Black hole thermodynamics in the presence of NLE fields

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Black hole thermodynamics

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0. $\kappa, \Phi, \ldots = \text{ const. on } H$

1. $\delta M = \frac{1}{8\pi} \kappa \, \delta \mathcal{A} + \Phi_{\mathsf{H}} \, \delta Q + \ldots$

Smarr $M = \frac{1}{4\pi} \kappa \, \mathcal{A} + \Phi_{\mathsf{H}} \, Q + \ldots$

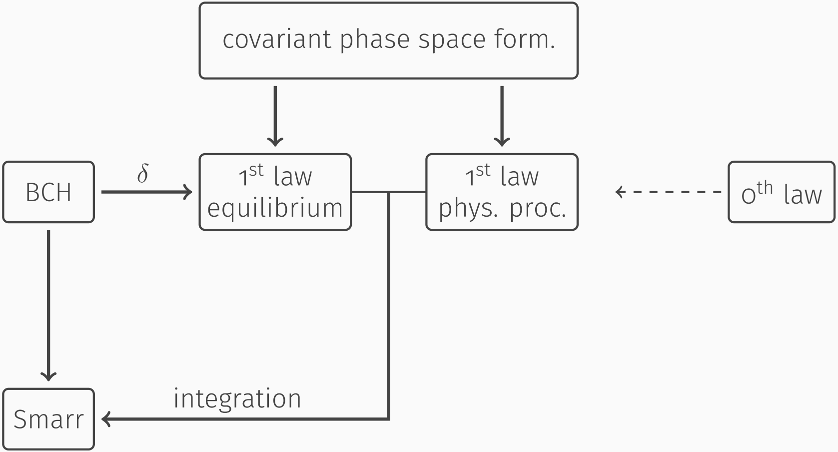
...and an auxiliary relation:

Bardeen–Carter–Hawking mass formula

$$M = \frac{1}{4\pi}\,\kappa\mathcal{A} + 2\Omega_{\mathsf{H}}J - 2\int_{\Sigma}\left(\star T(\chi) - \frac{1}{2}\,T\star\chi\right)$$

...for a Killing horizon $H[\chi]$ generated by χ^a

Relations among formulae



Going beyond Einstein and Maxwell

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- Einstein's GR

  $\rightarrow$ Lovelock, grav. Chern-Simons, $f(R)$ theories, ...

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- Maxwell's electrodynamics

  $\rightarrow$ **N**ON**L**INEAR **E**LECTRODYNAMICS

Basic elements of NLE

- two EM invariants: $\mathcal{F} := F_{ab} F^{ab}$ and $\mathcal{G} := F_{ab} \star F^{ab}$

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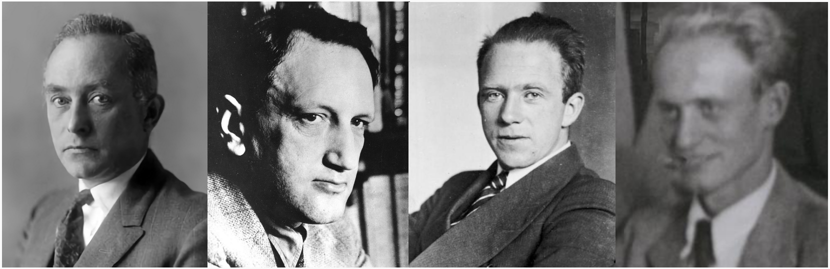
- auxiliary 2-form: $Z_{ab} := -4(\mathscr{L}_{\mathcal{F}} F_{ab} + \mathscr{L}_{\mathcal{G}} \star F_{ab})$
- generalized Maxwell's eqs.: $\mathrm{d}\mathbf{F} = 0$, $\mathrm{d} \star \mathbf{Z} = 0$

Inception of NLE in 1930s



Max Born, Leopold Infeld, Werner K. Heisenberg, Hans H. Euler

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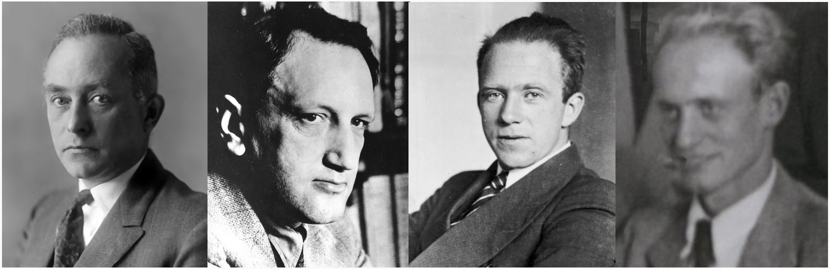

