Exact solutions with null matter in higher and infinite derivative gravity

IVAN KOLÁŘ

Van Swinderen Institute for Particle Physics and Gravity
University of Groningen, Groningen, Netherlands
i.kolar@rug.nl

Spanish Portuguese Relativity Meeting 2021 (EREP2021; online)

September 14, 2021

Based on:

- I. K., T. Málek, A. Mazumdar, *Exact solutions of nonlocal gravity in a class of almost universal spacetimes*, **arXiv:2103.08555**.
- I. K., T. Málek, S. Dengiz, E. Kilicarslan, *Exact gyratons in higher and infinite derivative gravity*, **arXiv:2107.11884**.

Generic quadratic curvature gravity

Action

General action quadratic in $\boldsymbol{R}$ (4-dim., parity-inv.):

$$S = \frac{1}{2}\int_M \mathfrak{g}^{\frac{1}{2}} \left[\varkappa^{-1}\big(R - 2\Lambda\big) + \boldsymbol{R}^{abcd}\boldsymbol{\mathcal{D}(\nabla)}_{abcd}{}^{efgh}\boldsymbol{R}_{efgh}\right] + S_{\mathrm{m}}$$

$$\boxed{\sim \frac{1}{2}\int_M \mathfrak{g}^{\frac{1}{2}} \left[\varkappa^{-1}\big(R - 2\Lambda\big) + R\mathcal{F}_1(\Box)R + \boldsymbol{S}^{ab}\mathcal{F}_2(\Box)\boldsymbol{S}_{ab} + \boldsymbol{C}^{abcd}\mathcal{F}_3(\Box)\boldsymbol{C}_{abcd}\right] + S_{\mathrm{m}}}$$

Analytic form-factors $\mathcal{F}_i(\Box) = \sum_{n=0}^{\infty} f_{i,n}\Box^n$:

- polynomial: (local) higher-derivative gravity
- non-polynomial: (non-local) infinite derivative gravity

Motivation:

- improved UV (renormalizability, regular spacetimes)
- no ghosts and extra d.o.f.

T. Biswas, E. Gerwick, T. Koivisto, A. Mazumdar, *Towards singularity and ghost free theories of gravity*, arXiv:1110.5249.

T. Biswas, A. Conroy, A. S. Koshelev, A. Mazumdar, *Generalized ghost-free quadratic curvature gravity*, arXiv:1308.2319.

Generic quadratic curvature gravity

Field equations

$$
\begin{aligned}
&\varkappa^{-1}\big(\boldsymbol{S_{ab}}-\tfrac{1}{4}R\boldsymbol{g_{ab}}+\Lambda\boldsymbol{g_{ab}}\big)+2\boldsymbol{S_{ab}}\mathcal{F}_1(\Box)R-2\big(\boldsymbol{\nabla_a}\boldsymbol{\nabla_b}-\boldsymbol{g_{ab}}\Box\big)\mathcal{F}_1(\Box)R+\big(\Box+\tfrac{1}{2}R\big)\mathcal{F}_2(\Box)\boldsymbol{S_{ab}}\\
&-2\boldsymbol{g_{d(a}}\big(\boldsymbol{\nabla^c}\boldsymbol{\nabla^d}-\boldsymbol{S^{cd}}\big)\mathcal{F}_2(\Box)\boldsymbol{S_{b)c}}+\boldsymbol{g_{ab}}\big(\boldsymbol{\nabla^c}\boldsymbol{\nabla^d}-\tfrac{1}{2}\boldsymbol{S^{cd}}\big)\mathcal{F}_2(\Box)\boldsymbol{S_{cd}}-4\big(\boldsymbol{\nabla^c}\boldsymbol{\nabla^d}+\tfrac{1}{2}\boldsymbol{S^{cd}}\big)\mathcal{F}_3(\Box)\boldsymbol{C_{d(ab)c}}\\
&-\boldsymbol{\Omega}_{1\boldsymbol{ab}}+\tfrac{1}{2}\boldsymbol{g_{ab}}\big(\boldsymbol{\Omega}_1{}^{\boldsymbol{c}}{}_{\boldsymbol{c}}+\mho_1\big)-\boldsymbol{\Omega}_{2\boldsymbol{ab}}+\tfrac{1}{2}\boldsymbol{g_{ab}}\big(\boldsymbol{\Omega}_2{}^{\boldsymbol{c}}{}_{\boldsymbol{c}}+\mho_2\big)-\boldsymbol{\Omega}_{3\boldsymbol{ab}}+\tfrac{1}{2}\boldsymbol{g_{ab}}\big(\boldsymbol{\Omega}_3{}^{\boldsymbol{c}}{}_{\boldsymbol{c}}+\mho_3\big)-2\boldsymbol{\Upsilon}_{2\boldsymbol{ab}}-4\boldsymbol{\Upsilon}_{3\boldsymbol{ab}}=\boldsymbol{T_{ab}}
\end{aligned}
$$

