

Spanish Portuguese Relativity

Meeting 2021

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EREP 2021

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Contributed Talk

Linking: causality and black holes
and cosmic censorship of smooth
structures

Based on joint works with Stefan
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events are causally related if you
can get from one event to another
without exceeding light speed

A spacetime is globally hyperbolic
if ① it does not have time travel
(no closed causal curves)

② there are no naked singularities
and the intersection of causal past
and causal future of any two points
 $J^+(p) \cap J^-(q)$ is compact

Hawking Ellis + Geroch

Globally hyperbolic spacetimes are homeomorphic to $\Sigma \times \mathbb{R}$ where each $\Sigma \times t$ is the so called Cauchy surface

Bernal + Sanchez Globally hyperbolic spacetime is diffeomorphic to $\Sigma \times \mathbb{R}$

Assume X^4 is 4 dimensional and is homeomorphic to $\Sigma^3 \times \mathbb{R}$ with Σ closed & oriented

Chernov Nemirovski then X is diffeomorphic to $\Sigma^3 \times \mathbb{R}$ with the unique smooth structure on Σ^3

Proof uses Thurston Geometrization (Perelman) and the results of Turaev

So exotic smooth structures are irrelevant for general relativity
For Σ^3 - not closed or nonorientable we do not currently know how to prove this fact.

So we have cosmic censorship page 3
of smooth structures

Remark This is false in higher
dimensions but is true if X^{m+1}
is contractible

Motivated by ideas of Penrose
Low (in the topological context)
and Natarro-Tod in Legendrian
context proposed conjectures
that say that P, Q are causally
related \Leftrightarrow the spheres of light
rays through P, Q are
linked in the manifold of all
light rays (contact structure
was discovered by Low)

Chernov Nemrovski proved this
in all dimensions for all cases
except when the Cauchy surface

Σ is either having a page 4 noncompact universal cover \tilde{M} or the integer cohomology of M is not isomorphic to the one of a CROSS compact rank one symmetric space.

So pretty much always you can completely reconstruct causality from this Legendrian link or which is the same from the intersection of light cones through p & q with a Cauchy surface.

Remarks When $\Sigma \cong$ a CROSS then linking \leftrightarrow causality and this is related to refocussing spacetimes introduced by Low

To study black holes one can consider Legendrian links with more than two components on the contact manifold of all light rays. Two components are given by the event horizon of a black hole with two possible orientations, one pointing inside the other pointing outside, but since this is a black hole effectively both coorienting vector fields point inside of a black hole.

The third Legendrian knot is a small cooriented sphere inside of a black hole.

If a black hole has a nonspherical or self-intersecting event horizon this should still work but the link components are not spheres and instead of the third link

component you take one page such
Legendrian sphere for each region
inside the black hole

This is a very promising direction
because as it follows from
our work Chernov + Nemirovskii
and from the work of Liu
these link components are nonloop
(somewhat analogous to tight Legendrian knots)

So this is where interesting
Legendrian link theory happens

For the good introduction to
non loose Legendrian higher
dimensional submanifold see
the work of Borner, Eliashberg,
Murphy whose work introduced
this notion

