Null shells: general matching across null boundaries and matching across Killing horizons

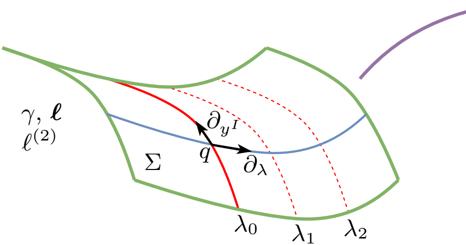
Miguel Manzano (joint work with Marc Mars)

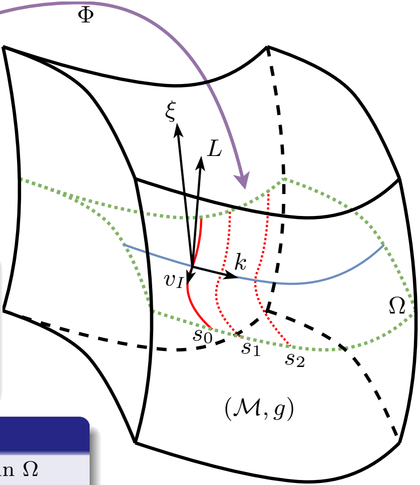
Fundamental Physics and Mathematics Institute
University of Salamanca

miguelmanzano06@usal.es
marc@usal.es

February 1, 2021

Overview





$(n+1)$ −dimensional manifold $(\mathcal{M}, g)$

- $(\mathcal{M}, g)$ time-oriented, Lorentzian $(-,+,\dots)$
- Ω null hypersurface: Φ, ξ, γ degenerate
- k null generator, geodesic

Spacelike section and foliation of Ω

Assumption: $\exists\ s : \Omega \to \mathbb{R}$ such that $k(s) \neq 0$ in Ω

Spacelike section: $S_{s_0} := \{p \in \Omega \mid s(p) = s_0,\ s_0 \in \mathbb{R}\}$

Tangent plane: $T_p S_{s(p)} := \{X \in T_p\Omega \mid X(s) = 0\}$

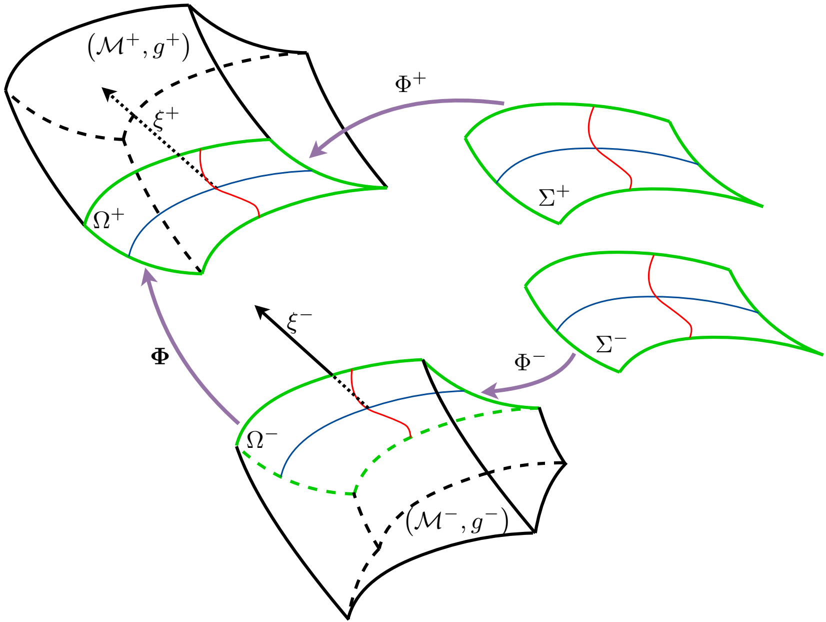
$\{S_s\}$ foliation of Ω; $h|_p$ induced metric of $S_{s(p)}$

