Geometric Justification of the Fundamental Interaction Fields for the Classical Long-Range Forces

Symmetry 2021, 13(3), 379; https://doi.org/10.3390/sym13030379 [arxiv:1907.05248]

Vesselin G. Gueorguiev
Ronin Institute & Institute for Advanced Physical Studies

Andre Maeder
Geneva Observatory, University of Geneva, Switzerland

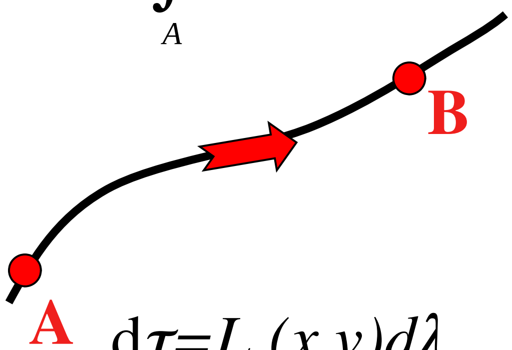
EREP2021-Spanish-Portuguese Relativity Meeting
13-16 September 2021, Aveiro, Portugal

Relativistic Particle

$$L_1(x,v)=\sqrt{g_{\mu\nu}v^{\mu}v^{\nu}}\,,\quad L_2(x,v)=g_{\mu\nu}v^{\mu}v^{\nu}$$

$$S[x(\tau)]=\int_A^B L(x,v)d\lambda$$

Real particles propagate with a **finite 3D speed**, **constant 4D speed**, and the corresponding Euler-Lagrange equations of motion are **equivalent** to the geodesic equations.



$d\tau=L_1(x,v)d\lambda$

$$\left(\frac{\delta S_1[x(\tau)]}{\delta x(\tau)}=0\right)\cong\left(\frac{\delta S_2[x(\tau)]}{\delta x(\tau)}=0\right)\cong\left(D_{\vec{v}}\vec{v}=0\right)$$

In proper time parameterization L=1 or 0 $\qquad g_{\mu\nu}v^{\mu}v^{\nu}=const$

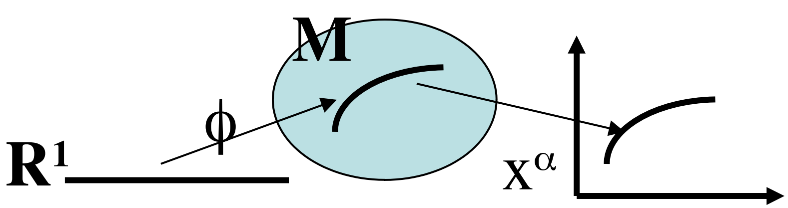
Should the smallest "distance" be the guiding principle?
If yes, how it should be defined in other situations?

Reparametrization Invariance

Leaving aside expectation that the spacetime is quantized …we ask:

- ***Should there be any preferred <u>trajectory parameterization</u> in the 4D spacetime?***
- ***Aren't we free to choose the <u>standard of distance</u> (time)?*** *(Using natural units c=1)*

<u>Smallest time interval</u> ?!… no more events under certain threshold?



$$\phi : R^1 \to M$$
$$\phi^* : R^1 \to TM$$
$$\phi_* : R^1 \leftarrow T^*M$$

$$\frac{d}{d\tau}\left(\frac{\partial L}{\partial v^\alpha}\right) = \frac{\partial L}{\partial x^\alpha}$$

Integrating using a *Volume* form is natural. Consider the pull-back of a one-form ω :
$\phi_*(\omega) = A_\alpha(x)v^\alpha d\tau = L(x,v)\, d\tau$,
this is <u>first order homogeneous Lagrangian</u>.

If $L(x,v)$ is homogeneous function of v then the Euler-Lagrange equations are invariant under τ-rescaling, $L(x,v)\sim H$ is constant.

