Quasinormal modes of AdS NUT black branes

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based on [2101.10652], *with* Pablo A. Cano (KU Leuven)

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EREP2021, September 2021

Introduction

Black holes in AdS/CFT :

Black holes	$\leftrightarrow$	"quark-gluon" plasma
Hawking temperature	$\leftrightarrow$	QFT temperature
$\mathcal{Z}^{E}_{\text{grav}}$	$\leftrightarrow$	$\mathcal{Z}^{\text{finiteT}}_{\text{CFT}}$
Electrostatic potential at $\mathcal{H}$	$\leftrightarrow$	Chemical potential

In particular, probe thermal CFTs

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- *Holographic hydrodynamics* : [Birmingham, Sachs, Solodukhin, Son, Starinets, Policastro, Herzog, Nuñez, Kovtun, Baier, Romatschke, Stephanov...]

  Hydrodynamics, correlation functions $\rightarrow$ BH perturbations

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- Characteristic value problem for $\omega \to$ discrete set of solutions only : ω are poles of thermal correlators

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AdS_5 planar black holes $\to$ CFT on $\mathbb{R}^{1,3}$ (most studied case)

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- Spherical BHs [Horowitz, Hubeny '00 ; Cardoso, Lemos '01 ; Cardoso, Konoplya, Lemos '03 ; Musiri, Siopsis '03]
- Planar BHs [Cardoso, Lemos '01 ; Miranda, Zanchin '06 ; Miranda, Morgan, Zanchin '08]
- Kerr-AdS BHs [Giammatteo, Moss '05 ; Dias, Reall, Santos '09 ; Dias, Santos '13 ; Cardoso, Dias, Hartnett, Lehner, Santos '13]

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Unexplored case : NUT charge

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"*Taub-NUT Space as a Counterexample to Almost Anything*"

[Misner '65]

$$ds^2 = -V(r)(dt+2n\cos\theta d\phi)^2+\frac{dr^2}{V(r)}+(r^2+n^2)\left(d\theta^2+\sin^2\theta d\phi^2\right)$$

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Euclidean : interesting for holography → CFTs on squashed spheres [Hawking, Hunter, Page '99 ; Chamblin, Emparan, Johnson, Myers '99 ; Imamura, Yokoyama '11 ; Martelli, Passias, Sparks '12 ; Bobev, Hertog, Vreys '16 ; Bobev, Bueno, Vreys '17 ; Bueno, Cano, Hennigar, Mann '18 ; Bueno, Cano, Hennigar, Penas, Ruipérez '20]

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Lorentzian : seemingly pathological properties

- CCC's (e.g. ∂_ϕ)
- Misner strings at $\theta = 0, \pi$ ("Unobservable" if $t \sim t + 8\pi n$)
- No 1st law apparently $dM \neq Td\left(\frac{A}{4}\right)$ [Astefanesei, Mann, Radu '04]

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- CTCs but no closed time-like geodesics Clément, Gal'tsov, Guenouche '15
- Freely falling observers do not observe Misner strings Clément, Gal'tsov, Guenouche '15
- Full cohomogeneity first law of thermodynamics (T and n independent) Hennigar, Kubiznak, Mann '19 ; Bordo, Gray, Hennigar, Kubiznak '19

$$dM = TdS + \psi dn \,, \quad S = A/4$$

- Holography : fluids with vorticity Leigh, Petkou, Petropoulos '11-'12. Scalar fluctuations interact with Misner string Kalamakis, Leigh, Petkou '20

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We approach these questions for **planar AdS-Taub-NUT Black Branes**.

Planar AdS NUT Black Branes

- **Planar AdS NUT Black Branes**
- Perturbation Theory
- Boundary Conditions
- Quasinormal Frequencies
- Conclusions

Planar AdS NUT Black Branes

- Horizon radius r_+, NUT charge n

$$ds^2 = -V(r)\left(dt + \frac{2n}{L^2}x dy\right)^2 + \frac{dr^2}{V(r)} + \frac{r^2+n^2}{L^2}\left(dx^2 + dy^2\right)$$

$$V(r) = \frac{(r - r_+)\left(3n^4 + 6n^2 r r_+ + r r_+\left(r^2 + r r_+ + r_+^2\right)\right)}{L^2 r_+\left(n^2 + r^2\right)}$$

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- Petrov type D, no Misner strings, several Killing horizons (CCC's but no CC geodesics).

Planar AdS NUT Black Branes

- Boundary metric (Nil space, Heisenberg's group manifold)

$$d\hat{s}^2 = -\left(dt + \frac{2n}{L^2}xdy\right)^2 + dx^2 + dy^2$$

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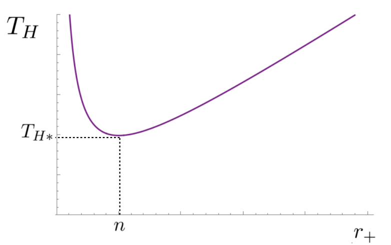
- Two BHs for every $T = \frac{3(n^2+r_+^2)}{4\pi L^2 r_+} > T_* = \frac{3|n|}{2\pi L^2}$. Dominant saddle $\rightarrow r_+ \geq |n|$.



