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TÉCNICO LISBOA

Collapse of axisymmetric Gravitational Waves in vacuum

Isabel Suárez Fernández

In collaboration with:
Daniela Cors Agulló, Sarah Renkhoff, Prof. David Hilditch & Prof. Bernd Brügmann

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FCT
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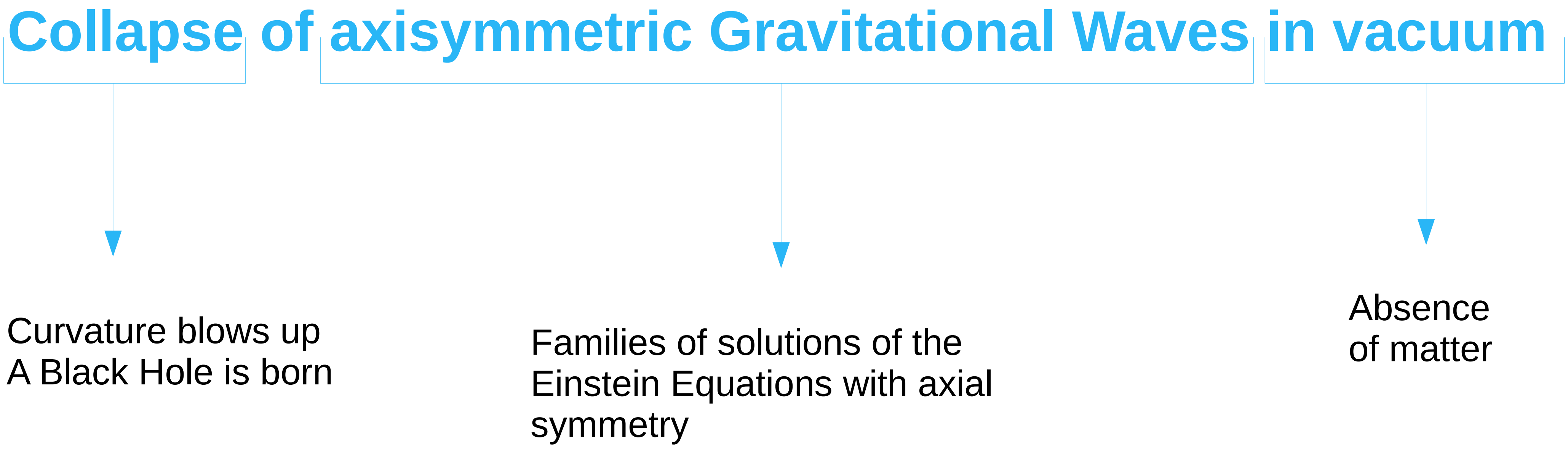


Collapse of axisymmetric Gravitational Waves in vacuum

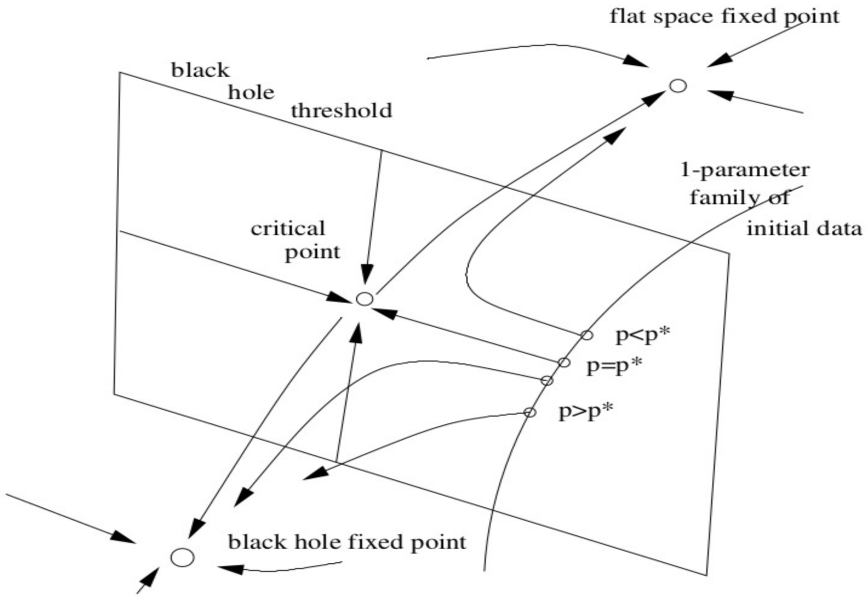
Curvature blows up
A Black Hole is born

Families of solutions of the Einstein Equations with axial symmetry

Absence
of matter

Critical phenomena

What are Critical Phenomena?



- Universality
- Self-similar behavior
- Power law behavior near the threshold

[1] Gundlach, C., Martín-García, J.M. Critical Phenomena in Gravitational Collapse. Living Rev. Relativ. 10, 5 (2007)
[2] M. Choptuik. Universality and scaling in gravitational collapse of a massless scalar field Phys. Rev. Lett. 70, 9. (1993)

Roadmap of the talk

- Axisymmetric Gravitational Waves as Initial Data → Brill Waves

- Evolution of different families with the pseudospectral code **BAMPS**
  - Use of Adaptive Mesh Refinement (AMR)

- Bisection search
  - Apparent horizon finder, **Ahloc3d**

- Study of critical phenomena
  - Scaling of the Kretsmann scalar, topology of the Apparent Horizon, lapse collapse...