$$
\boldsymbol{\Omega}_{1\boldsymbol{ab}}=\sum_{n=1}^{\infty}f_{1,n}\sum_{l=0}^{n-1}\boldsymbol{\nabla_a}\Box^l R\,\boldsymbol{\nabla_b}\Box^{n-l-1}R\,,\qquad \mho_1=\sum_{n=1}^{\infty}f_{1,n}\sum_{l=0}^{n-1}\Box^l R\,\Box^{n-l}R\,,
$$

$$
\boldsymbol{\Omega}_{2\boldsymbol{ab}}=\sum_{n=1}^{\infty}f_{2,n}\sum_{l=0}^{n-1}\boldsymbol{\nabla_a}\Box^l\boldsymbol{S^{cd}}\,\boldsymbol{\nabla_b}\Box^{n-l-1}\boldsymbol{S_{cd}}\,,\qquad \mho_2=\sum_{n=1}^{\infty}f_{2,n}\sum_{l=0}^{n-1}\Box^l\boldsymbol{S^{cd}}\,\Box^{n-l}\boldsymbol{S_{cd}}\,,
$$

$$
\boldsymbol{\Omega}_{3\boldsymbol{ab}}=\sum_{n=1}^{\infty}f_{3,n}\sum_{l=0}^{n-1}\boldsymbol{\nabla_a}\Box^l\boldsymbol{C^{cdef}}\,\boldsymbol{\nabla_b}\Box^{n-l-1}\boldsymbol{C_{cdef}}\,,\quad \mho_3=\sum_{n=1}^{\infty}f_{3,n}\sum_{l=0}^{n-1}\Box^l\boldsymbol{C^{cdef}}\,\Box^{n-l}\boldsymbol{C_{cdef}}\,,
$$

$$
\boldsymbol{\Upsilon}_{2\boldsymbol{ab}}=\sum_{n=1}^{\infty}f_{2,n}\sum_{l=0}^{n-1}\boldsymbol{\nabla_c}\big[\Box^l\boldsymbol{S^{cd}}\,\boldsymbol{\nabla_{(a}}\Box^{n-l-1}\boldsymbol{S_{b)d}}-\boldsymbol{\nabla_{(a}}\Box^l\boldsymbol{S^{cd}}\,\Box^{n-l-1}\boldsymbol{S_{b)d}}\big]\,,
$$

$$
\boldsymbol{\Upsilon}_{3\boldsymbol{ab}}=\sum_{n=1}^{\infty}f_{3,n}\sum_{l=0}^{n-1}\boldsymbol{\nabla_c}\big[\Box^l\boldsymbol{C^{cdef}}\,\boldsymbol{\nabla_{(a}}\Box^{n-l-1}\boldsymbol{C_{b)def}}-\boldsymbol{\nabla_{(a}}\Box^l\boldsymbol{C^{cdef}}\,\Box^{n-l-1}\boldsymbol{C_{b)def}}\big]
$$

