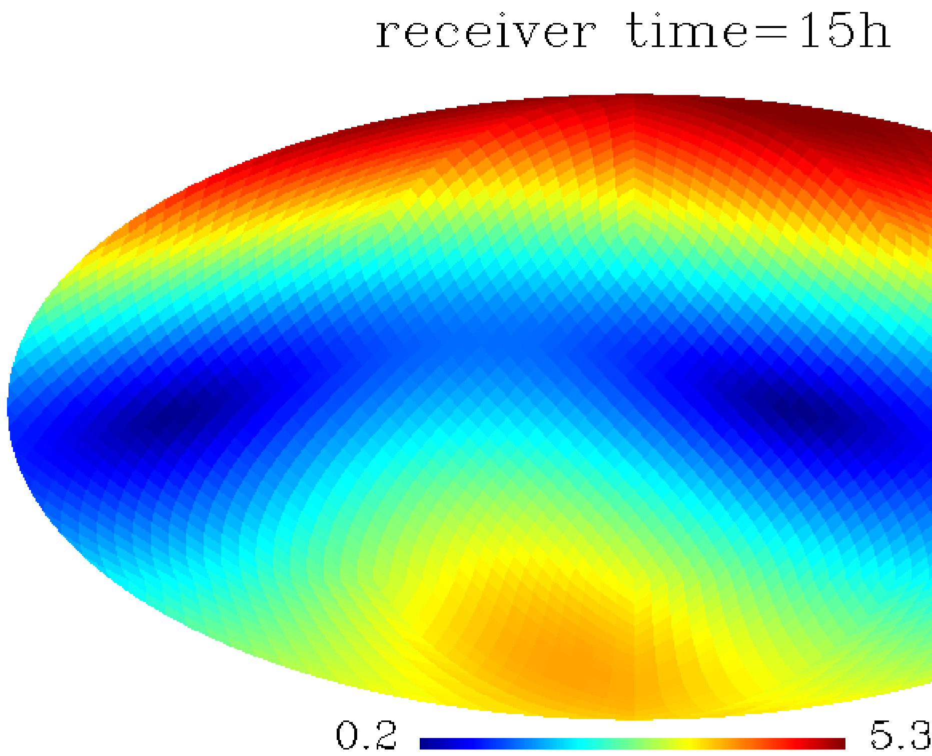
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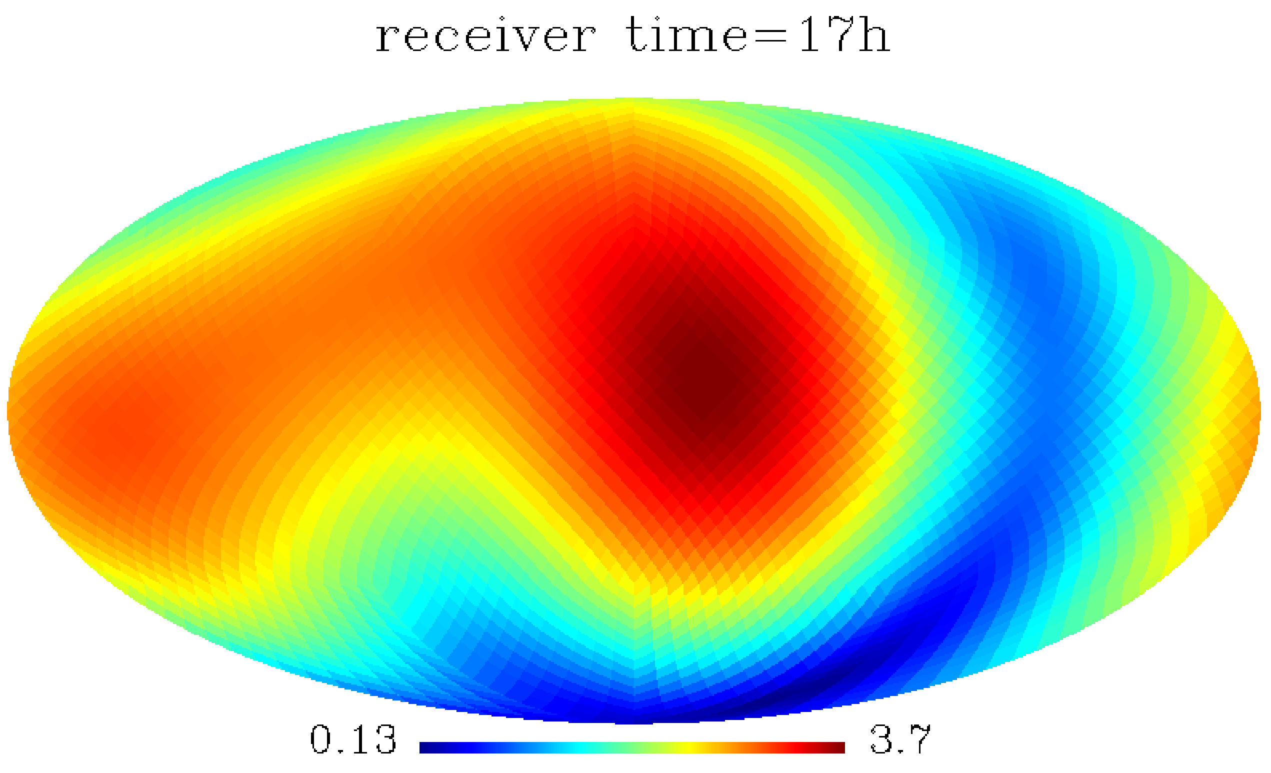
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