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- Max Born (1933): an upper limit b for the field strength

$$\mathscr{L}^{(\mathrm{Born})} = b^2 \left(1 - \sqrt{1 + \frac{\mathcal{F}}{2b^2}} \right)$$

Inception of NLE in 1930s

- Born–Infeld (1934)

$$\mathscr{L}^{(\mathrm{BI})} = b^2 \left(1 - \sqrt{1 + \frac{\mathcal{F}}{2b^2} - \frac{\mathcal{G}^2}{16b^4}} \right)$$

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- Euler–Heisenberg (1935): one-loop QED corrections to classical Maxwell

$$\mathscr{L}^{(\mathrm{EH})} = -\frac{1}{4}\,\mathcal{F} + \frac{\alpha^2}{360 m_e^4} \left(4\mathcal{F}^2 + 7\mathcal{G}^2 \right) + O(\alpha^3)$$

This talk: recent results

BJS 2021

A. Bokulić, T. Jurić and I.S.:
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PRD **103** (2021) 124059 [**2102.06213**]

THIS TALK: RECENT RESULTS

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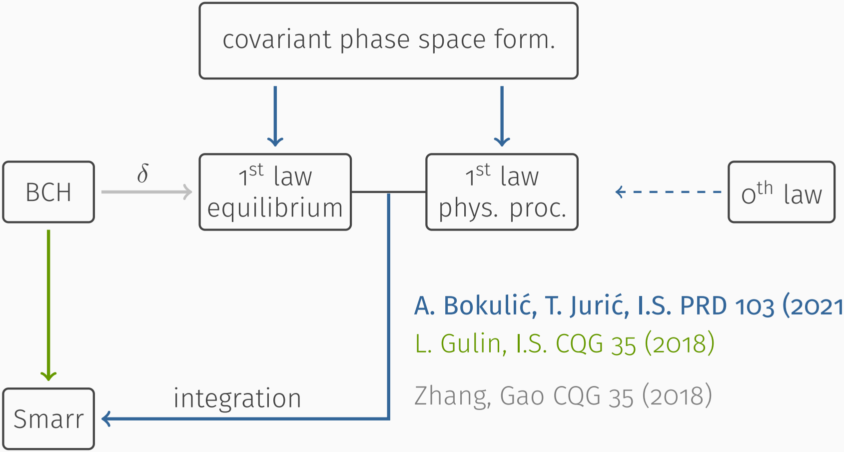
GS 2018

L. Gulin and I.S.: **Generalizations of the Smarr formula for black holes with NLE fields**
CQG **35** (2018) 025015 [**1710.04660**]

Recent results



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Zeroth law(s)

Zeroth law(s) on a Killing horizon H[K]

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surface gravity $K^b \nabla_b K^a \overset{H}{=} \kappa K^a$

electric potential $E := -i_K F = -\mathrm{d}\Phi$

magnetic potential $H := i_K \star Z = -\mathrm{d}\Psi$

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$$\kappa, \Phi, \Psi \overset{H}{=} \text{const.}$$

proof via ...	κ	Φ, Ψ for Maxwell
Einstein's EOM	Bardeen et al. 1973	Carter 1973
bifurcation surface	Kay & Wald 1991	Gao 2003
Frobenius' theorem	Carter 1972	I.S. 2012, 2014

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⋆ static case, $k_{[a}\nabla_b k_{c]} = 0$

$$E_a = 0 \text{ or } B_a = 0$$

$$D_a = K^b Z_{ab} = 0 \text{ or } H_a = K^b \star Z_{ba} = 0$$

$$\text{if } (\mathscr{L}_{\mathcal{F}})^2 + (\mathscr{L}_{\mathcal{G}})^2 \neq 0$$

First law

- variation of a field ϕ

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- subtle issue: variation of coupling constants

$$\delta\beta_i := \frac{\partial\beta_i(\lambda)}{\partial\lambda}\Big|_{\lambda=0}$$

$$\delta \mathbf{L}[\phi;\beta] = \mathbf{E}[\phi;\beta]\,\delta\phi + \boldsymbol{\Lambda}^i[\phi;\beta]\,\delta\beta_i + \mathrm{d}\boldsymbol{\Theta}[\phi,\delta\phi;\beta]$$

