Hyperboloidal massless scalar field in 3D

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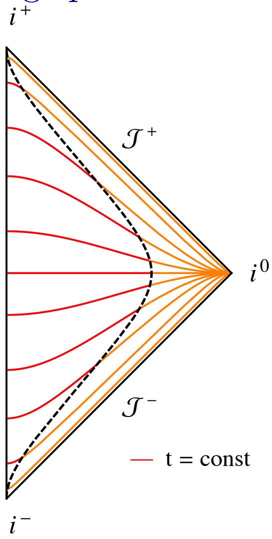
Slicing spacetime to reach future lightlike infinity



Standard slicing options for the initial value formulation of the Einstein equations, to solve them as an evolution in time:

- Standard Cauchy slices

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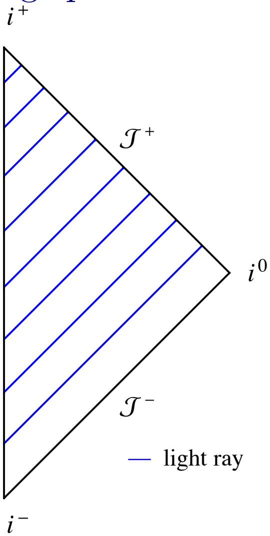

Standard slicing options for the initial value formulation of the Einstein equations, to solve them as an evolution in time:

- Standard Cauchy slices
- Null slices

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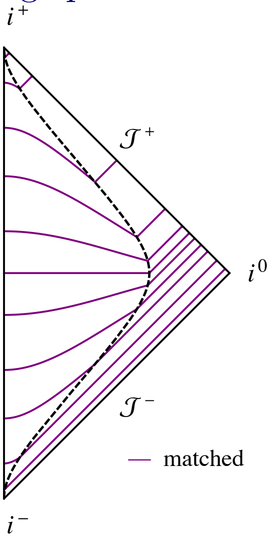

Standard slicing options for the initial value formulation of the Einstein equations, to solve them as an evolution in time:

- Standard Cauchy slices
- Null slices
- Cauchy-Characteristic matching / extraction

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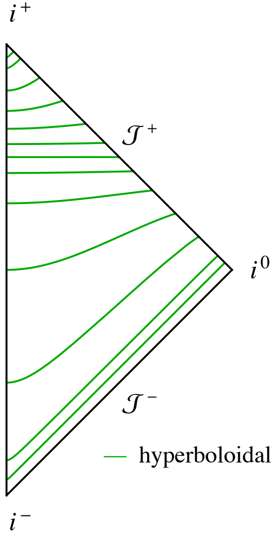

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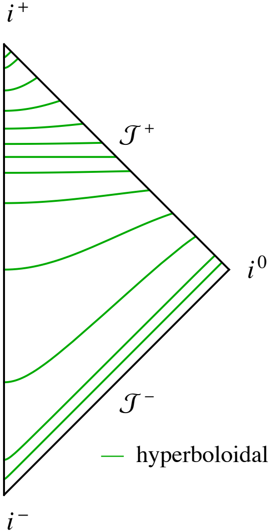

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- Cauchy-Characteristic matching / extraction
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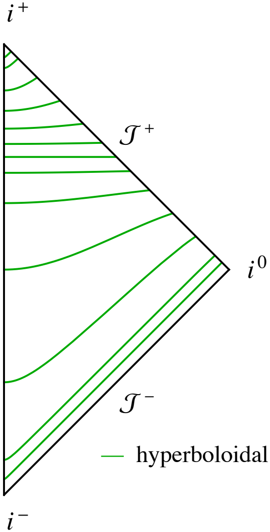
Advantages of the hyperboloidal approach:

- Extraction at $\mathscr{I}^+$, no approximations.
- Slices spacelike & smooth everywhere.
- More resolution for the central part.

Slicing spacetime to reach future lightlike infinity



Future null infinity ($\mathscr{I}^+$) is a region of spacetime of interest

- for the study of global properties of spacetimes and
- for the extraction of gravitational waves (only well described at $\mathscr{I}^+$, where observers are located).

A possible approach: Penrose's conformal compactification of spacetime.
The physical metric $\tilde{g}_{\mu\nu}$ is rescaled

$$g_{\mu\nu} \equiv \Omega^2 \tilde{g}_{\mu\nu}, \qquad (1)$$

with $\Omega|_{\mathscr{I}^+} = 0$ to keep $g_{\mu\nu}$ finite there.

