Ringdown of Compact objects

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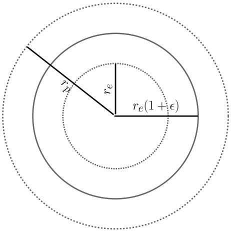
[In collaboration with Arpan Bhattacharyya]

Exotic Compact Objects (ECO):

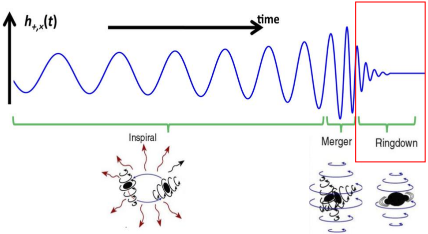
- slightly larger than BHs,
  $\epsilon = (R - r_e)/r_e$
- has finite reflection coefficient



Need for ECOs:

- Theoretical aspects: Fundamental problem like existence of singularity and Cauchy horizon, Black hole information paradox problem
- Observational aspects: To test whether the observed object is BH or not?

Ringdown properties of these objects generally differs from that of the BH



[M. Favata/SXS/K. Thorne]

Quasinormal modes

- According to perturbation theory, RDs are consists of exponentially damped sinusoidal waves ($\approx \mathrm{Exp}[i\omega t]$, $\omega = \omega_R + i\omega_I$): Quasinormal modes

- QNM carries information about the internal structure about the emitting source
- For BH, it depends on BH parameters M, Q and a (No-hair theorem)
- For ECO, ω additionally depend on the properties of the compact object

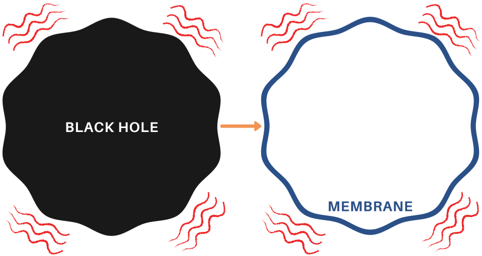
An Interpretation of QNMs in Geometrical-Optics limit:

- QNMs are interpreted in terms of photons trapped in unstable photon orbit $r_{\rm ph}$ and slowly leaking out.
- ω_R is related to the angular velocity of the photons at $r_{\rm ph}$
- ω_I is related to the decay time scale $t_{\rm d}$ of null geodesics at the unstable photon orbit

We study the ringdown and QNMs of charged ECOs in static spacetime using memebrane paradigm.

Membrane Paradigm

External static observer can replace the interior of a BH with a fluid membrane at EH.



[Phys. Rev. D 102, 064053]

- The energy momentum tensor of the membrane

$$\tau_{ab} = (K^+ h_{ab} - K^+_{ab}) = \rho u_a u_b + (p - \zeta\Theta)\gamma_{ab} - 2\eta\sigma_{ab}$$

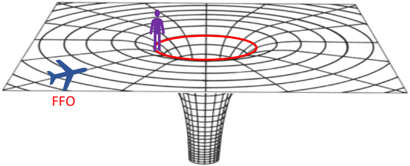
[Israel Equation]

- Membrane paradigm predicts the existence of surface 4-current j_s^μ

$$E^\perp_{\text{FIDO}} = 4\pi j_s^0 = 4\pi\sigma_e\,, \qquad (\vec{B}^{\parallel}_{\text{FIDO}})^A = 4\pi(\vec{j}_s \times \hat{n})^A$$

[Maxwell's Equation]

- Consider two sets of observers:



- To FFOs, FIDOs are moving outward with velocity

$$v = \sqrt{1 - f(R)}\ .$$

The fields measured by FFO at $\mathcal{S}$ are finite.

- However, a FIDO measurement is Lorentz boosted

$$E^{\theta}_{\mathrm{FIDO}} = \gamma\left(E^{\theta}_{\mathrm{FFO}} - vB^{\phi}_{\mathrm{FFO}}\right)\ , \qquad B^{\phi}_{\mathrm{FIDO}} = \gamma\left(B^{\phi}_{\mathrm{FFO}} - vE^{\theta}_{\mathrm{FFO}}\right)\ ,$$

- For a compact enough object, $v \approx 1 - (1/2)f + \mathcal{O}(f^2)$.