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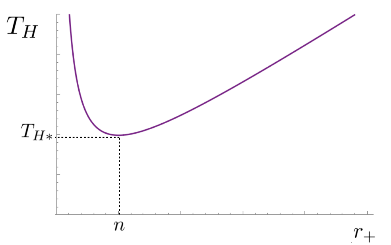

- Boundary theory characterised by

$$T \quad \text{and} \quad \frac{n}{L^2}$$

Perturbation Theory

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- Defining an NP frame and emulating Teukolsky's derivation [Teukolsky '73]

$$\left[(D - 4\rho - \bar{\rho})(\Delta - 4\gamma + \mu) - \delta\bar{\delta} - 3\Psi_2\right] \Psi_0^{(1)} = 0$$

$$\left[(\Delta + 3\gamma - \bar{\gamma} + 4\mu + \bar{\mu})\,(D - \rho) - \bar{\delta}\delta - 3\Psi_2\right] \Psi_4^{(1)} = 0$$

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- Planar operators realise Heisenberg algebra

$$\left.\begin{aligned}\mathbf{a}^\dagger &= \frac{L}{\sqrt{4n\omega}}\left(-i\partial_x+ik+\frac{2ni\omega}{L^2}x\right)\\ \mathbf{a} &= \frac{L}{\sqrt{4n\omega}}\left(-i\partial_x-ik-\frac{2ni\omega}{L^2}x\right)\end{aligned}\right\}[\mathbf{a},\mathbf{a}^\dagger]=1$$

Perturbation Theory

The equations separate :

- Radial :

$$\left[\mathcal{D}_{-1}\Delta\mathcal{D}_1^\dagger + 6\left(\frac{r^2+n^2}{L^2} \pm i\omega r\right) - 4\omega n\left((q_\pm \mp 2) + \frac{1}{2}\right)\right] R^{\omega,q_\pm}_{(\pm 2)}(r) = 0$$

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- Planar (momentarily, $e := \omega$, $B := 2n/L^2$) :

$$\left\{-\frac{1}{2}\partial_x^2 + \frac{e^2B^2}{2}\left(\frac{k}{eB} + x\right)^2\right\}\mathcal{H}_{q_\pm}(x) = eB\left(q_\pm + \frac{1}{2}\right)\mathcal{H}_{q_\pm}(x)$$

Boundedness $\Rightarrow q_\pm \in \mathbb{Z}$.

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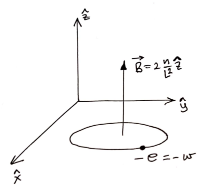
- Landau quantisation ! $\to q_\pm$ Landau Levels.



(recently : [Elinos, Patiño])

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$$
\begin{aligned}
h_{\mu\nu} =&\{-k_\mu k_\nu \bar{\delta}\bar{\delta} - \bar{m}_\mu \bar{m}_\nu (D-\bar{\rho})(D+3\bar{\rho}) \\
&+ k_{(\mu}\bar{m}_{\nu)}\left[(D-\bar{\rho}+\rho)\bar{\delta} + \bar{\delta}(D+3\bar{\rho})\right]\}\bar{\varphi}^{IRG} \\
&+ \{-l_\mu l_\nu \delta\delta - m_\mu m_\nu (\Delta - 3\bar{\gamma} + \gamma + \bar{\mu})(\Delta - 4\bar{\gamma} - 3\bar{\mu}) \\
&+ l_{(\mu} m_{\nu)}\left[\delta(\Delta - 4\bar{\gamma} - 3\bar{\mu}) + (\Delta - 3\bar{\gamma} - \gamma + \bar{\mu} - \mu)\delta\right]\}\bar{\varphi}^{ORG}
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$$

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$$\Psi_0 = \frac{e^{-4i\arctan(r/n)}}{V^2\sqrt{n^2+r^2}} e^{-i\omega t+iky} \mathcal{H}_{q_{+2}}(x) Y_{+2}(r)$$

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Dirichlet BC on these variables $a_{+2} = a_{-2} = 0$? **Wrong !**

- Impose Dirichlet BC on $h_{\mu\nu}$ $\rightarrow$ read off $\lambda_{\pm 2} = \frac{b_{\pm 2}}{a_{\pm 2}}$

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$$\mathcal{D}^\dagger_{-1}\Delta\mathcal{D}^\dagger_0\mathcal{D}^\dagger_0\Delta\mathcal{D}^\dagger_1 R^{\omega,q_+}_{(+2)} = C_{(-2)} R^{\omega,q_+ -4}_{(-2)}$$
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- At infinity (gauge $h_{r\mu} = 0$) : $h_{ab} = \frac{1}{r}h^{(1)}_{ab} + r^2\left(h^{(2)}_{ab} + \mathcal{O}(r^{-1})\right) \quad \text{when} \quad r \rightarrow \infty$

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- From $h^{(2)}_{ab} = 0 \rightarrow \lambda_{-2} = \frac{M'_q + P'_q\lambda_{+2}}{Q'_q + S'_q\lambda_{+2}}$.