$\longrightarrow$ very few exact solutions (most studies in linearized theory)

Ansatzes:

- recursive curvature ($\Box R = r_1 R + r_2$) $\Longrightarrow$ # of derivatives reduced (localization) $\longrightarrow$ bouncing cosmologies
- **almost universal spacetimes** $\Longrightarrow$ effective linearization ('$\boldsymbol{R}^2$' absent/simpler) $\longrightarrow$ gen. gravitational waves

Almost universal spacetimes

Types of $[\boldsymbol{S},\boldsymbol{C}]$

Null orthonormal (co)frame (NP formalism):

$$\boldsymbol{g} = -\boldsymbol{l} \vee \boldsymbol{n} + \boldsymbol{m} \vee \bar{\boldsymbol{m}} \,, \quad \boldsymbol{l}^\sharp \cdot \boldsymbol{n} = -1 \,, \quad \boldsymbol{m}^\sharp \cdot \bar{\boldsymbol{m}} = 1$$

PP along $\boldsymbol{l}^\sharp$: $\mathrm{D}\boldsymbol{l} = \mathrm{D}\boldsymbol{n} = \mathrm{D}\boldsymbol{m} = 0$, $\mathrm{D} \equiv \boldsymbol{l}^\sharp \cdot \boldsymbol{\nabla}$

Boost ($\boldsymbol{l} \to e^{\varpi}\boldsymbol{l}$, $\boldsymbol{n} \to e^{-\varpi}\boldsymbol{n}$, $\boldsymbol{m} \to \boldsymbol{m}$):

boost weight b of q	$q \to e^{b\varpi} q$
boost order $\mathfrak{bo}_{\boldsymbol{l}}$ of $\boldsymbol{t}$	maximum b of non-zero comp. of $\boldsymbol{t}$
types of $\boldsymbol{t}$ based on $\mathfrak{bo}$	$\boldsymbol{l}$ with minimum $\mathfrak{bo}_{\boldsymbol{l}} \equiv \mathfrak{bo}$
k-balanced $\boldsymbol{t}$	$\begin{cases} \boldsymbol{t}^{\langle b \rangle} = 0, & b \geq -k \\ \mathrm{D}^{-b-k}\boldsymbol{t}^{\langle b \rangle} = 0, & b < -k \end{cases}$

$[\boldsymbol{S},\boldsymbol{C}]$ of types III/N/O ($\mathfrak{bo} = -1, -2$):

$$\begin{aligned} \boldsymbol{S} &= -2\boldsymbol{l} \vee \left(\Phi_{21}\boldsymbol{m} + \bar{\Phi}_{21}\bar{\boldsymbol{m}}\right) + 2\Phi_{22}\,\boldsymbol{l}\boldsymbol{l} \,, \\ \boldsymbol{C} &= \Psi_3(\boldsymbol{l} \wedge \boldsymbol{m}) \vee (\boldsymbol{n} \wedge \boldsymbol{l} + \boldsymbol{m} \wedge \bar{\boldsymbol{m}}) + \bar{\Psi}_3(\boldsymbol{l} \wedge \bar{\boldsymbol{m}}) \vee (\boldsymbol{n} \wedge \boldsymbol{l} + \bar{\boldsymbol{m}} \wedge \boldsymbol{m}) \\ &\quad + \Psi_4(\boldsymbol{l} \wedge \boldsymbol{m})(\boldsymbol{l} \wedge \boldsymbol{m}) + \bar{\Psi}_4(\boldsymbol{l} \wedge \bar{\boldsymbol{m}})(\boldsymbol{l} \wedge \bar{\boldsymbol{m}}) \end{aligned}$$

$$R = \text{const.}$$

M. Ortaggio, V. Pravda, A. Pravdová, *Algebraic classification of higher dimensional spacetimes based on null alignment*, arXiv:1211.7289.