Initial data → Brill Waves

Non-linear vacuum solution to Einstein's constraint equations → solving elliptic equation

Arbitrary seed function q(ρ,z) → typically a Gaussian profile

Axisymmetric Brill Waves

$$dl^2 = \Psi^4 \left[e^{2q}(d\rho^2 + dz^2) + \rho^2 d\phi^2\right]$$

$$q(\rho, z) = A\rho^2 e^{-[(\rho-\rho_0)^2/\sigma_\rho^2 + (z-z_0)^2/\sigma_z^2]}$$

Families

$z_0 = 0$

$\sigma_z = 1$

$\sigma_\rho = 1$

$\rho_0 = 0, 4, 5$

$A > 0 \longrightarrow$ Prolate

$A < 0 \longrightarrow$ Oblate

Evolution with BAMPS

- Pseudospectral code
- Adaptive Mesh Refinement (AMR)

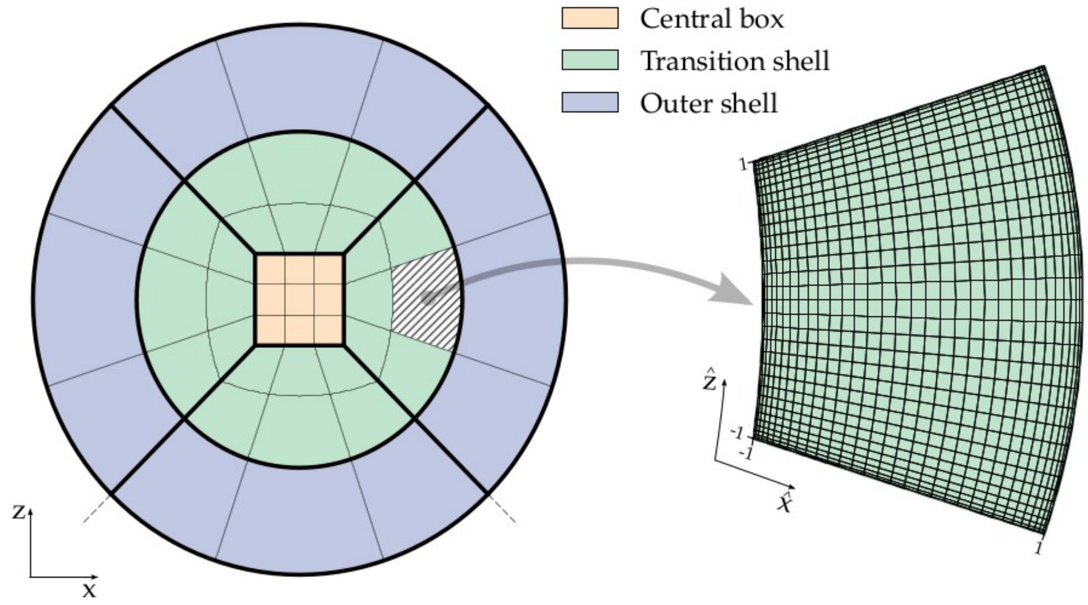
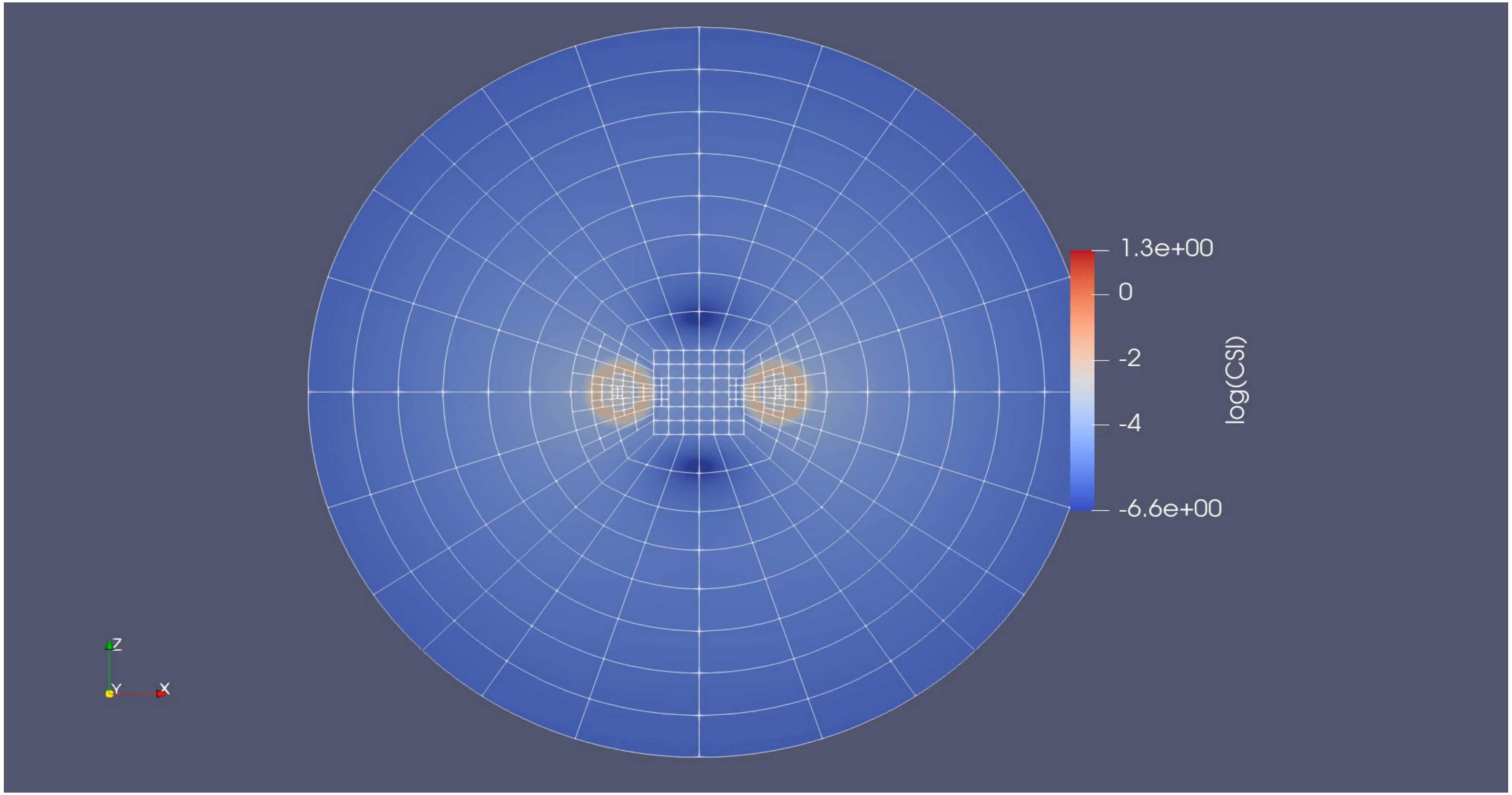