Equations for the conformal metric

The Einstein equations including cosmological constant are

$$G[\tilde{g}]_{ab} + \tilde{g}_{ab}\Lambda = 8\pi T[\tilde{g}]_{ab}.$$

Expressing them in terms of the rescaled metric $g_{ab} = \Omega^2 \tilde{g}_{ab}$ gives

$$G[g]_{ab} + \frac{2}{\Omega}(\nabla_a\nabla_b\Omega - g_{ab}\Box\Omega) + \frac{3}{\Omega^2}g_{ab}(\nabla_c\Omega)(\nabla^c\Omega) + \frac{1}{\Omega^2}g_{ab}\Lambda = 8\pi T[\tfrac{g}{\Omega^2}]_{ab}.$$

Extra formally divergent terms at $\mathscr{I}$ appear in the equations.

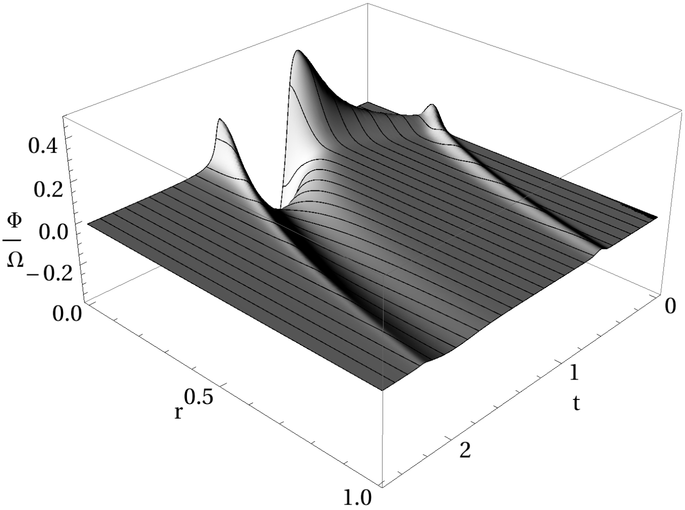
Evaluating the previous expression multiplied by Ω^2 at $\mathscr{I}$ yields

$$(\nabla_c\Omega)(\nabla^c\Omega)\big|_{\mathscr{I}} = -\Lambda,$$

so that

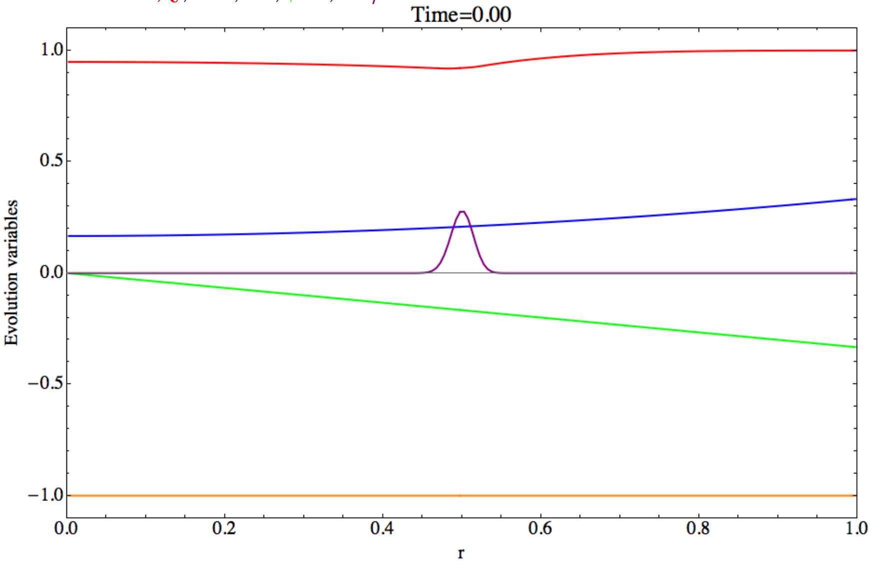
- $\Lambda = 0$: $\mathscr{I}$ is null,
- $\Lambda > 0$: $\mathscr{I}$ is spacelike,
- $\Lambda < 0$: $\mathscr{I}$ is timelike.