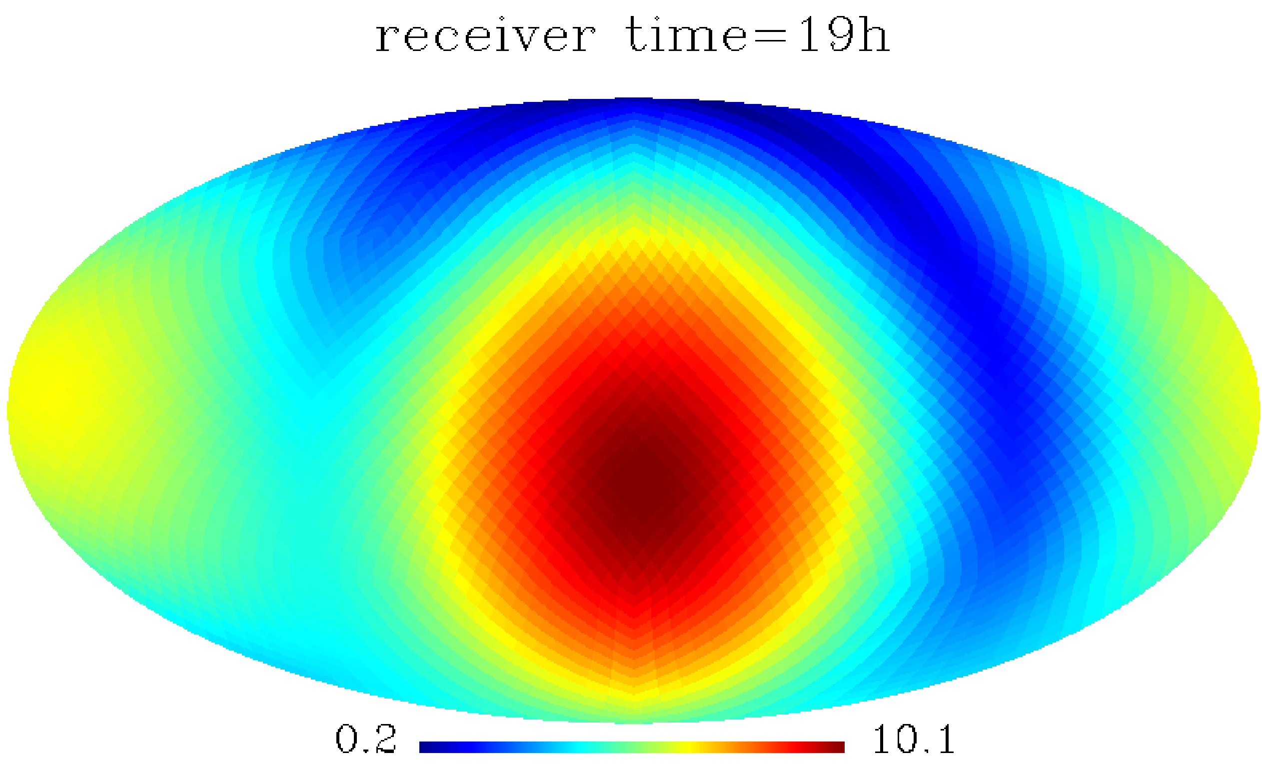
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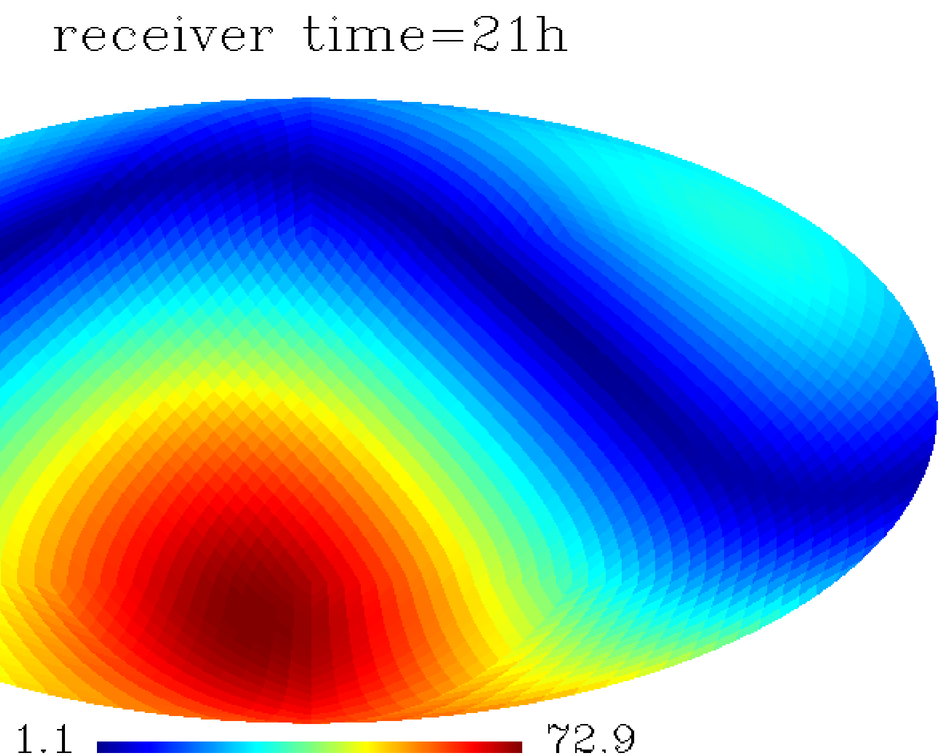