FIG. 1: The left part of the diagram gives a two dimensional sketch of the `bamps` cubedball grid layout. The ball is built up of several transformed cubes. These patches can further be divided in subpatches. In the example shown we have $\mathcal{N}_{\mathrm{cu}} = 3, \mathcal{N}_{\mathrm{cs}} = 2$ and $\mathcal{N}_{\mathrm{ss}} = 1$. On the right is shown that each subpatch is covered by Gauss-Lobatto grids ranging from -1 to 1 in local coordinates.

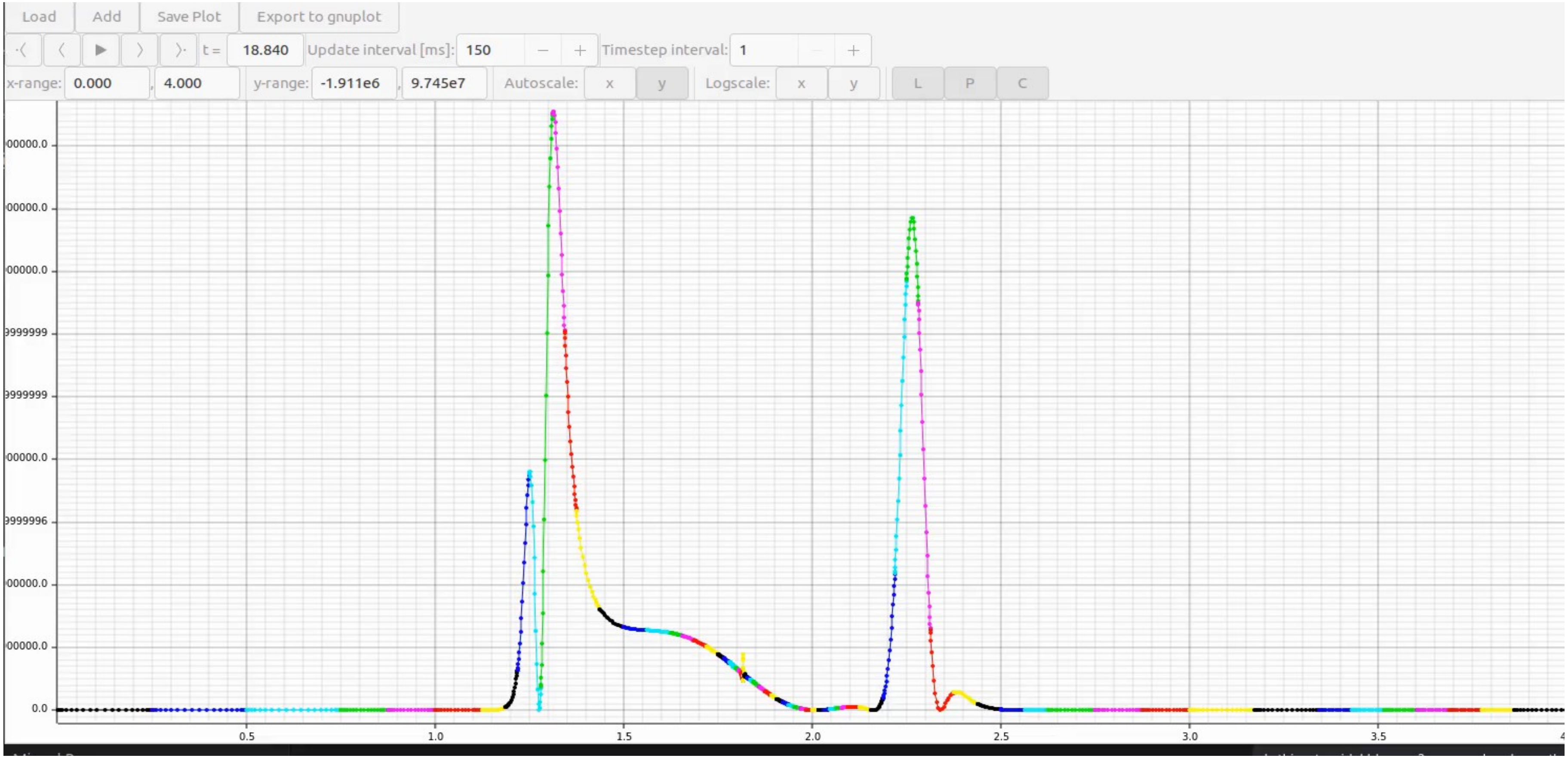
[3] Hilditch, Weyhausen, Brügmann. Pseudospectral method for gravitational wave collapse. Phys. Rev. D93 (2016)