What is the general mathematical expression for homogeneous function of first order?

$$f(\vec{v}) = \sum_i v^i \frac{\partial}{\partial v^i} f(\vec{v})$$

Is there a unique procedure to extract A, g, S … from L:

$$L(x,v) = A_\alpha(x)v^\alpha + \sqrt{g_{\mu\nu}(x)v^\mu v^\nu} + \ldots + \sqrt[n]{S(v,v,\ldots,v)} + \ldots$$

First Order Homogeneous Lagrangians

$$L = \sum_{\alpha} v^{\alpha} \frac{\partial L}{\partial v^{\alpha}}$$

Cons

- Constraints among the Euler-Lagrange equations
  - ➢ $(x,v) \to (x,p)$ problem
  - ➢ The Hamiltonian function is ZERO
- Some researchers do not like it…

$$H = v^{\alpha} p_{\alpha} - L,\ p_{\alpha} = \frac{\partial L}{\partial v^{\alpha}}$$

Pros

- ➢ Reparametrization invariance
- ➢ Parameterization independent pathintegral quantization
- ➢ May help in dealing with singularities
- ➢ Any $L(x,v) \to L(x,v/v_0)v_0$ a reparametrization invariant one.
- ➢ Easily generalized to p-branes

Canonical form:

$$L(x,v) = qA_{\alpha}(x)v^{\alpha} + m\sqrt{g_{\mu\nu}\left(x\right) v^{\mu}v^{\nu}} + \ldots + \varepsilon \sqrt[n]{S\left(v, v, \ldots, v\right)} + \ldots$$

Coordinate time parameterization c=1:

$$L(x,v) = qA_0(x) + q\vec{A}\cdot\vec{v} + m\sqrt{1 - \vec{v}\cdot\vec{v}} + \ldots$$

Important Lagrangians

- 0-brane (*point particle*): $\omega^{\Gamma} \to \dfrac{dx^{\alpha}}{d\tau} = v^{\alpha}$,

$$L(x,\omega) = A_{\Gamma}(x)\omega^{\Gamma} + \sqrt{g_{\Gamma\Lambda}(x)\omega^{\Gamma}\omega^{\Lambda}} \to L(x,v) = eA_{\alpha}(x)v^{\alpha} + m\sqrt{g_{\mu\nu}(x)v^{\mu}v^{\nu}}$$

- 1-brane (strings): $\omega^{\Gamma} \to \Upsilon^{\alpha\beta} = \dfrac{\partial(x^{\alpha},x^{\beta})}{\partial(\tau,\sigma)}$,

$$\Upsilon^{\alpha\beta} = \det\begin{pmatrix} \partial_{\tau}x^{\alpha} & \partial_{\sigma}x^{\alpha} \\ \partial_{\tau}x^{\beta} & \partial_{\sigma}x^{\beta} \end{pmatrix} = \partial_{\tau}x^{\alpha}\partial_{\sigma}x^{\beta} - \partial_{\sigma}x^{\alpha}\partial_{\tau}x^{\beta},$$

String theory Lagrangian: $L(x^{\alpha},\partial_{i}x^{\beta}) = \sqrt{Y^{\alpha\beta}Y_{\alpha\beta}}$

- p-brane: ***Dirac-Nambu-Goto Lagrangian:*** $L(x^{\alpha},\partial_{D}x^{\beta}) = \sqrt{Y^{\Gamma}Y_{\Gamma}}$

One Time Physics, Causality, and m>0

Assume *that the gravity like term is always present:* $m\sqrt{g_{\alpha\beta}(x)v^{\alpha}v^{\beta}}$

$$g_{\alpha\beta}v^{\alpha}v^{\beta}\geq 0$$

Positivity guarantees well defined proper time! Thus, causality!
Select a local coordinate system where g is (+…+,-…-), call the (+) coordinates time coordinates and the (-) space coordinates.

***Cases*:**

1. ~~*No time coordinates.*~~ $g_{\alpha\beta}v^{\alpha}v^{\beta}=-\sum_i (v^i)^2\leq 0$
2. ~~*Two and more time coordinates.*~~ $g_{\alpha\beta}v^{\alpha}v^{\beta}=(V)^2+(v^0)^2-\sum_i (v^i)^2,$ $\Rightarrow V^2+1\geqq\vec{v}^2$
3. ***One time coordinate.*** $g_{\alpha\beta}v^{\alpha}v^{\beta}=(v^0)^2-\sum_i (v^i)^2\rightarrow 1-\vec{v}^2\Rightarrow 1\geq\vec{v}^2$

m>0, Minkowski signature, and Common Arrow of time! (m>0 <=> |φ|>0 [*])

[*] Reparametrization Invariance and Some of the Key Properties of Physical Systems
V G. Gueorguiev and A. Maeder in Symmetry 2021, 13, 522. (10.3390/sym13030522) [ArXiV:1903.02483]

Key New Results & Concepts

- Un-proper time parametrization as source of **extra forces/accelerations**.
- **Pathological behavior** in S_n interactions for n>2.
- Resolving **ambiguities for the interaction fields** using the canonical form of *L*.