Almost universal spacetimes

Definition & relation to Kundt

(Almost) universal spacetimes T-III/N/O:

tracefree parts of all $\boldsymbol{B}_{ab}(\boldsymbol{\nabla}\cdots\boldsymbol{\nabla}\boldsymbol{R})$ are of type III/N/O ($\mathfrak{bo} = -1, -2$)

$$\boxed{\boldsymbol{B} = \bar{\psi}\,\boldsymbol{l} \vee \boldsymbol{m} + \psi\,\boldsymbol{l} \vee \bar{\boldsymbol{m}} + \omega\,\boldsymbol{ll} + \xi\,\boldsymbol{g}\,, \quad \xi = \text{const.}}$$

LHS of all possible field equations

Sufficient conditions:

Kundt [O,III/N/O] & $\underline{\boldsymbol{K} = 0}$ $\Longrightarrow$ T-O *universal*

Kundt [N,III/N/O] $\Longrightarrow$ T-N *almost universal*

Kundt [III,III/N/O] $\Longrightarrow$ T-III *almost universal*

where $\boldsymbol{K}_{ab} \equiv \boldsymbol{\nabla}_a \boldsymbol{C}_{cdef} \boldsymbol{\nabla}_b \boldsymbol{C}^{cdef}$

S. Hervik, V. Pravda, A. Pravdová, *Type III and N universal spacetimes*, arXiv:1311.0234.
M. Kuchynka, T. Málek, V. Pravda, A. Pravdová, *Almost universal spacetimes in higher-order gravity theories*, arXiv:1810.02178.

Kundt: non-expanding, non-shearing, non-twisting null geodesic congruence $\boldsymbol{l}^\sharp = \boldsymbol{\partial}_r$

$$\boldsymbol{g} = -\mathbf{d}u \vee \left[\mathbf{d}r + H(u,r,\zeta,\bar{\zeta})\mathbf{d}u + \left(W(u,r,\zeta,\bar{\zeta})\mathbf{d}\zeta + \text{c.c.}\right)\right] + P^{-2}(u,\zeta,\bar{\zeta})\mathbf{d}\zeta \vee \mathbf{d}\bar{\zeta}$$

$\boldsymbol{\nabla t}$ is k-balanced if $\boldsymbol{t}$ is k-balanced; pp-waves ($\boldsymbol{\nabla l} = 0$) $\subset$ Kundt

Studied cases:

Spacetime	Sym. rank-2 tensors	Matter content
(1) Kundt [N,III/N] & $\underline{\boldsymbol{K} = 0}$ arXiv:2103.08555	T-N; $\boldsymbol{B} = \mathcal{B}(\Box)\boldsymbol{S} + \xi\,\boldsymbol{g}$	$\boldsymbol{T} = 2\Xi_{22}\boldsymbol{ll}$, $\Lambda \neq 0$
(2) pp-waves [III,III] arXiv:2107.11884	T-III; quad. in $\boldsymbol{S}, \boldsymbol{C}$, many cont. vanish	$\boldsymbol{T} = -2\boldsymbol{l} \vee \left[\Xi_{21}\boldsymbol{m} + \bar{\Xi}_{21}\bar{\boldsymbol{m}}\right] + 2\Xi_{22}\boldsymbol{ll}$, $\Lambda = 0$

(1): $\boldsymbol{S}$ is 1-bal., $\boldsymbol{C}$ is 0-bal. $\Longrightarrow \boldsymbol{\nabla}\cdots\boldsymbol{\nabla S}$ has $\mathfrak{bo} = -2$, $\boldsymbol{\nabla}\cdots\boldsymbol{\nabla C}$ has $\mathfrak{bo} = -1$
$\Longrightarrow \boldsymbol{B} \propto$ '$\boldsymbol{S}$', '$\boldsymbol{C}$', '$\boldsymbol{C}^2$' $\Longrightarrow$ '$\boldsymbol{C}^2$' $= 0$ due to $\boldsymbol{K} = 0$; '$\boldsymbol{S}$', '$\boldsymbol{C}$' $\propto \mathcal{B}(\Box)\boldsymbol{S} + \xi\,\boldsymbol{g}$