Adaptive Mesh Refinement (AMR)



$\rho_0 = 5, A = 0.12$

Bisection search: Subcriticality $\rho_0 = 0, A = 4.69667$

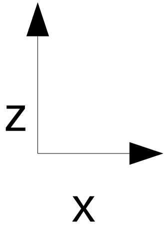


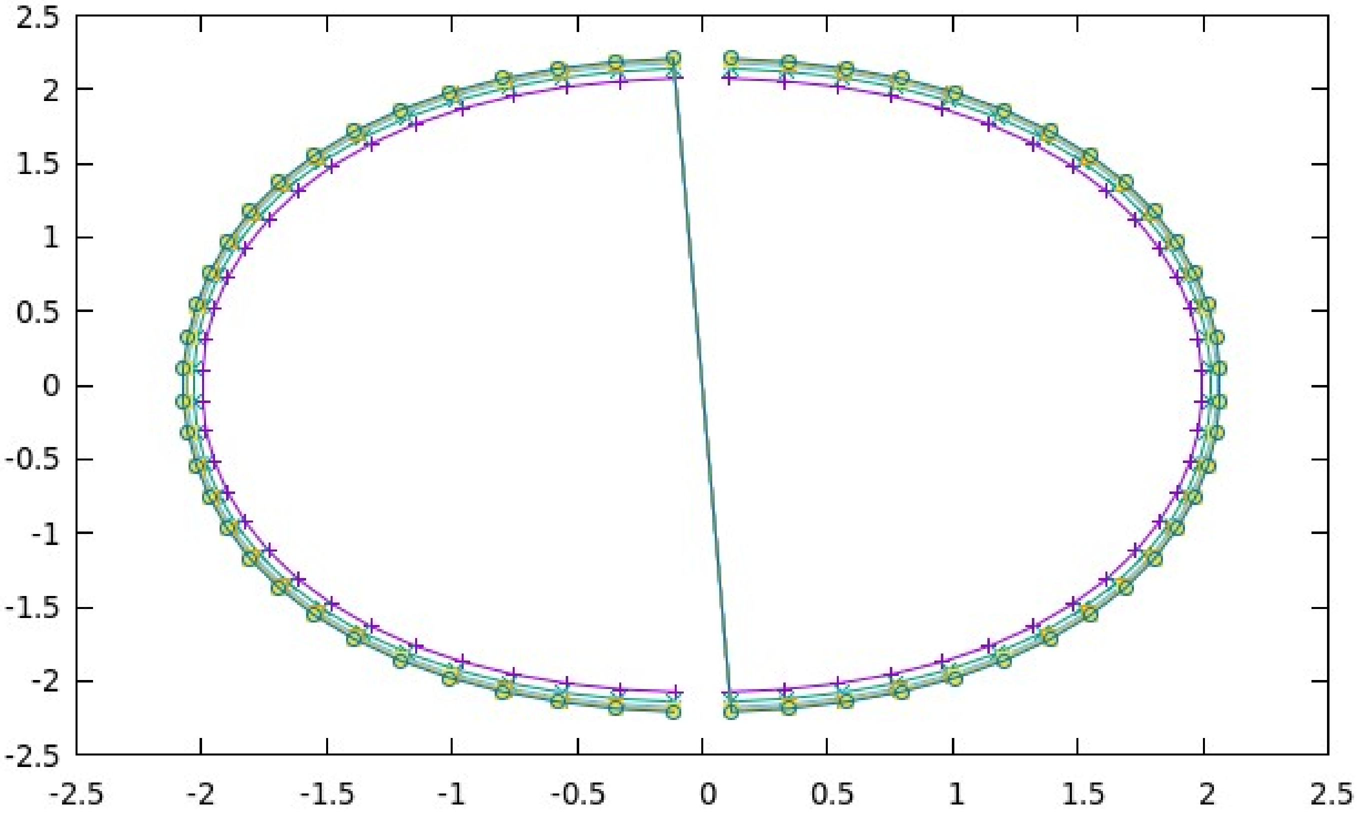
t = 18.840 Update interval [ms]: 150 Timestep interval: 1

x-range: 0.000, 4.000 y-range: -1.911e6, 9.745e7

[4] Sarah Renkhoff, Muninn, (2020) git repository https://git.tpi.uni-jena.de/srenkhoff/muninn

Bisection search: Supercriticality





Ahloc3d → Apparent horizon finder

$$\rho_0 = 0, A = -4.4$$

[5] Sarah Renkhoff, Ahloc3d, (2020) git repository https://git.tpi.uni-jena.de/srenkhoff/ahloc3d

Study of Critical Phenomena: Context

[6] Evolutions of centered Brill waves with a pseudospectral method, David Hilditch, Andreas Weyhausen, and Bernd Brügmann, Phys. Rev. D 96, 104051

Evolution of one family of **centered, prolate brill waves** with BAMPS, and no AMR