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$$\mathbf{J}_\xi := \boldsymbol{\Theta}[\phi,\pounds_\xi\phi;\beta] - i_\xi \mathbf{L}[\phi;\beta]$$

$$\mathrm{d}\mathbf{J}_\xi \approx 0\ , \quad \mathbf{J}_\xi \approx \mathrm{d}\mathbf{Q}_\xi$$

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$$\Omega_\Sigma[\phi,\delta\phi,\pounds_\xi\phi;\beta] = \int_\Sigma i_\xi(\mathbf{E}\,\delta\phi + \delta\mathbf{C}) + \\ + \int_{\partial\Sigma} (\delta\mathbf{Q}_\xi - i_\xi\boldsymbol{\Theta}[\phi,\delta\phi;\beta]) - K^i_\xi(\beta)\,\delta\beta_i$$

$$K^i_\xi(\beta) := -\int_\Sigma i_\xi \boldsymbol{\Lambda}^i[\phi;\beta]$$

NLE first law (BJS 2021)

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$$\delta M = \frac{\kappa}{8\pi}\,\delta\mathcal{A} + \Omega_{\mathsf{H}}\,\delta J + \Phi_{\mathsf{H}}\,\delta Q + \sum_i K^i_\chi\,\delta\beta_i$$

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N.B.: Legendre transformation $M = \mathcal{E} + K^i_\chi\,\beta_i$

Smarr formula

Generalized NLE Smarr (GS 2018)

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$$M = \frac{\kappa \mathcal{A}}{4\pi} + 2\Omega_{\mathsf{H}} J + \Phi_{\mathsf{H}} Q + \Psi_{\mathsf{H}} P + \Delta$$

$$\Delta = \frac{1}{2} \int_{\Sigma} T \star \boldsymbol{\chi}$$

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...if $\mathscr{L} = \sigma^{-1} f(\sigma \mathcal{F}, \sigma \mathcal{G})$ then $\Delta = \sigma \mathcal{C}$

First law → Smarr formula

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- rescaled field configurations $(\lambda^2 g_{ab}, \lambda^\nu \mathbf{A})$

First law $\to$ Smarr formula

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First law $\rightarrow$ Smarr formula

- rescaled field configurations $(\lambda^2 g_{ab}, \lambda^\nu \mathbf{A})$
- consistency: $\beta_i \to \lambda^{b_i} \beta_i$
- using $\delta\mathscr{Q} = \mathrm{d}\mathscr{Q}/\mathrm{d}\lambda\,|_{\lambda=1}$

$$M = \frac{\kappa}{4\pi}\mathcal{A} + 2\Omega_{\mathsf{H}} J + \Phi_{\mathsf{H}} Q + \sum_i b_i K_\chi^i \beta_i$$

Linearity of the Smarr formula

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- crucial issue:
  turning $T{\star}\boldsymbol{\chi}$ into $\mathrm{d}(\Phi{\star}\mathbf{Z})$, $\mathrm{d}(\Phi\mathbf{F})$, $\mathrm{d}(\Psi\mathbf{F})$ and $\mathrm{d}(\Psi{\star}\mathbf{Z})$

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  $$\mathscr{L} = a(\mathscr{L}_\mathcal{F}\mathcal{F} + \mathscr{L}_\mathcal{G}\mathcal{G}) + b\left(2\mathscr{L}_\mathcal{F}\mathscr{L}_\mathcal{G}\mathcal{F} + (\mathscr{L}_\mathcal{G}^2 - \mathscr{L}_\mathcal{F}^2)\mathcal{G}\right) + c\mathcal{G}$$

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- NLE with Maxwellian weak field limit,
  $\mathscr{L}_\mathcal{F}(0,0) = -1/4$ and $\mathscr{L}_\mathcal{G}(0,0) = 0 \Rightarrow$ ...Maxwell!

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- EM duality invariant case $\Rightarrow b = 0 = c \Rightarrow$
  $\mathscr{L} = \mathcal{F}^{1/a} f(\mathcal{G}/\mathcal{F})$ for $\mathcal{F} \neq 0$ or
  $\mathscr{L} = \mathcal{G}^{1/a} g(\mathcal{F}/\mathcal{G})$ for $\mathcal{G} \neq 0$

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- thermodynamic role of coupling parameters
- EiBI gravity w/ Maxwell $\longleftrightarrow$ GR w/ BI NLE

Thank you for the attention!

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