Regular black holes in three dimensions

Pablo Bueno

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1. Introduction

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- It shares many of the properties of higher-D black holes, including thermodynamics, holography, etc.

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- Einstein gravity plus non-linear electrodynamics. [Cataldo, Garcia; Myung, Kim, Park;...] Examples of singularity-free black holes for special choices of the modified Maxwell Lagrangian. [Cataldo, Garcia; He, Ma; Mazharimousavi, Halilsoy, Tahamtan]

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A similar setup will be the one for our new $D = 3$ black holes...

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- ◇ They admit static and spherically symmetric solutions with $g_{tt}g_{rr} = -1$, *i.e.*,

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Their properties have been studied in ~ 80 papers with many interesting results...
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- Where the function $f(r)$ again satisfies an equation which may be algebraic (Quasi-topological) or differential of order 2 (Generalized Quasi-topological).
- Can be alternatively described in dual frame, in which $f(r)$ satisfies the same equation and $F^{\text{elec.}} \sim \Phi'(r)\mathrm{d}t\wedge\mathrm{d}r$.

2. Electromagnetic Quasi-topological gravities in three dimensions

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- In general, there exists a dual "electric" frame

$$\mathcal{L}^{\mathrm{EMQT}}[g^{ab}, R_{ab}, \partial_a \phi] \quad \Longleftrightarrow \quad \mathcal{L}^{\mathrm{EMQT}}_{\mathrm{dual}}[g^{ab}, R_{ab}, F_{ab}]\,,$$

with solutions $F \propto \mathrm{d}t \wedge \mathrm{d}r$.

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EM-QT gravities in $D = 3$

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$$I_{\text{EMQT}} = \frac{1}{16\pi G}\int \mathrm{d}^3x\sqrt{|g|}\left[R + \frac{2}{L^2} - \mathcal{Q}\right],$$

where

$$\boxed{\begin{aligned}\mathcal{Q} \equiv &+\sum_{n=1}\alpha_n L^{2(n-1)}(\partial\phi)^{2n}\\ &-\sum_{m=0}\beta_m L^{2(m+1)}(\partial\phi)^{2m}\left[(3+2m)R^{bc}\partial_b\phi\partial_c\phi - R\,(\partial\phi)^2\right],\end{aligned}}$$

where $(\partial\phi)^2 \equiv (g^{ab}\partial_a\phi\partial_b\phi)$, and where the α_n, β_m are arbitrary dimensionless constants.

Equations and static solutions

The full non-linear equations of this theory evaluated for the ansatz

$$ds^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\varphi^2\,, \quad \phi = p\varphi\,,$$

reduce to a single independent equation for $f(r)$ which can be integrated once and solved. The result is

$$\boxed{f(r) = \frac{\left[\frac{r^2}{L^2} - \mu - \alpha_1 p^2 \log(r/L) + \sum_{n=2} \frac{\alpha_n p^{2n} L^{2(n-1)}}{2(n-1)r^{2(n-1)}}\right]}{\left[1 + \sum_{m=0} \frac{\beta_m p^{2(m+1)}(2m+1)L^{2(m+1)}}{r^{2(m+1)}}\right]}\,.}$$

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- Multiparametric continuous generalization of BTZ. Reduces to it for $\alpha_{n\geq 1}=\beta_{m\geq 0}=0$.
- Fully analytic. Simple dependence on radial coordinate.
- When only α_1 is on, this is the charged BTZ metric with $Q^2\equiv 2\alpha_1 p^2$.