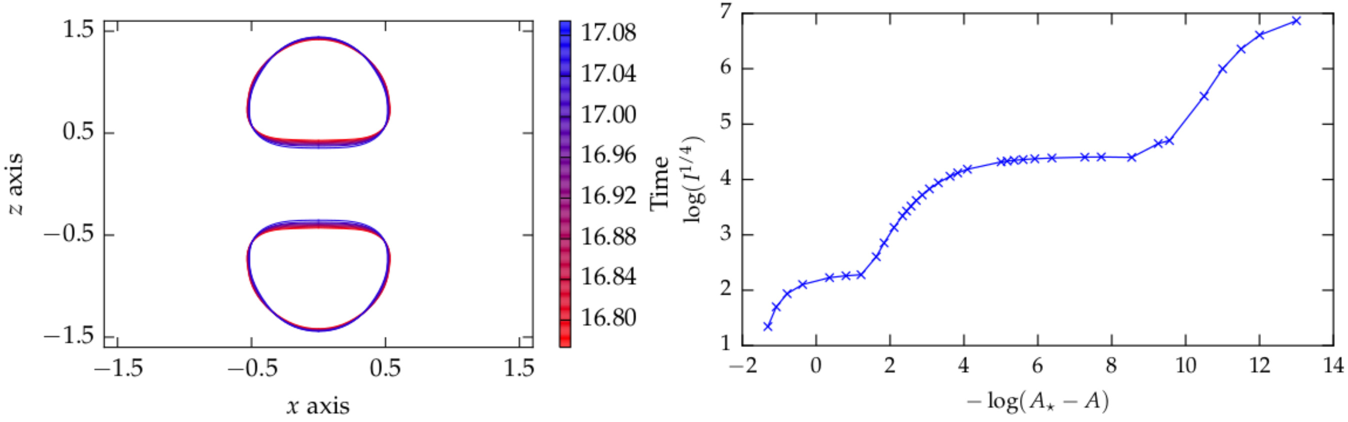


FIG. 4. In the left panel the apparent horizons at different times in $A = 4.698$ centered Brill wave initial data, as obtained in sweep 3 with $\Delta A = (0.1)^3$, are plotted. Evidently two apparent horizons appear in the data, each around the observed peaks in the Kretschmann scalar at $z = \pm z_{\text{peak}}$, indicating the likelihood that the family results in head-on binary black hole spacetime near the critical amplitude. This behavior is robust in that in weaker supercritical data that we can successfully classify, we always find such horizons. In the right hand panel we plot the logarithm of the absolute value of the Kretschmann scalar against $-\log(A_\star - A)$ taking $A_\star = 4.6966953125$ as the critical amplitude. The result can be well-fitted by a straight-line with gradient ~ 0.37 plus a function of period ~ 8. This is indicative of critical behavior [41], but since we see only one full period, starting from around $-\log(A_\star - A) = 2$, we do not consider the result conclusive.

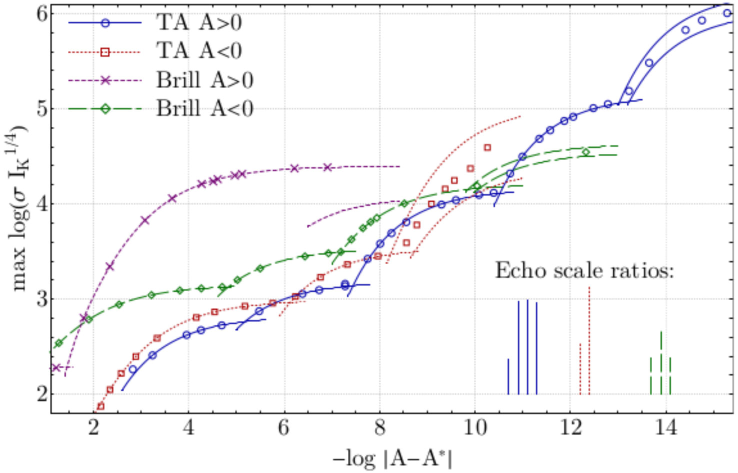
Study of Critical Phenomena: Context

[7] Universality of curvature invariants in critical vacuum gravitational collapse, Tomáš Ledvinka, Anton Khirnov, arXiv:2102.09579

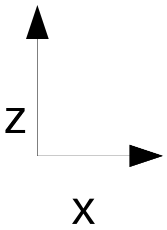
Evolution of different families of axisymmetric GW → Only **centered** families

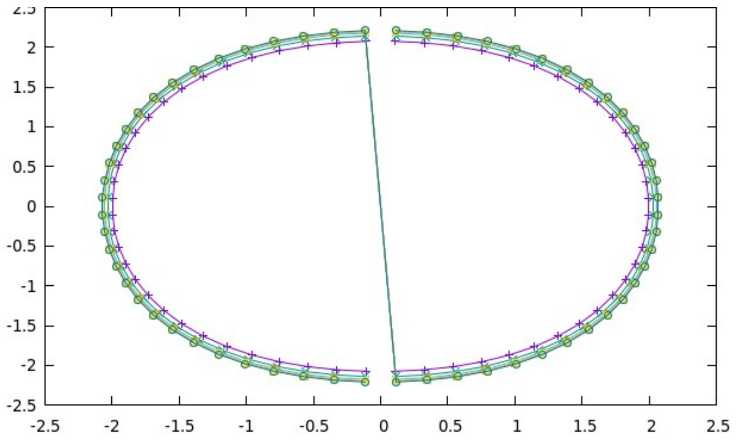
“For different families of initial data we observe **universal ‘echoes’** in the form of irregularly repeating approximate scaled copies of the same piece of spacetime.”

“We did not observe a universal and regularly self-similar solution in $A \rightarrow A_*$ limit and we find the **dimensionless characteristics** of near-critical behavior [...], **depend on the ID family**.”