Scalar field

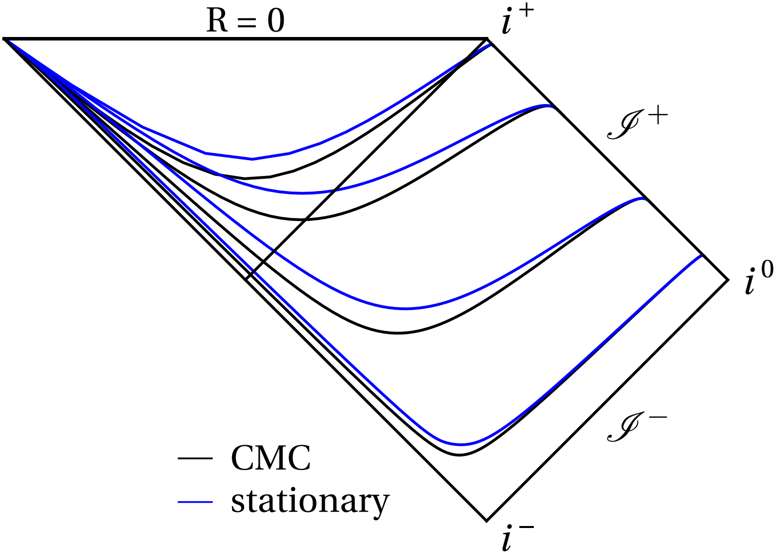


Vañó-Viñuales, Husa and Hilditch. *Class. Quant. Grav.* 32.17 (2015).

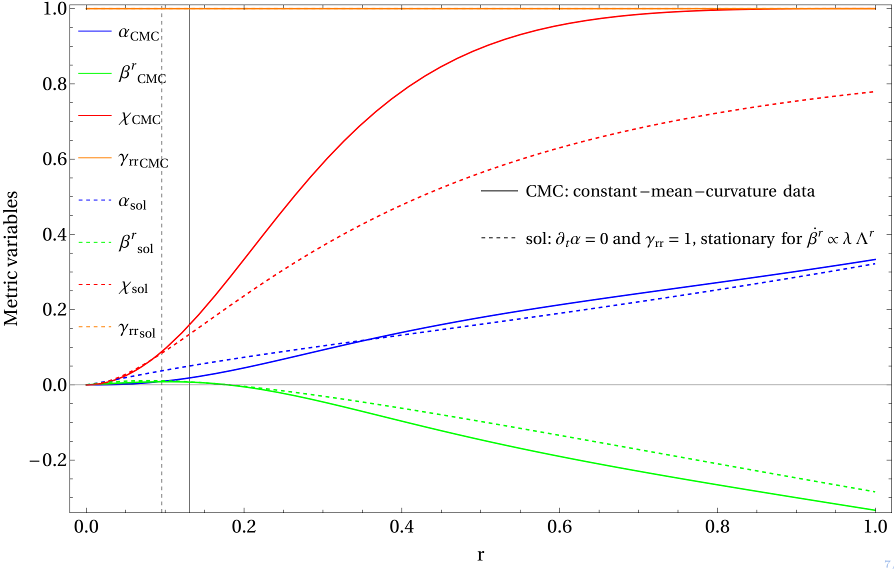
Evolution: χ, $\tilde{K}$, α, β^r, Φ/Ω x



Trumpet dynamics



Stationary initial data



Wave equation as toy model

Einstein equations are complicated and non-linear, start with simpler models:

This approach

- Non-conformally invariant:

$$\tilde{\Box}\tilde{\Phi} = 0 \qquad \rightarrow$$

$$\Box\Phi + \Phi\left(\frac{\Box\Omega}{\Omega^2} - \frac{2\nabla_a\Omega\nabla^a\Omega}{\Omega}\right) = 0$$

- 2nd order in space
- no matching on slice
- only Minkowski background, Schwarzschild trumpet planned
- by default use spherical coords