Equivalent Lagrangians

$$v^i \frac{\partial L(x,\vec{v})}{\partial v^i} = nL(x,\vec{v}), \quad h = pv - L \rightarrow h = (n-1)L$$

$$L \rightarrow f(L), \quad \frac{d}{d\tau}\left(\frac{\partial L}{\partial v}\right) = \frac{\partial L}{\partial x} \rightarrow \frac{d}{d\tau}\left(f'\frac{\partial L}{\partial v}\right) = f'\frac{\partial L}{\partial x}$$

0

$$\frac{f''}{f'}\left(\frac{\partial L}{\partial v}\right)\frac{dL}{d\tau} + \frac{d}{d\tau}\left(\frac{\partial L}{\partial v}\right) = \frac{\partial L}{\partial x}$$

Fictitious forces/accelerations in
un-proper time parametrization

$$\frac{dp_\mu}{dt} = \partial_\mu L - \frac{f''}{f'}\dot{L}p_\mu, \; p_\mu = \frac{\partial L}{\partial v^\mu} \quad \Rightarrow \quad \frac{dp_\mu}{dt} = \partial_\mu L + \frac{\dot{L}}{2L}p_\mu, \text{ when } f(L) = \sqrt{L}.$$

$$\boxed{\frac{dv_\mu}{dt} = \partial_\mu L + \kappa_0 v_\mu,} \text{ when } L = g_{\mu\nu}v^\mu v^\nu.$$

for $L=\lambda^{-2}L_{GR}$ *and* $k_0=-d\lambda/(\lambda dt)$.

Pathology of S_n interactions for n >2

$$L = \left(S_n\left(v\right)\right)^{1/n}$$

parametrization gauge $S_n\left(v\right) = const$

$S_n(r,w,u) = \psi(r)w^n + \phi(r)u^n.$ where $w = dx^0/d\tau$ and $u = dr/d\tau$

$$\begin{aligned}\frac{du}{d\tau} &= -\frac{u^2\phi'(r)}{(n-1)\phi(r)} + \frac{1}{u^{n-2}}\frac{w^n\psi'(r)}{n(n-1)\phi(r)},\\ \frac{dw}{d\tau} &= -\frac{wu\psi'(r)}{(n-1)\psi(r)}.\end{aligned}$$

For n>2, using *u/w=v/c and dividing by w^2, one see a **pathology at v → 0** (no co-moving rest frame):*

$$\frac{(n-1)}{w^2}\frac{du}{d\tau} = -\frac{v^2\phi'(r)}{c^2\phi(r)} + \frac{c^{n-2}}{v^{n-2}}\frac{\psi'(r)}{n\phi(r)}.$$

DM QM & Inflation

Summary

- ✓ First order homogeneous ***matter Lagrangians*** in ***canonical form*** naturally contain ***electromagnetic and gravitational*** interactions.

$$A_\mu(x) = \tfrac{1}{2}(\tilde{L}(x,v) - \tilde{L}(x,-v))_{/\mu}\Big|_{v=(1,\vec{0})}$$

$$L = v^\mu A_\mu(x) + \sqrt{g_{\alpha\beta}v^\alpha v^\beta}$$

$$g_{\alpha\beta}(x) = \tfrac{1}{4}(\tilde{L}(x,v) + \tilde{L}(x,-v))^2_{/\alpha/\beta}\Big|_{v=(1,\vec{0})}$$

- ✓ The framework predicts fictitious forces/accelerations and specific new interactions beyond the ***electromagnetic and gravitational*** interactions!

$$\frac{(n-1)}{w^2}\frac{du}{d\tau} = -\frac{v^2\phi'(r)}{c^2\phi(r)} + \frac{c^{n-2}}{v^{n-2}}\frac{\psi'(r)}{n\phi(r)}.$$

$$\frac{dv_\mu}{dt} = \partial_\mu L + \kappa_0 v_\mu, \text{ when } L = g_{\mu\nu}v^\mu v^\nu.$$

