Asymptotic quasinormal modes of string-theoretical d-dimensional black holes

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Overview

In this presentation, I will display some analytical results, concerning the Callan Myers Perry black hole.

I will provide analytical expressions for some quasinormal frequencies in this black hole.

- Eikonal limit $\longrightarrow$ Already done earlier today
- Asymptotic limit $\longrightarrow$ Target of this presentation

Quasinormal modes and frequencies

Master equation

$$\frac{d^2\psi}{dx^2} + \left(\omega^2 - V\right)\psi = 0 \tag{1}$$

- $\lim_{x \to +\infty} \psi(x) \propto e^{-i\omega x}$
- $\lim_{x \to -\infty} \psi(x) \propto e^{i\omega x}$

Callan Myers Perry black hole

Metric tensor field

$$ds^2 = -fdt \otimes dt + \frac{1}{f}dr \otimes dr + r^2 d^2\Omega_{d-2} \tag{2}$$

- $f(r) = f_0(r)\left(1 + \lambda'\delta f\right)\ ,\ \lambda' \propto \dfrac{\alpha'}{R_h^2}$

- $f_0(r) = 1 - \left(\dfrac{R_h}{r}\right)^{d-3}$

- $\delta f(r) = -\dfrac{(d-3)(d-4)}{2}\left(\dfrac{R_h}{r}\right)^{d-3}\left[\dfrac{1-\left(\frac{R_h}{r}\right)^{d-1}}{1-\left(\frac{R_h}{r}\right)^{d-3}}\right]$

Scalar and gravitational perturbations

Scalar perturbations $\longrightarrow$ V_s [Cardoso and Lemos 2002]

Tensor type gravitational perturbations $\longrightarrow$ V_g [Moura and Schiappa 2006]

- $V_s(r) = f\left(\frac{l(l+d-3)}{r^2} + \frac{(d-2)(d-4)f}{4r^2} + \frac{(d-2)f'}{2r}\right)$

- $V_g(r) = V_s(r) + \lambda' C(r)$

Asymptotic limit

- The asymptotic limit targets quasinormal frequencies ω such that $|\Re(\omega)| \ll |\Im(\omega)|$.

- This limiting case poses an operational problem in imposing the appropriate boundary conditions.

- Solution: Monodromy method. [Motl and Neitzke 2003] [Natário and Schiappa 2004]

Monodromy method I

- Analytic continuation to the complex r-plane.

- We pick two closed homotopic curves in the complex r-plane, enclosing the event horizon. We consider the monodromy of the perturbation, associated with a full loop around these curves.

- In one monodromy, we encode the information of the boundary condition in the event horizon. In the other one, we encode the information of the boundary condition in spatial infinity.

Monodromy method II

Monodromy theorem

$$\Downarrow$$

Homotopic curves share the same monodromy

Final result

Equation to solve for the quasinormal frequencies.

Perturbative approach

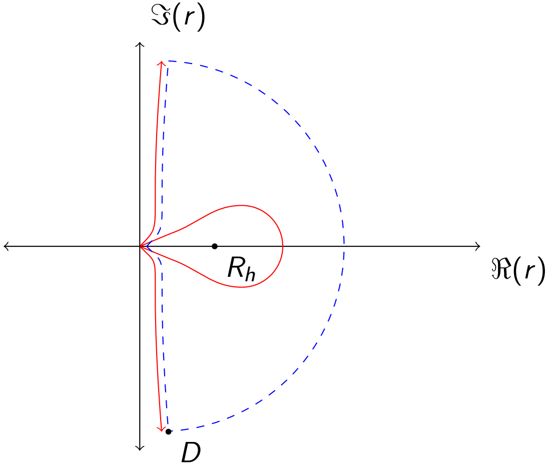
Perturbative approach

$$\psi(z) = \psi_0 + \lambda' \psi_1 \quad ; \quad V(z) = V_0(z) + \lambda' V_1(z) \quad ; \quad x \mapsto z \tag{3}$$

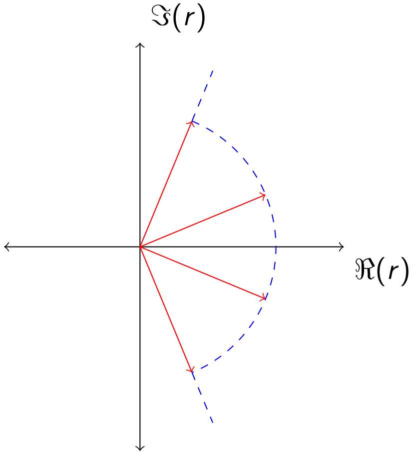
- $\dfrac{d^2\psi_0}{dz^2}(z) + (\omega^2 - V_0(z))\psi_0(z) = 0$

- $\dfrac{d^2\psi_1}{dz^2}(z) + (\omega^2 - V_0(z))\psi_1(z) = \xi(z)$

Big contour and Stokes lines



Zooming near the origin



Behaviour near the origin I

Expanding $V_0(r)$ and $\xi(r)$ near $r = 0$ yields

$$\psi_0(z) = A_+\sqrt{2\pi}\sqrt{\omega z}J_{\frac{j}{2}}(\omega z) + A_-\sqrt{2\pi}\sqrt{\omega z}J_{-\frac{j}{2}}(\omega z) \tag{4}$$

for some $A_\pm \in \mathbb{C}$. Using the Wronskian method yields

$$\psi_1(z) = A_+\sqrt{2\pi}\sqrt{\omega z}J_{\frac{j}{2}}(\omega z)\mathcal{C}_+(z) + A_-\sqrt{2\pi}\sqrt{\omega z}J_{-\frac{j}{2}}(\omega z)\mathcal{C}_-(z). \tag{5}$$

Behaviour near the origin II

- We are interested in a $\frac{3\pi}{d-2}$ Radians rotation in the complex r-plane.