LlamaWaveHyperboloidal*

- Conformally invariant ($\tilde{\Phi} = \Omega\Phi$):

$$(\tilde{\Box} - \tilde{R}/6)\tilde{\Phi} = 0 \quad \rightarrow$$

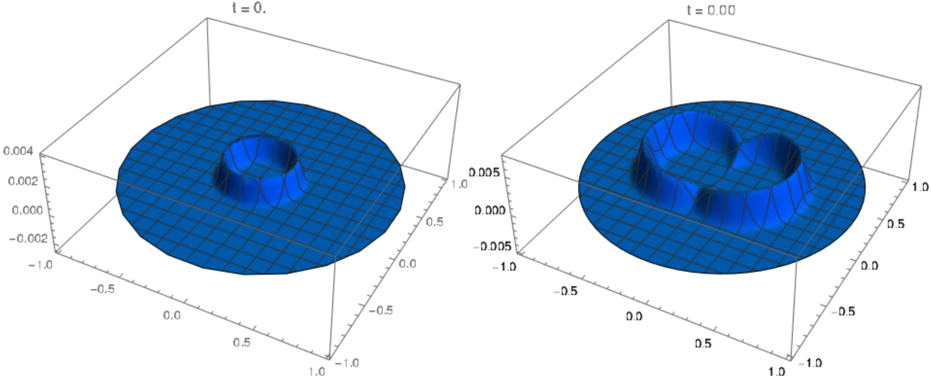
$$\left(\Box - \frac{R}{6}\right)\Phi = 0$$

- 1st order, flux-conservative
- matching: inner Cauchy slice and outer hyperboloidal one
- up to Kerr background
- Cartesian coords matched

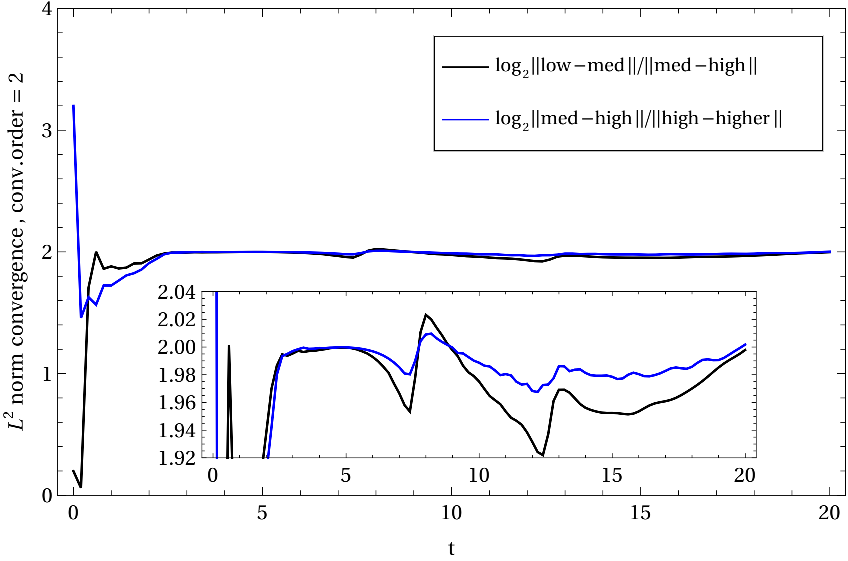
*Thorn for the Einstein Toolkit code: Jasiulek. *Class. Quant. Grav.* 29 (2012).

Massless scalar field equation

Evolution of the massless wave equation on a hyperboloidal slice of Minkowski spacetime using curvilinear coordinates in NRPy+ l r.



Convergence: axisymmetric initial data and evolution



Implementation into the Einstein Toolkit

Implementation as Einstein Toolkit thorn:

- The Einstein Toolkit is an open-source modular code optimised for numerical relativity with a large community of users.
- Follow LlamaWaveHyperboloidal thorn: interior Cartesian coordinates matched to outer spherical shell.
- Wave equation suitable for spherical coordinates everywhere, but infrastructure better suited to Cartesian ones.

Future tests in 3D (NRPy+/Einstein Toolkit):

- Schwarzschild and Kerr trumpet backgrounds for the scalar field.
- Implement Good-Bad-Ugly (GBU) model (Gasperin et al. *Class. Quant. Grav.* 37.3 (2020)), semilinear model of the Einstein eqs.
- Linerised Einstein equations before non-linear implementation.

Summary

- Successful spherically symmetric results of the hyperboloidal initial value problem → start long-awaited 3D implementation.
- Current 3D implementation of the massless wave equation on a regular hyperboloidal background used as toy model for GR.
- More numerical experiments (other hyperboloidal backgrounds and models) before attempting the full Einstein equations.

Thank you for your attention!

Questions?