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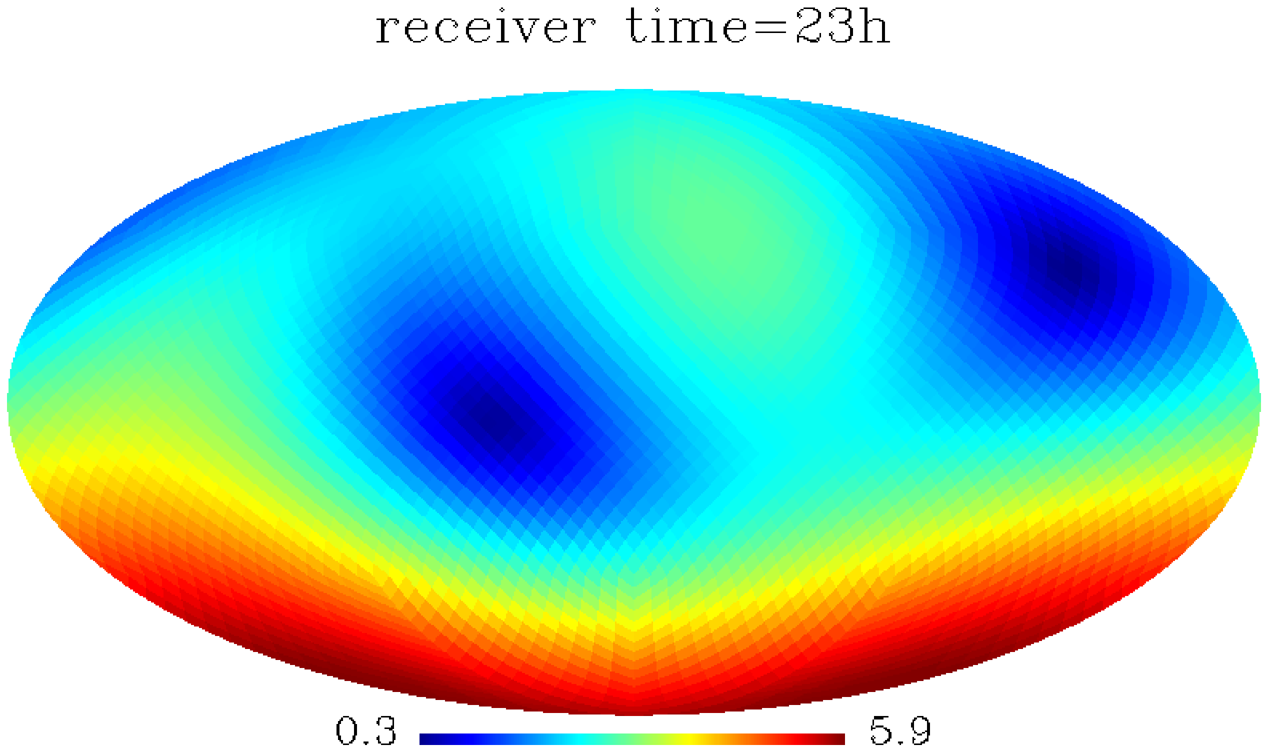
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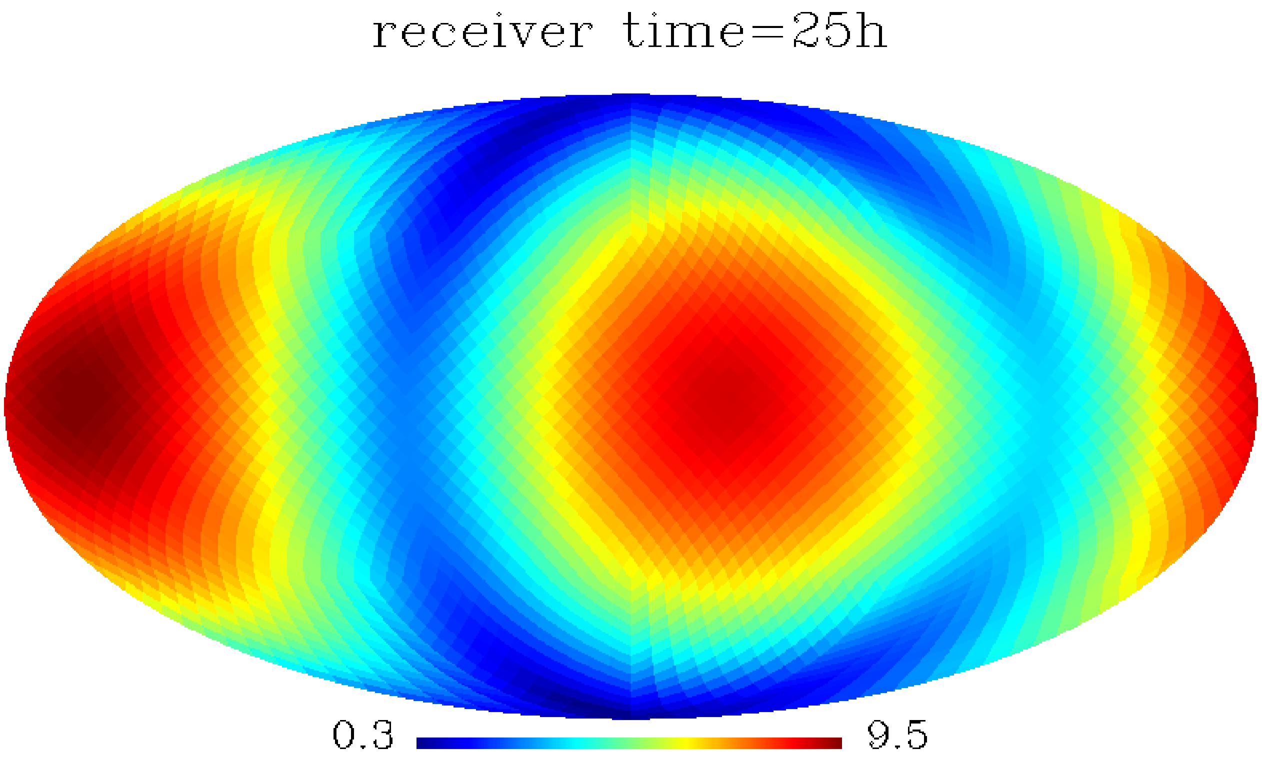