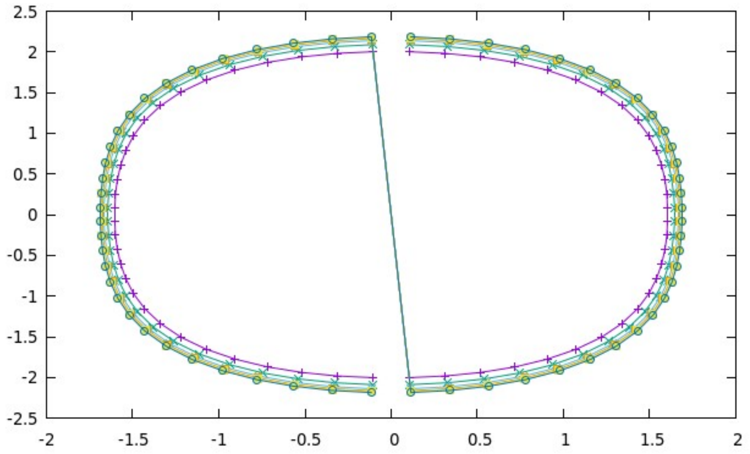


Results: Oblate $\rho_0 = 0$

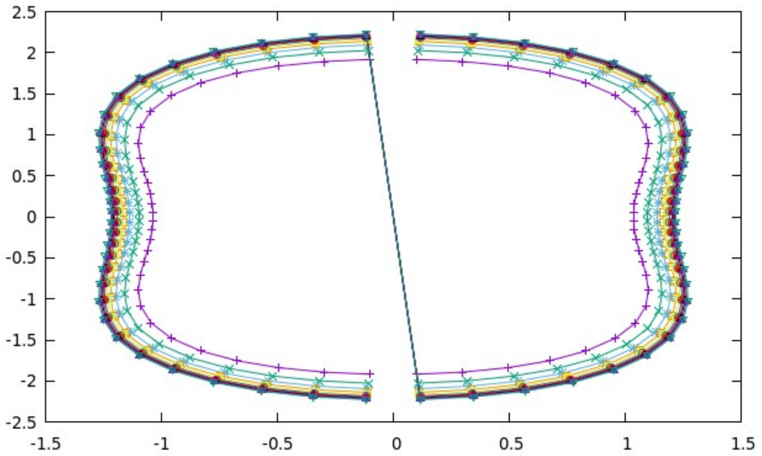




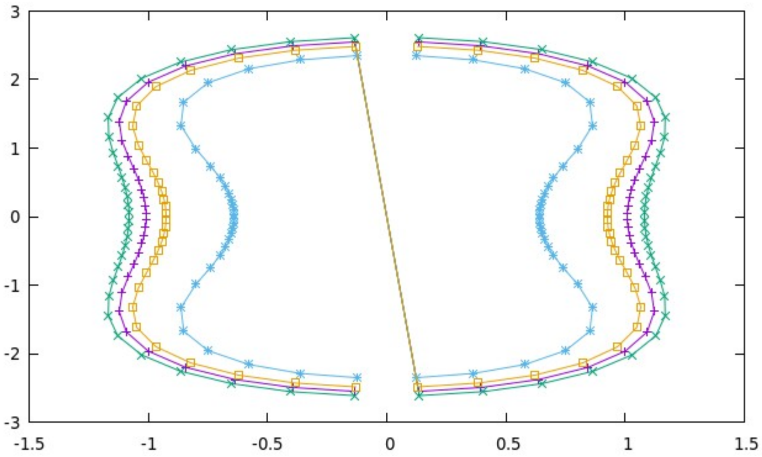
A = -4.4



A = -4.3

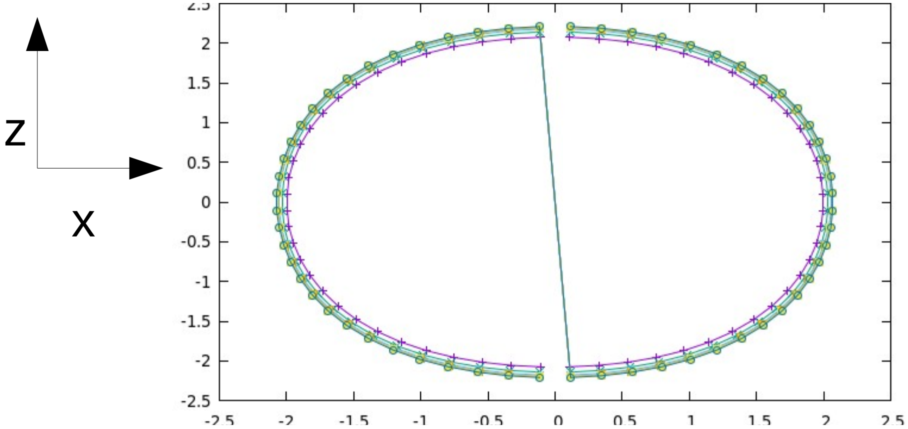


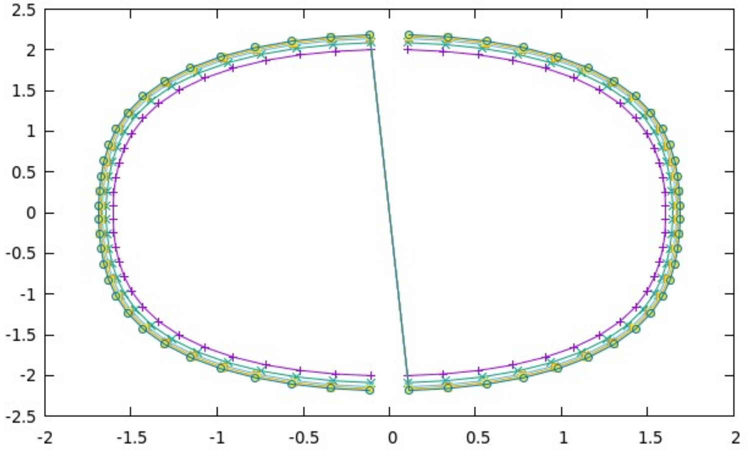
A = -4.2



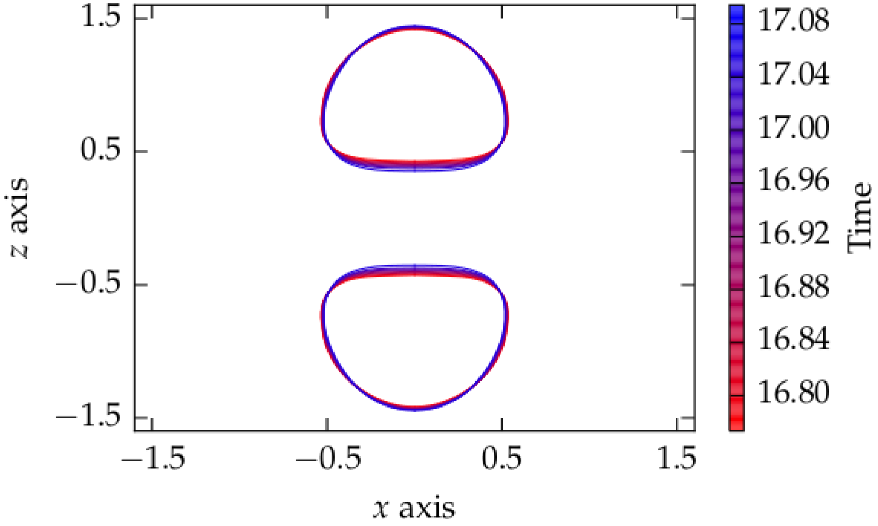
A = -4.1

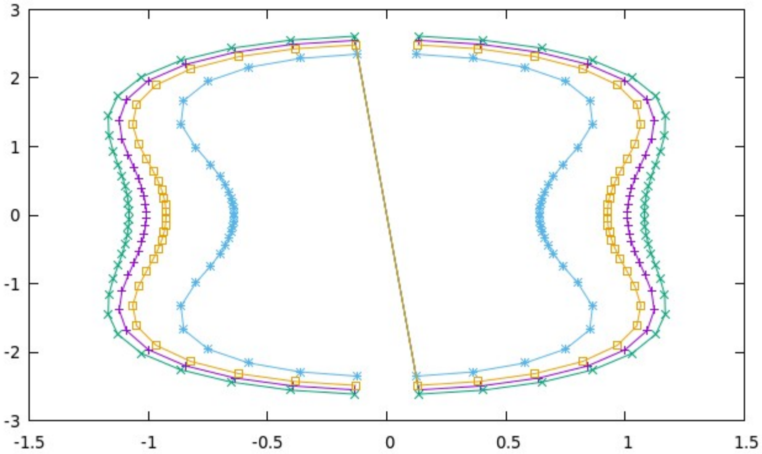
Results: Oblate $\rho_0 = 0$





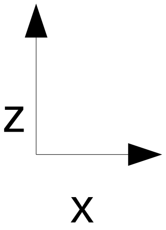
A = -4.3

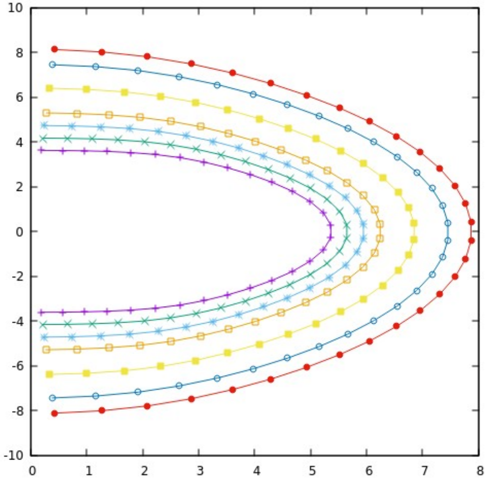