$$(\hat{n} \times B^{||}_{\mathrm{FIDO}})^{\theta} = -B^{\phi}_{\mathrm{FIDO}} \approx \gamma\left(E^{\theta}_{\mathrm{FFO}} - vB^{\phi}_{\mathrm{FFO}}\right) - \frac{f}{2}\gamma\left(E^{\theta}_{\mathrm{FFO}} + B^{\phi}_{\mathrm{FFO}}\right) + \mathcal{O}(f^{\frac{3}{2}})$$
$$= E^{\theta}_{\mathrm{FIDO}} - E^{\theta}_{\mathrm{Cor}}$$

Boundary Conditions

$$\vec{E}^{\parallel}_{\mathrm{FIDO}} = (\hat{n} \times \vec{B}^{\parallel}_{\mathrm{FIDO}}) + \vec{E}^{\parallel}_{\mathrm{Cor}} \, .$$

- In BH limit $f \to 0$, $\vec{E}^{\parallel}_{\mathrm{Cor}}$ vanishes.

Axial perturbation:

- Only non-vanishing component of $\vec{E}^{\parallel}_{\mathrm{FIDO}}$ is E^{ϕ}_{FIDO}
- Surface Current, $j^{\phi} = \sigma_e \delta u^{\phi}$
- From Ohm's law, $$\rho_s = \frac{E^{\phi}_{\mathrm{FIDO}}}{j^{\phi}} = \frac{-4\pi \tilde{F}^{r\phi} n_r}{\sigma_e \delta u^{\phi}} + \rho_{\mathrm{Cor}}$$
- Redefining $\rho_S = \rho_s - \rho_{\mathrm{Cor}}$ and choose it as free parameter
- Replacing the expression of $\tilde{F}^{r\phi}$, n_r, σ_e and δu^{ϕ}, we obtain boundary condition for electromagnetic perturbation
- Similarly, by replacing the value of perturbed extrinsic curvature and δu^{ϕ} in Israel eqn, we obtain the boundary condition for gravitational perturbation

Perturbation Equation

The perturbation equations of the compact object:

$$\frac{d^2 Z_i}{dr_*^2} + \left[\omega^2 - V_i(r, \Theta_1)\right] Z_i = 0 \,, \quad i \in (1, 2)$$

where, $\Theta_1 = (M, Q, \ell)$

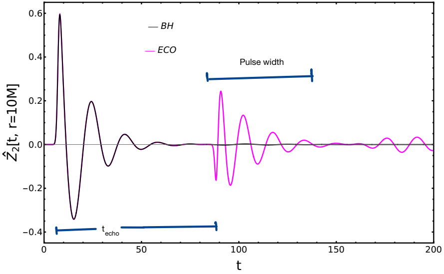
Boundary Conditions:

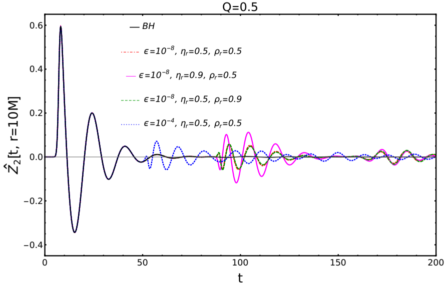
- At the surface of the compact object

$$\left.\frac{dZ_i}{dr_*}\right|_R = \sum_{j=1}^{2} F_{ij}(\omega, \Theta_1, \Theta_2)\, Z_j$$

Θ_2	Black holes	Compact object
Compactness Parameter ϵ	0	$0 < \epsilon \leq \epsilon_{ph}$
Shear Viscosity(η)	$\eta_{BH} = 1/16\pi$	Free Parameter
Resistivity,ρ_S	$\rho_{SBH} = 4\pi$	Free Parameter

Ringdown of highly compact objects ($\epsilon \equiv \frac{R-r_{eh}}{r_{eh}} \lesssim 0.01$) :





Echo time=t_{echo}=Time separation between two consecutive signal

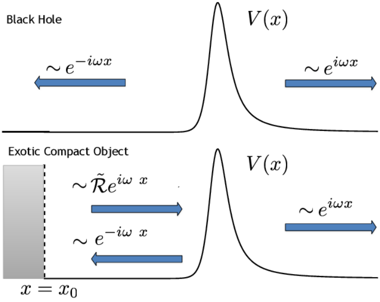
Observations :

- Echo time decreases with the increase of ϵ
- Echo time increases with the increase of Q
- Amplitude of the echo signal decreases with the increase of η
- Ringdown signal has a very weak dependence on the resistivity ρ_S