3. Black holes

Black holes

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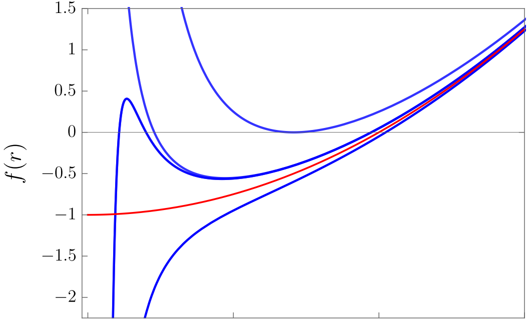
- Depending on the values of the α_n and the β_m, this describes different kinds of solutions. Firstly, the number of horizons depends on the number of positive zeros of the numerator which in turn depends on the values and signs of the α_n.
- As $r\to 0$, the spacetime can look very different, depending on the value of the combination $m_{\max}+2-n_{\max}$, where $n_{\max}$ and $m_{\max}$ are the largest values of n and m corresponding to non-vanishing α_n's and β_m's.

Black holes with curvature singularities

- Whenever $f(r)$ contains at least a real zero and $n_{\max} > m_{\max} + 2 \Rightarrow$ curvature singularity. [BTZ plotted in red]



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- The second, "BTZ like", corresponds to a singularity in the causal structure at $r = 0$, where spacetime ceases to be Haussdorf.

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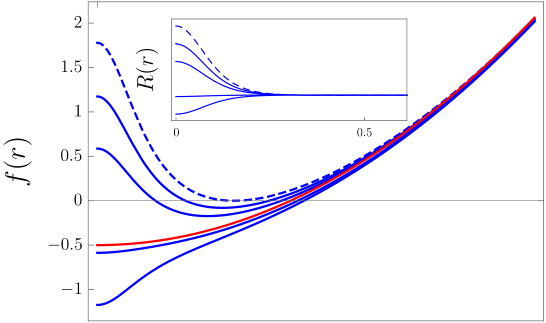

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- Recall for BTZ: $f(r) \overset{r\to 0}{=} -\mu$, which is only regular for $\mu = -1$.

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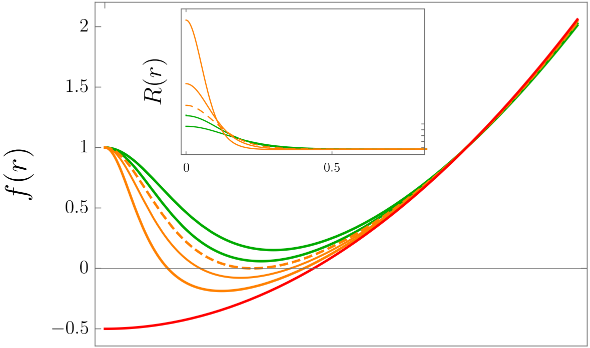

[In orange, regular black holes with two horizons and a extremal one (dashed). The green curves correspond to horizonless solutions which are completely regular everywhere.]

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- Here, the point $r = 0$ becomes a sort of new asymptotic region.

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where all the constants: μ, p, β_0 and L^2 are free parameters.

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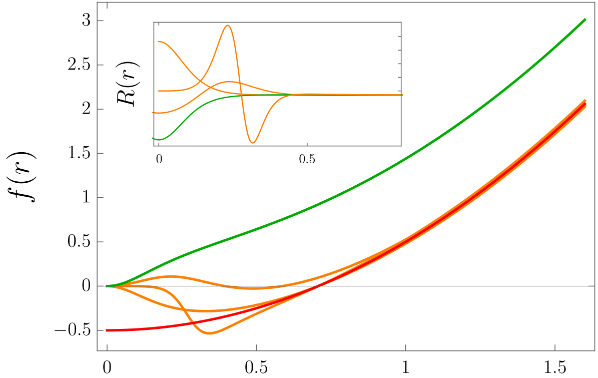

[Examples with one and two horizons shown in orange. Globally regular horizonless solutions with this behavior also exist (green).]

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◇ In some cases the black holes have no singularity at all. This happens for $f(r) \overset{r\to 0}{\to} 1$ and for $f(r) \overset{r\to 0}{\to} \mathcal{O}(r^{2s})$. In the latter case, this is achieved without imposing any condition between the parameters.

Obrigado

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Simons Collaboration on
Quantum Fields, Gravity and Information

Instituto
Balseiro

BONUS SLIDES

Generalized Quasi-topological gravities

Criterion: 2^{nd}-order eqs on certain backgrounds.