- In the complex z-plane this rotation amounts to 3π Radians.

- We only need to compute the change of the ψ_0 and ψ_1 under this rotation.

Asymptotic matching

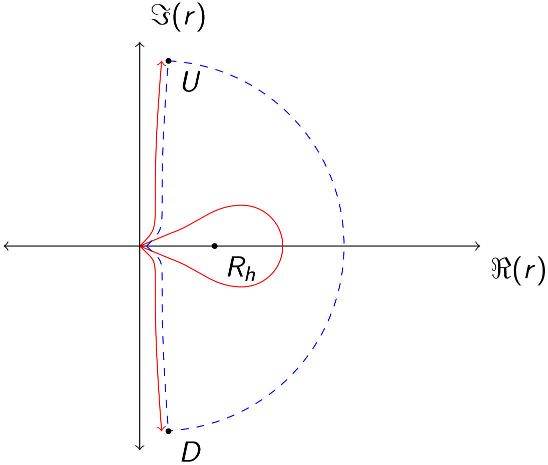
WKB approximation

$$\psi(z) \sim C_+ e^{i\omega z} + C_- e^{-i\omega z} \tag{6}$$

Using known asymptotic expansions agrees with the WKB approximation!

$$\psi(z) = \psi_0(z) + \lambda' \psi_1(z) \sim \Omega_+(A_+, A_-)e^{i\omega z} + \Omega_-(A_+, A_-)e^{-i\omega z} \tag{7}$$

Boundary condition



Closing the contour

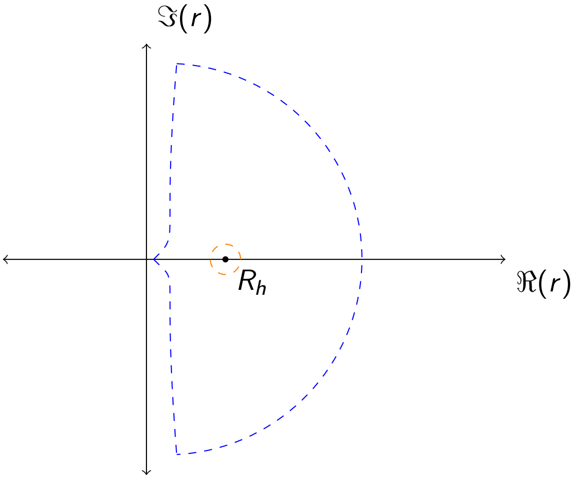
- As we abandon the Stokes line we can no longer rely on the WKB approximation to track the behaviour of ψ.

- Small corrections become important given that $|e^{i\omega x}| \ll 1$.

- WKB theory tells us that the dominant term (proportional to $e^{-i\omega x}$) remains unchanged.

- Using this information we can finally close the contour and compute the monodromy of ψ.

Small contour



Solution near R_h and monodromy

Expanding V near R_h allow us to analytically solve the master equation yielding

$$\psi(x) = B_+ e^{i\omega x} + B_- e^{-i\omega x} \tag{8}$$

for some $B_\pm \in \mathbb{C}$. Imposing the boundary condition yields

$$B_- = 0 \implies \psi(x) = B_+ e^{i\omega x}. \tag{9}$$

Now we can compute the monodromy of ψ around the small contour.

Important

The tortoise variable has a branch point in the event horizon R_h.

Equating monodromies I

Final result

$$\frac{\omega}{T_{\mathcal{H}}} = \ln(3) + (2k+1)\pi i + \lambda\delta \, , \ k \in \mathbb{N} \tag{10}$$

- Gravitational perturbations $\longrightarrow \delta = \left(\frac{4\pi}{d-3}\right)^2 T_{\mathcal{H}}^2 \left[\frac{d-3}{d-2}\frac{(2k+1)}{4}\right]^{\frac{d-1}{d-2}} \Pi_{\mathsf{T}} \, \mathrm{e}^{\frac{d-5}{2(d-2)}\pi i}$
- Scalar test fields $\longrightarrow \delta = \left(\frac{4\pi}{d-3}\right)^2 T_{\mathcal{H}}^2 \left[\frac{d-3}{d-2}\frac{(2k+1)}{4}\right]^{\frac{d-1}{d-2}} \Pi_{\mathsf{S}} \, \mathrm{e}^{\frac{d-5}{2(d-2)}\pi i}$

Equating monodromies II

Both constants Π_S and Π_T depend heavily on the dimension.

- $$\Pi_\mathrm{T} = \frac{2\sqrt{\pi}}{3}\frac{(d(d-5)+2)(d-4)}{d-1}\frac{\Gamma\left(\frac{1}{2(d-2)}\right)\Gamma\left(\frac{d-3}{2(d-2)}\right)}{\Gamma\left(\frac{d-1}{2(d-2)}\right)^2}\sin\left(\frac{\pi}{d-2}\right)$$

- $$\Pi_\mathrm{S} = \frac{8}{3}\frac{(d-4)(d-3)(d-2)}{d-1}\frac{\pi^2}{2^{\frac{1}{d-2}}}\frac{\Gamma\left(\frac{1}{d-2}\right)}{\left[\Gamma\left(\frac{d-1}{2d-4}\right)\right]^4}\sin\left(\frac{\pi}{2(d-2)}\right)$$

Conclusion

- The results obtained do converge to the uncorrected ones under the limit $\lambda' \to 0$.

- It would be interesting to compare these results with numerical data.

Thank you for your attention!

References

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