(2): $\boldsymbol{\nabla l} = 0 \Longrightarrow \mathrm{D}\boldsymbol{R} = 0$, $\boldsymbol{l}^\sharp \cdot \boldsymbol{R} = 0 \Longrightarrow \mathrm{D}\Psi_3 = \mathrm{D}\Phi_{21} = \mathrm{D}\Psi_4 = \mathrm{D}\Phi_{22} = 0$, $[\mathrm{D}, \boldsymbol{\nabla}] = 0$
$\Longrightarrow$ contractions of $\boldsymbol{\nabla}\cdots\boldsymbol{\nabla S\nabla}\cdots\boldsymbol{\nabla S}$ vanish if at least one $\boldsymbol{S}$ has no free index

Gyratons within pp-waves [III,III] ($\Lambda = 0$):

$$\boldsymbol{g} = -\mathbf{d}u \vee \big(\mathbf{d}r + H(u,\rho,\varphi)\,\mathbf{d}u + J(u,\rho,\varphi)\,\mathbf{d}\varphi\big) + \mathbf{d}\rho\mathbf{d}\rho + \rho^2\mathbf{d}\varphi\mathbf{d}\varphi$$

$\longrightarrow$ eqns. reduce to two partly decoupled & linear PDEs for H & J

Axial symmetry $\pounds_{\boldsymbol{\partial}_\varphi}\boldsymbol{g} = 0$ ($\Xi_{21} = -\bar{\Xi}_{21}$):

$$\big[1 + \varkappa\mathcal{F}_2(\triangle_1)\triangle_1\big]\Phi_{21} = \varkappa\Xi_{21}$$

$$\big[1 + \varkappa\left(\mathcal{F}_2(\triangle_0) + 2\mathcal{F}_3(\triangle_0)\right)\triangle_0\big]\Phi_{22} + \varkappa\sum_{n=1}^{\infty}\sum_{k=0}^{n-1}\triangle_0^k(f_{2,n-1}\diamondsuit + f_{3,n}\heartsuit)\triangle_1^{n-k-1}\Phi_{21} = \varkappa\Xi_{22}$$

$$\Phi_{21} = -\bar{\Phi}_{21} = \circledcirc J\,, \qquad \triangle_w \equiv \partial_\rho^2 + \frac{1}{\rho}\partial_\rho - \frac{w^2}{\rho^2}\,, \qquad \diamondsuit \equiv \frac{4i}{\sqrt{2}}\left(\frac{J_{,\rho}}{\rho}\partial_\rho + \frac{J_{,\rho\rho}}{2\rho} + \frac{J_{,\rho}}{2\rho^2}\right)$$

$$\Phi_{22} = \frac{1}{2}\triangle_0 H + \frac{(J_{,\rho})^2}{4\rho^2}\,, \qquad \circledcirc \equiv -\frac{i}{4\sqrt{2}}\partial_\rho\left(\frac{1}{\rho}\partial_\rho\right)\,, \qquad \heartsuit \equiv -4i\sqrt{2}\left(2\frac{\rho J_{,\rho\rho} - J_{,\rho}}{\rho^2}\partial_\rho^2 + \dots\right)$$

$\longrightarrow$ eqns. reduce to two ODEs

- lin. in J; indep. of H; solution $J = J_{\mathrm{hom}} + J_{\mathrm{part}}[\Xi_{21}]$
- lin. in H; quad. in J; solution $H = H_{\mathrm{hom}} + H_{\mathrm{part}}[J, \Xi_{22}]$