2^{nd}-order eqs $\forall$ backgrounds $\Rightarrow$
Lovelock gravities [Lanczos; Lovelock]

2^{nd}-order traced eqs $\forall$ backgrounds $\Rightarrow$
Quasi-topological gravities I [Oliva, Ray]

2^{nd}-order eqs on max. sym. backg. and $g_{tt}g_{rr} = -1$ BHs + algebraic eq for $f(r) \equiv g_{tt} \Rightarrow$
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All these only exist for $D \geq 5$

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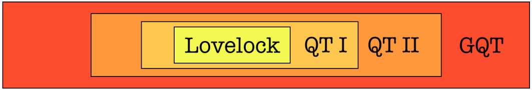

Some general properties of (G)QTs

$\sim$ 65-75 related papers since 2016

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- Phenomenological deviations from Einstein gravity: shadows, etc. [Hennigar, Poshteh, Mann]

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- $D=4$ GQTs define holographic toy-models of $d=3$ CFTs (analogously to *e.g.*, Gauss-Bonnet and QT in $D\geq 5$). [PB, Cano, Ruiperez]

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Some general properties of (G)QTs

$\sim$ 60-70 related papers since 2016

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And more...

Adding rotation

- The static solutions can be easily generalized into rotating ones by performing a boost in the t and φ coordinates,

$$t \to \gamma t - L\omega\varphi\,, \quad \varphi \to \gamma\varphi - \omega t/L\,.$$

We get

$$\mathrm{d}s^2 = -\frac{r^2 f}{\rho^2}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f} + \rho^2\left[\mathrm{d}\varphi + N^{\varphi}\mathrm{d}t\right]^2\,, \quad \phi = p\left[\gamma\varphi - \frac{\omega t}{L}\right]\,,$$

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with the $f(r)$ given before.
- We can think of new solutions as "magnetic" or "electric" depending on which frame we choose.

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◇ Dual description in terms of EM field

$$\mathcal{L}[g^{ab}, R_{ab}, \partial_a \phi] \quad \Longleftrightarrow \quad \mathcal{L}_{\text{dual}}[g^{ab}, R_{ab}, F_{ab}]\,,$$

where the dual field strength and Lagrangian are defined by

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$$\mathcal{L}_{\rm dual} = R + \frac{2}{L^2} - \frac{F^2}{2\alpha_1} + L^2\left[\frac{3\beta_0}{\alpha_1^2}F_a{}^c F^{ab} R_{\langle cb\rangle} - \frac{\alpha_2}{4\alpha_1^4}(F^2)^2\right] + \mathcal{O}(L^4)\,.$$

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What are GQT gravities?

Consider some purely gravitational action $I[g_{ab}, R_{abcd}]$ and a general static and spherically symmetric ansatz,

$$\mathrm{d}s^2|_{N,f} = -N(r)f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega^2_{(D-2)}\,.$$

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- We say that $I[g_{ab}, R_{abcd}]$ is of the "Generalized Quasi-topological" class if the Euler-Lagrange equation of $f(r)$ associated to I_f vanishes identically, *i.e.*, if

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This is equivalent to $\sqrt{|g|}\mathcal{L}[g_{ab}, R_{ab}]$ becoming a total derivative when evaluated on $\mathrm{d}s^2|_f$.

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$$\mathrm{d}s^2|_f = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2(\mathrm{d}\theta^2 + \sin\theta^2 \mathrm{d}\phi)\,, \quad F^{\text{mag.}} = P \sin\theta \mathrm{d}\theta \wedge \mathrm{d}\phi$$

the Euler-Lagrange eq. of $f(r)$ for the on-shell Lagrangian $\sqrt{|g|}\mathcal{L}\Big|_{f, F^{\text{mag}}}$ vanishes identically.

- Then, the theory admits solutions with g_{ab} and F_{ab} given by those expressions with $f(r)$ satisfying a differential equation of order ≤ 2.