$\varphi(p) := -\langle L, k\rangle_g\big|_p \neq 0, \qquad \psi_I(p) := -\langle L, v_I\rangle_g\big|_p$

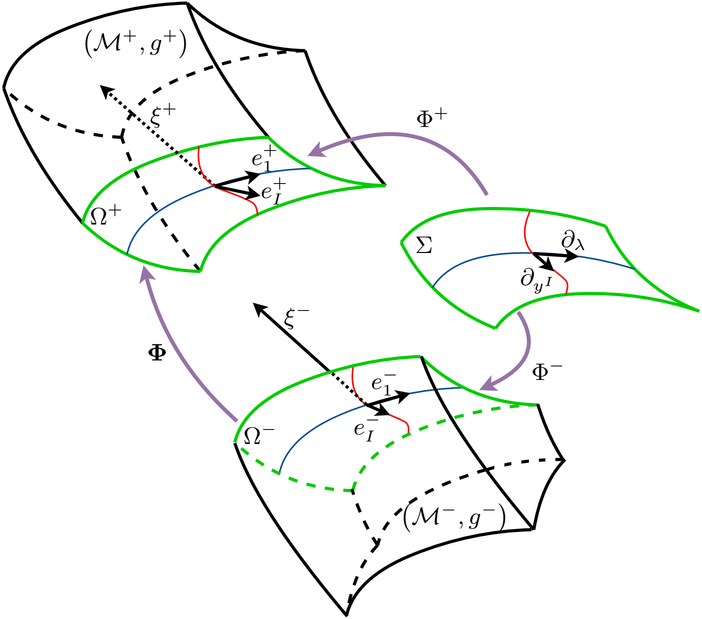
$\{L, k, v_I\}$ basis of $\Gamma(T\mathcal{M})|_\Omega$

- $k(s) = 1, \quad v_I|_p \in T_p S_{s(p)}$,
- $L \notin T_p\Omega\ \forall p \in \Omega$, null

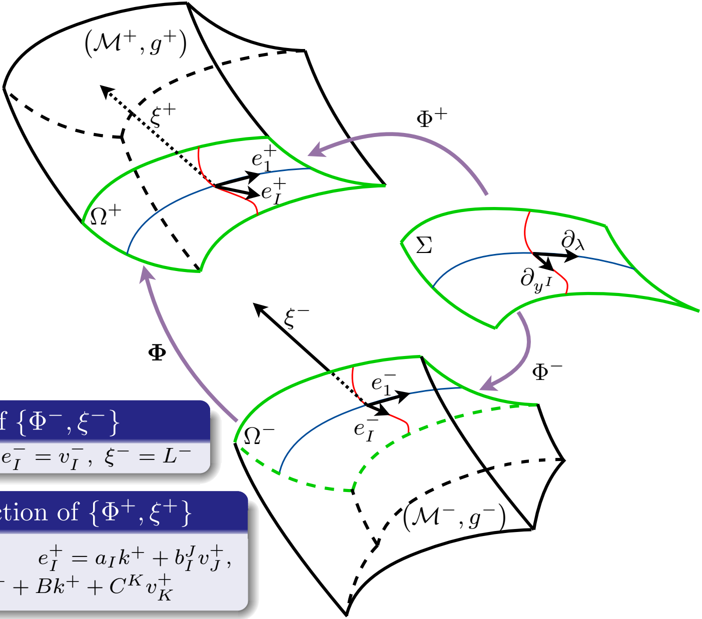
General matching of two spacetimes across their null boundaries