Explanation for the dependence of ϵ and Q

For the radiations generated at r_{ph}

- QNMs as null geodesics trapped at photon sphere and slowly leaking out
- Due to finite reflectivity, a part got reflected back.
- Echo signal= Prompt ringdown+ a series of pulses



[Phys. Rev. D 96, 084002 (2017)]

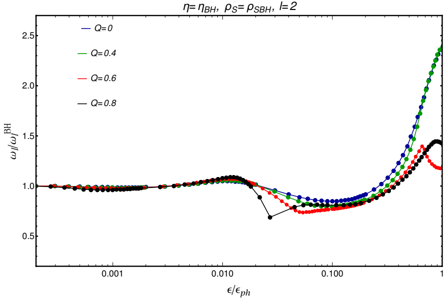
Time for this to and fro motion

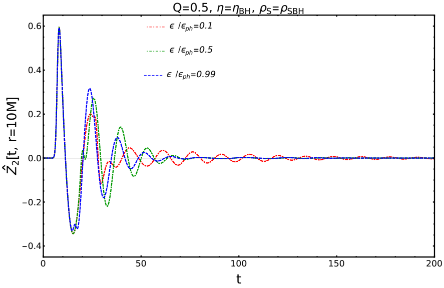
$$t_{\rm echo} = 2\int_R^{r_{\rm ph}} \frac{dr}{f(r)} = r_e(\epsilon_{\rm ph} - \epsilon) - \frac{1}{\kappa_e}\log[\frac{\epsilon}{\epsilon_{\rm ph}}] + \frac{1}{\kappa_c}\log[\frac{\epsilon+\varepsilon}{\epsilon_{\rm ph}+\varepsilon}]\,,$$

Observation:

- $t_{\rm echo}\downarrow$ as $\epsilon\uparrow$
- $t_{\rm echo}\uparrow$ as $Q\uparrow$

Interpretation of QNMs





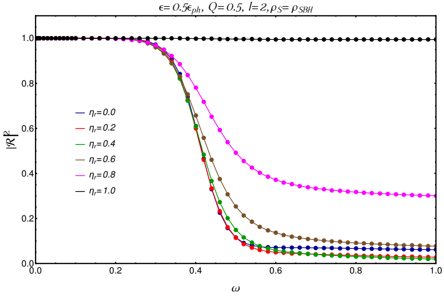
Interpretation of QNM:

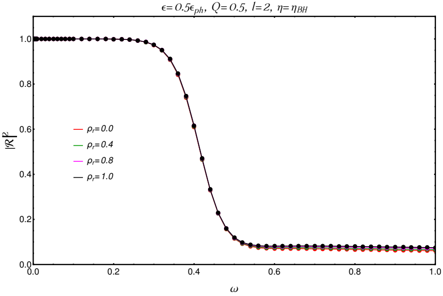
- QNMs will modified if $t_{\mathrm{echo}} <$ decay timescale of null geodesics at photon sphere(t_d)
- For RN BH, $t_d \approx 10M$
- gets modification for less compact objects

Interpretation on the dependence of η and ρ_S

The reflection coefficient turns out to be

$$|\mathcal{R}|^2 = \left[\frac{1-\eta/\eta_{\rm BH}}{1+\eta/\eta_{\rm BH}}\right]^2 - \frac{8Q}{3}\left[\frac{(1-\eta/\eta_{\rm BH})}{(1+\eta/\eta_{\rm BH})^2}\right] + \frac{2Q^2}{9\pi R}\left[\frac{3(\rho_S-\rho_{\rm SBH})(1-\eta/\eta_{\rm BH})}{(1+\eta/\eta_{\rm BH})^3}\right]$$



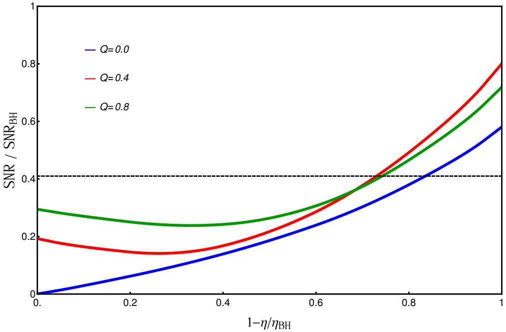


$$\eta_r = 1 - \eta/\eta_{\rm BH}$$

$$\rho_r = 1 - \rho_S/\rho_{\rm SBH}$$

Detectibility

SNR calculation:



Conclusion:

- We have studied the perturbations of compact objects in model independent way.
- This model can get a glimpse of the ringdown properties of astrophysical relevant objects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