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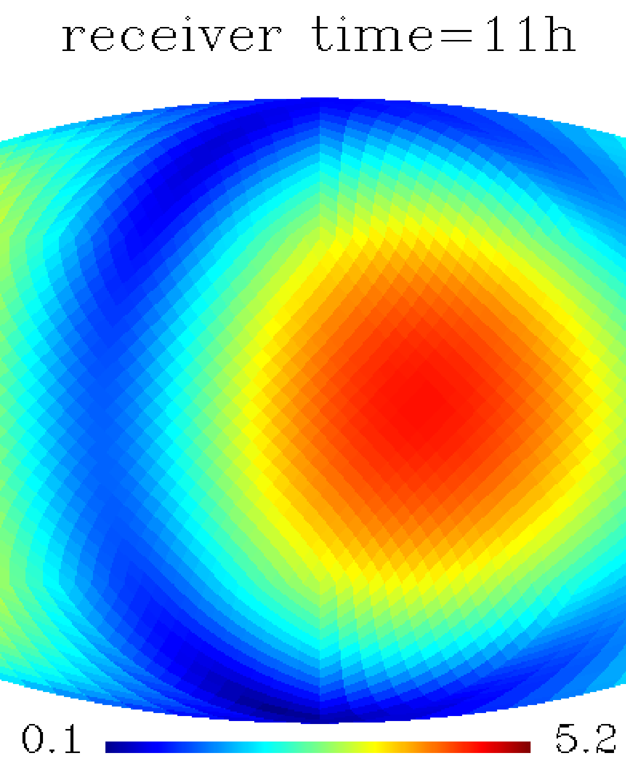
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