General matching of two spacetimes across their null boundaries



General matching of two spacetimes across their null boundaries



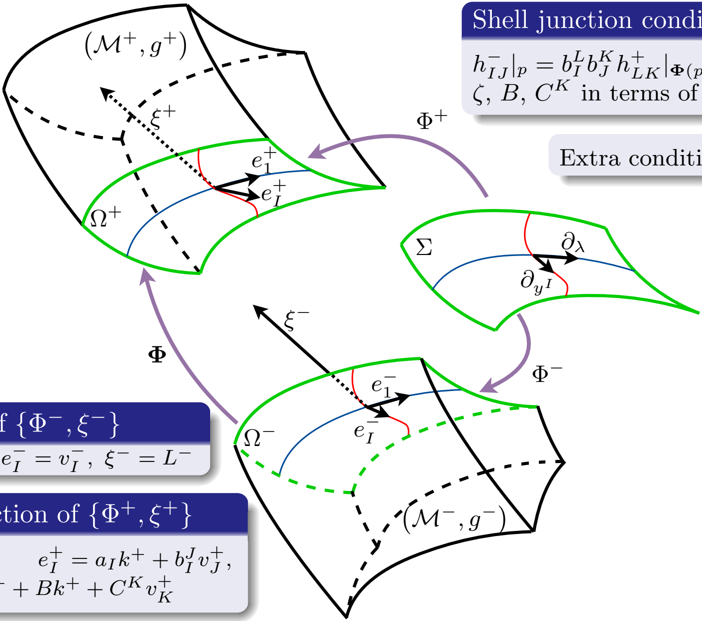
Choice of $\{\Phi^-, \xi^-\}$

$e_1^- = k^-, \; e_I^- = v_I^-, \; \xi^- = L^-$

Construction of $\{\Phi^+, \xi^+\}$

$e_1^+ = \zeta k^+, \qquad e_I^+ = a_I k^+ + b_I^J v_J^+,$
$\xi^+ = \frac{1}{A} L^+ + B k^+ + C^K v_K^+$

General matching of two spacetimes across their null boundaries



Shell junction conditions

$h^-_{IJ}|_p = b_I^L b_J^K h^+_{LK}|_{\boldsymbol{\Phi}(p)}$,
ζ, B, C^K in terms of A, a_I, b_I^J

Extra condition: $A > 0$

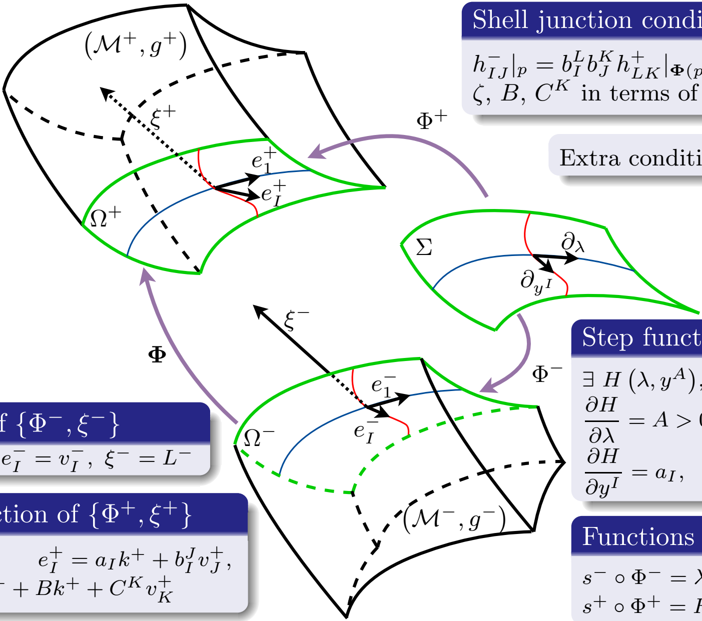
Choice of $\{\Phi^-, \xi^-\}$

$e_1^- = k^-$, $e_I^- = v_I^-$, $\xi^- = L^-$

Construction of $\{\Phi^+, \xi^+\}$

$e_1^+ = \zeta k^+$, $\quad e_I^+ = a_I k^+ + b_I^J v_J^+$,
$\xi^+ = \frac{1}{A} L^+ + B k^+ + C^K v_K^+$