Boundary Conditions

We get two classes of boundary conditions

$$\lambda_{+2}^{(\pm)} = \frac{i}{(3-2\hat{q})\hat{\omega}\epsilon + \hat{\omega}^2 - 2\epsilon^2}\Big(2\hat{q}^2\hat{\omega}\epsilon^2 + 2\hat{q}\hat{\omega}\epsilon^2 - 4\hat{q}\hat{\omega}^2\epsilon + \hat{\omega}^3 - 5\hat{\omega}\epsilon^2$$
$$+ 8\epsilon^3 + 2\hat{\omega}^2\epsilon \mp 2\epsilon^2\sqrt{(\hat{q}-3)(\hat{q}-2)(\hat{q}-1)\hat{q}\hat{\omega}^2 + 9\epsilon^2}\Big)$$

corresponding to two distinct polarisations of $h_{\mu\nu}$*, which reduce to the standard even and odd polarisations in the planar case* $n = 0$.

Quasinormal Frequencies

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$$\omega(\sigma T, \sigma n) = \sigma \omega(T, n)$$

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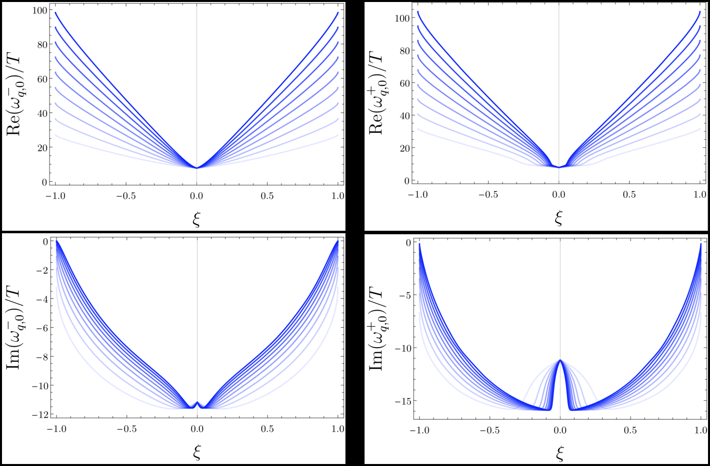
- Symmetries :
  1. $\tilde{\omega}^{(\pm)}_{q,m}(-\xi) = -\left(\tilde{\omega}^{(\pm)}_{q,m}(\xi)\right)^*$
  2. $\left\{\tilde{\omega}^{(\pm)}_{q,m}(-\xi)\right\}_{q,m} = \left\{\tilde{\omega}^{(\pm)}_{q,m}(\xi)\right\}_{q,m}$ (hidden symmetry)

Quasinormal Frequencies

Ordinary QNMs : lowest overtones



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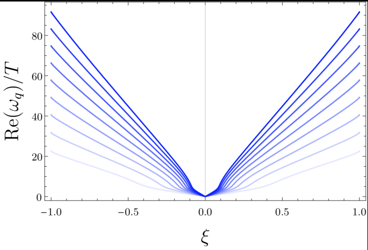

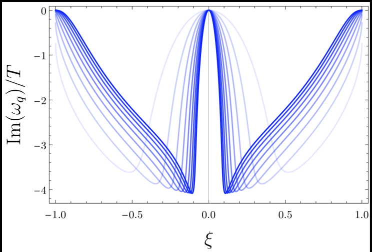

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- Analytic approximation for small ξ (i.e. $T \gg \frac{3nq}{2\pi L^2}$)

$$\omega_q \approx \frac{n a_q}{L^2} - i\frac{3 b_q n^2}{4\pi T L^4}$$

$$a_q = \frac{1}{2}\left(2q + 5 + \sqrt{4q^2 + 20q + 33}\right)$$

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- Reduces to plasma sound wave mode ($\eta/s = 1/(4\pi)$).

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- For higher spins no analogue proof.
- Detailed numerical analysis suggests full (mode-) stability (even for $\epsilon > 1$!)

Conclusions

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Outlook

- Studied QNMs of Taub-NUT spacetimes for the first time
- Discrete spectrum. Landau levels
- All modes have $\text{Im}(\omega) < 0 \Rightarrow$ linear (mode-) stability !
- CFT : perturbations of a plasma in the geometry

$$d\hat{s}^2 = -\left(dt + \frac{2n}{L^2} x dy\right)^2 + dx^2 + dy^2$$

- QFT on spacetime with CTCs must make sense !
- Definite prediction for large temperature $\omega_q \approx \frac{n a_q}{L^2} - i \frac{3 b_q n^2}{4\pi T L^4}$. Compare now with plasma perturbations.

Conclusions

Future directions

- Overtone structure. Proof of stability.
- Electromagnetic perturbations (and other spins).
- Compare holographic predictions with CFT computations (e.g. partition function, correlators).
- Extension to spherical Taub-NUTs : interplay with Misner string.

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Thank you !