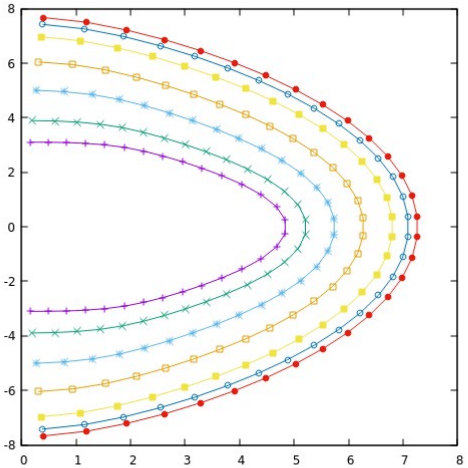
A = -4.1

Results Prolate $\rho_0 = 4$

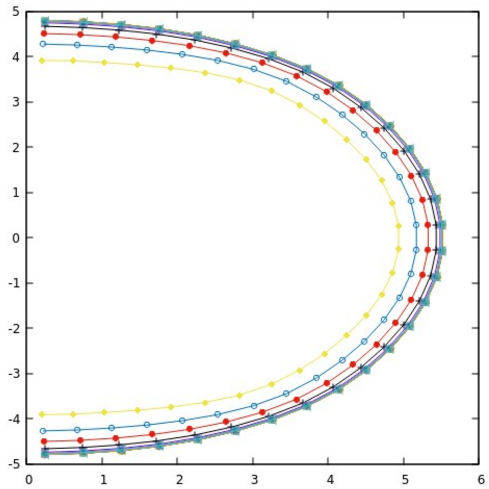




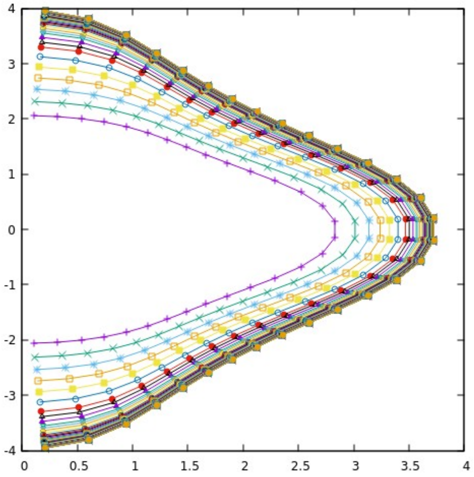
A = 0.15



A = 0.14

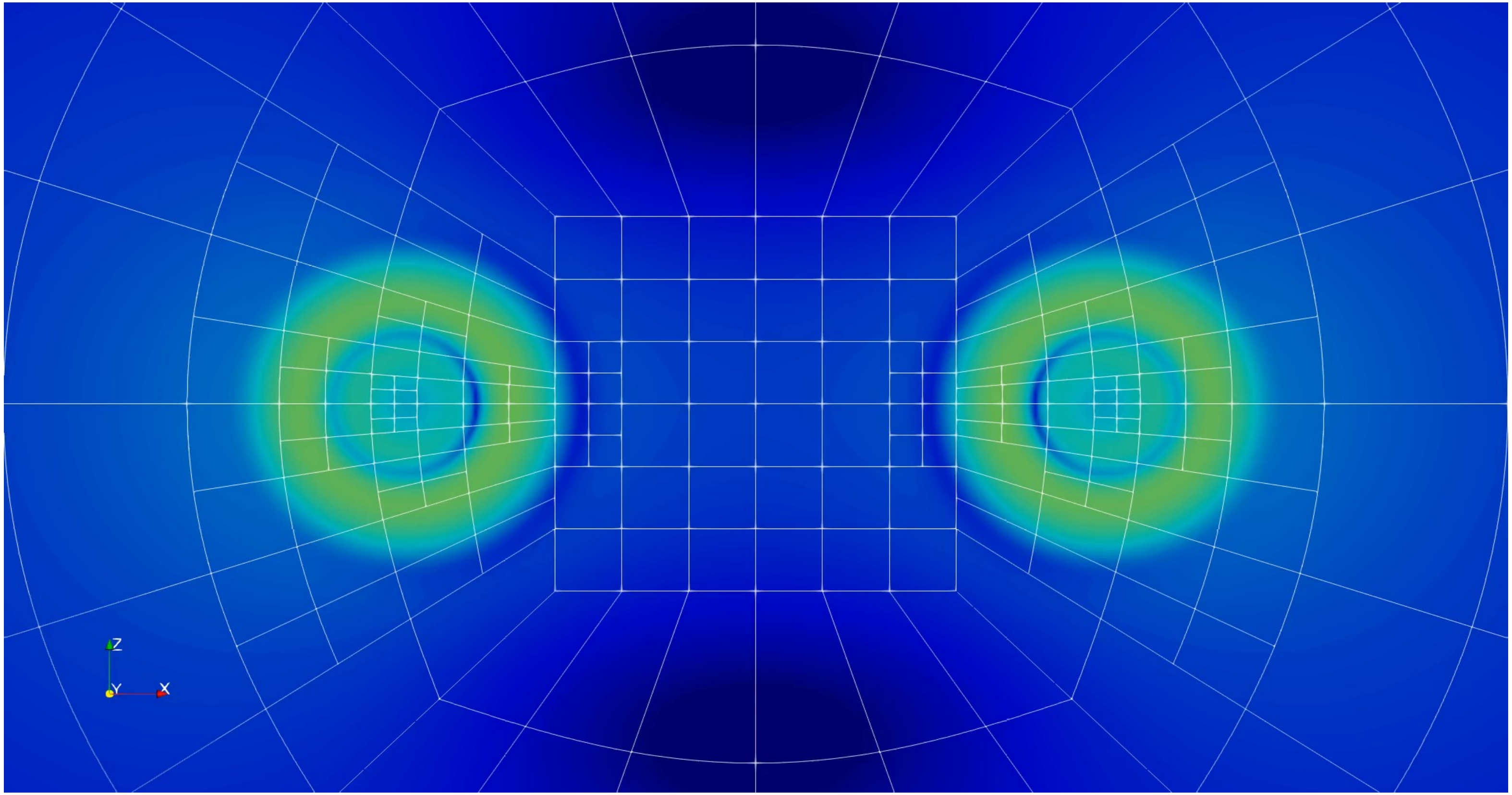


A = 0.12



A = 0.11

Results



$\rho_0 = 5, A = 0.12$

Conclusions

- We were able to reproduce previous results [6] with prolate, centered Brill Waves and we are already pushing forward.
- Less computational resources due to the use of AMR.

	ρ0	**Amplitude**	
		Subcritical	**Supercritical**
Prolate	0	4.6966875	4.696703125
	4	0.096	0.11
	5	0.063	0.081
Oblate	0	-3.50	-3.51
	4	-0.075	-0.083
	5	-0.048	-0.055

- Hints to new topologies, in agreement with [8].
- Looking forward to create and extend the scaling Kretsman scalar plot.

[8] I. Suárez Fernández, R. Vicente and D.Hilditch. Phys. Rev. D 103, 044016 (2021)

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Thank you

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