General matching of two spacetimes across their null boundaries



Shell junction conditions

$h^-_{IJ}|_p = b_I^L b_J^K h^+_{LK}|_{\boldsymbol{\Phi}(p)}$,
ζ, B, C^K in terms of A, a_I, b_I^J

Extra condition: $A > 0$

Choice of $\{\Phi^-, \xi^-\}$

$e_1^- = k^-$, $e_I^- = v_I^-$, $\xi^- = L^-$

Construction of $\{\Phi^+, \xi^+\}$

$e_1^+ = \zeta k^+, \qquad e_I^+ = a_I k^+ + b_I^J v_J^+,$
$\xi^+ = \frac{1}{A} L^+ + B k^+ + C^K v_K^+$

Step function H

$\exists\ H(\lambda, y^A),\ h^I(y^A)$:
$$\frac{\partial H}{\partial \lambda} = A > 0,$$
$$\frac{\partial H}{\partial y^I} = a_I, \quad b_I^K = \frac{\partial h^K}{\partial y^I}$$

Functions $s^\pm$

$s^- \circ \Phi^- = \lambda + \text{const.}$
$s^+ \circ \Phi^+ = H + \text{const.}$

Matching across Killing horizons: Killing vectors identified

Previous considerations

- $\mathscr{H}_\eta^\pm$ Killing horizons with respect to the Killing vector fields $\eta^\pm$
- Requirement: $\Omega^\pm := \overline{\mathscr{H}}_\eta^\pm$ are smooth connected (null) hypersurfaces
- Assumption: $\kappa_\eta^\pm$ constant on $\Omega^\pm$ (with full generality, we take $\kappa_\eta^\pm \geq 0$)

Matching: Killings $\eta^\pm$ identified

- $\eta^\pm \overset{\Omega^\pm}{=} F^\pm k^\pm, \qquad \text{where} \qquad F^\pm \overset{\Omega^\pm}{=} f^\pm + \kappa_\eta^\pm s^\pm, \qquad k^\pm\left(f^\pm\right) \overset{\Omega^\pm}{=} 0,$
- The map $\mathbf{\Phi}$ relates $\eta^\pm \quad \Longrightarrow \quad \mathrm{d}\mathbf{\Phi}\left(\eta^-\right) \overset{\Omega^+}{=} a\eta^+, \quad a \neq 0, \quad a \in \mathbb{R}$

$$F^- \overset{\Omega^\pm}{=} \frac{aF^+}{\partial_\lambda H} \quad \Longrightarrow \quad f^- + \kappa_\eta^- \lambda \overset{\Omega^+}{=} \frac{a\left(f^+ + \kappa_\eta^+ H\right)}{\partial_\lambda H}$$

- First consequence: zeroes of $\eta^\pm$ must be mapped to each other via $\mathbf{\Phi}$
- Out of the zeroes of $\eta^\pm$, this yields a 1^{st}−order PDE for H (Cond. $\partial_\lambda H > 0$)
- We separately study (a) $\kappa_\eta^\pm = 0$, (b) $\kappa_\eta^\pm \neq 0$
- $\eta^\pm$ degenerate: connected zero-sets (if any) are null subsets defined by $f^\pm = 0$
- $\eta^\pm$ non-degenerate: one unique connected zero-set defined by $f^\pm + \kappa_\eta^\pm s^\pm = 0$

Matching across Killing horizons: Killing vectors identified

$\eta^{\pm}$ degenerate

$$f^{-} \overset{\Omega^{+}}{=} \frac{af^{+}}{\partial_{\lambda} H} \quad \Longrightarrow \quad H(\lambda, y^{A}) \overset{\Omega^{+}}{=} \beta(y^{A})\lambda + \mathcal{H}(y^{A}), \quad \beta(y^{A}) := \frac{af^{+}}{f^{-}} > 0$$

- Condition: same number of connected components of zero-sets on both sides
- Since $\mathbf{\Phi}$ is continuous, the zero-sets of $\eta^{\pm}$ cannot be arbitrarily identified
- Killing vectors to be identified must be either future or past