On the Canonical Form of the First Order Homogeneous Lagrangians

$$L(x,v)=\frac{h_{\alpha\beta}(x)v^{\alpha}v^{\beta}}{\sqrt{g_{\mu\nu}(x)v^{\mu}v^{\nu}}},$$

Both fields (*g* and *h*) have same type of matter sources **~*vv***.

$$\frac{\delta L(x,v)}{\delta h_{\mu\nu}}=\frac{L(x,v)}{\left(h_{\alpha\beta}(x)v^{\alpha}v^{\beta}\right)}v^{\mu}v^{\nu},\qquad \frac{\delta L(x,v)}{\delta g_{\mu\nu}}=\frac{L(x,v)}{\left(g_{\alpha\beta}(x)v^{\alpha}v^{\beta}\right)}v^{\mu}v^{\nu}.$$

The canonical form defines fields for each source type *vvv…v*.

$$L(x,v)=qA_{\alpha}(x)v^{\alpha}+m\sqrt{g_{\mu\nu}(x)\,v^{\mu}v^{\nu}}+\ldots+\varepsilon\sqrt[n]{S(v,v,\ldots,v)}+\ldots$$

Background Interaction Fields

Matter should provide the sources for the interaction fields!

Electromagnetic interaction ***Gravitational interaction*** ***Do these exist?***

$$L(x,v) = qA_\alpha(x)v^\alpha + m\sqrt{g_{\mu\nu}(x)\,v^\mu v^\nu} + \ldots + \varepsilon\sqrt[n]{S(v,v,\ldots,v)} + \ldots$$

Constructing Lagrangians for the interaction fields.

- ***The "never ending process":***
  Consider each background field (S_n) as M-brane theory since $S_n : M \to \mathcal{S}_n\mathcal{M}$.
- ***External algebra method:*** *(A-type fields)*
  Constructing external n-forms proportional to the volume form using <u>external derivatives</u> and the <u>Hodge dual</u> operation: $\boldsymbol{A \wedge {*A}}$ and $\boldsymbol{dA \wedge {*dA}}$.
- ***Gauge symmetry method:***
  Use the Euler-Lagrange equations or their integral (geodesic) deviation.

Field Lagrangians

Lorentz equation: $\vec{a} = \hat{F} \cdot \vec{v},\ \hat{\mathrm{F}} = \mathrm{dA} + [\mathrm{A}, \mathrm{A}]$

Geodesic equation: $\vec{a} = \vec{v} \cdot \Gamma \cdot \vec{v},\ \Gamma \rightarrow \Gamma + \partial\Lambda$

Geodesic deviation: $\frac{d}{d\tau}\frac{d}{d\tau}\vec{\xi} = \hat{\mathrm{R}}\vec{\xi},\ \hat{\mathrm{R}} = \mathrm{d}\Gamma + [\Gamma, \Gamma]$

*A∧*A does not obey the symmetries of the matter equations,*
when F∧F=d(A∧F) is a boundary term…(Q-Hall effect)

$$L(A, \partial A) = \int_M F \wedge {*F}$$

R is a ***special*** *curvature two-form with values in Lie(TM).*

$$R_{ij,kl} = -R_{ij,lk} = -R_{lk,ij} = R_{kl,ij},\quad R^{*} = R_{ij,*(jk)},\quad \boxed{R^{\wedge} = R_{i[j,k]l}}\ ?\ (4\mathrm{D})$$

Einstein-Cartan action: $L(g, \partial g) = \int_M \hat{R}_{\alpha\beta} \wedge {*(dx^{\alpha} \wedge dx^{\beta})}$

Summary

- ✓ First order homogeneous ***matter Lagrangians*** in ***canonical form*** naturally contain ***electromagnetic and gravitational*** interactions.

- ✓ Gravitational term with Minkowski signature and positive mass is needed to assure well-defined proper time and causality; thus, common arrow of time.

- ✓ Unique construction of ***Lagrangians for the interaction fields***.

- ✔ The framework predicts specific new interactions beyond the ***electromagnetic and gravitational*** interactions!