$\eta^{\pm}$ non-degenerate (Assumption: both Killing horizons are complete)

$$H(\lambda, y^{A}) \overset{\Omega^{+}}{=} = \frac{1}{a\kappa_{\eta}^{+}} \left(\alpha(y^{A}) \left(f^{-}(y^{A}) + \kappa_{\eta}^{-}\lambda \right)^{\hat{\kappa}} - af^{+}(y^{A}) \right), \quad \hat{\kappa} := \frac{a\kappa_{\eta}^{+}}{\kappa_{\eta}^{-}} \neq 0,$$

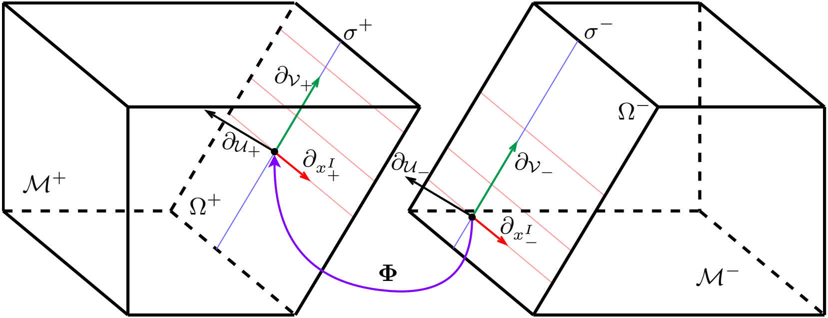
- The bifurcation surfaces must be mapped to each other
- $\alpha(y^{A}) \neq 0$ arbitrary, $\operatorname{sign}(\alpha(y^{A}))$ must be the same for all null generators
- Full matching requires $\hat{\kappa} = 1$, i.e. surface gravities of η^{-}, $a\eta^{+}$ must coincide

Energy-momentum tensor: $\tau^{11} = \rho, \quad \tau^{1I} = j^{I}, \quad \tau^{IJ} = \gamma^{IJ} p$ [e.g. Poisson, 2004]

- The identification of Killings always results in shells with vanishing pressure p

Explicit examples: plane-fronted impulsive wave

- Regions $\{\mathcal{U}_+ > 0\}$, $\{\mathcal{U}_- < 0\}$ of Minkowski spacetime
- Metrics: $\mathrm{d}s^2_\pm = -2\mathrm{d}\mathcal{V}_\pm \mathrm{d}\mathcal{U}_\pm + \delta_{AB} dx^A_\pm dx^B_\pm$, Foliation defining functions: $s^\pm = \mathcal{V}_\pm$
- General shell: $H\left(\lambda, y^A\right) = \alpha\left(y^A\right) \int \exp\left(-\int p\left(\lambda, y^A\right) \mathrm{d}\lambda\right) \mathrm{d}\lambda + \mathcal{H}\left(y^A\right)$, $\alpha\left(y^A\right) > 0$



Matching identifying Killings: $\eta^- \longleftrightarrow a\eta^+$

(a) Degenerate: $\eta^\pm \overset{\Omega^\pm}{=} \partial_{\mathcal{V}_\pm} \Longrightarrow H\left(\lambda, y^A\right) \overset{\Omega^+}{=} a\lambda + \mathcal{H}(y^A)$, $a > 0$

(b) Non-degenerate: $\eta^\pm \overset{\Omega^\pm}{=} -\mathcal{V}_\pm \partial_{\mathcal{V}_\pm} \Longrightarrow H\left(\lambda, y^A\right) \overset{\Omega^+}{=} \alpha(y^A)\lambda$, $a = 1$, $\alpha(y^A) > 0$

Energy-momentum tensor: (a) $\rho \neq 0$, $j^I = 0$, $p = 0$; || (b) $\rho \neq 0$, $j^I \neq 0$, $p = 0$