Gyratons (spinning null matter)

Field equations

Homogeneous parts:

$$\left[1+\varkappa\mathcal{F}_2(\triangle_1)\triangle_1\right]\circledcirc J_{\mathrm{hom}}=0$$

$$\left[1+\varkappa\left(\mathcal{F}_2(\triangle_0)+2\mathcal{F}_3(\triangle_0)\right)\triangle_0\right]\triangle_0 H_{\mathrm{hom}}=0$$

# of extra sol. given by zeros of $[\dots]$ in s-space ($\mathcal{H}_w[\triangle_w\phi]=-s^2\mathcal{H}_w[\phi]$)

Particular parts:

$$\Phi_{21}^{\mathrm{part}}=\varkappa\mathcal{H}_1^{-1}\left[\frac{\mathcal{H}_1[\Xi_{21}]}{1-\varkappa\mathcal{F}_2(-\tilde{s}^2)\tilde{s}^2}\right]$$

$$\Phi_{22}^{\mathrm{part}}=\varkappa\mathcal{H}_0^{-1}\left[\frac{\mathcal{H}_0[\Xi_{22}]}{1-\varkappa\left(\mathcal{F}_2(-s^2)+2\mathcal{F}_3(-s^2)\right)s^2}\right]$$
$$-\varkappa\sum_{n=1}^{\infty}\sum_{k=0}^{n-1}\mathcal{H}_0^{-1}\left[\frac{(-s^2)^k\mathcal{H}_0\left[\left(f_{2,n-1}\diamondsuit+f_{3,n}\heartsuit\right)\mathcal{H}_1^{-1}\left[(-\tilde{s}^2)^{n-k-1}\mathcal{H}_1[\Phi_{21}]\right]\right]}{1-\varkappa\left(\mathcal{F}_2(-s^2)+2\mathcal{F}_3(-s^2)\right)s^2}\right]$$

$$\circledcirc J_{\mathrm{part}}=\Phi_{21}^{\mathrm{part}}\implies J_{\mathrm{part}}=2\sqrt{2}i\int_0^\rho d\rho'\,(\rho^2-\rho'^2)\Phi_{21}^{\mathrm{part}}$$

$$\triangle_0 H_{\mathrm{part}}=2\Phi_{22}^{\mathrm{part}}-\frac{\left(J_{,\rho}\right)^2}{2\rho^2}\implies H_{\mathrm{part}}=\int_0^\infty d\rho'\,\rho'\begin{Bmatrix}\log\left(\rho'/\rho_0\right), & \rho<\rho'\\ \log\left(\rho/\rho_0\right), & \rho>\rho'\end{Bmatrix}\left(2\Phi_{22}^{\mathrm{part}}-\frac{\left(J_{,\rho}\right)^2}{2\rho'^2}\right)$$

Green's function of $\triangle_0$; Cauchy formula for repeated integration

Field equations:

$$\mathcal{F}_1(\Box) = \mathcal{F}_2(\Box) = \mathcal{F}_3(\Box) = 0 \implies \boxed{\Phi_{21} = \varkappa\Xi_{21}\ , \quad \Phi_{22} = \varkappa\Xi_{22}}$$

Homogeneous parts:

$$\circledcirc J_{\mathrm{hom}} = 0\ , \quad \triangle_0 H_{\mathrm{hom}} = 0$$

$$J_{\mathrm{hom}} = c_1(u)\rho^2 + c_2(u)\ , \quad H_{\mathrm{hom}} = c_3(u)\log\rho + c_4(u)$$

Solutions:

Vacuum (source: $\rho = 0$)	Gaussian beam ($\delta(x) \to \frac{e^{-x^2/4\epsilon^2}}{2\sqrt{\pi}\epsilon}$)
$J = \frac{\varkappa\chi_{\mathrm{J}}(u)}{4\pi}$ $H = \frac{\varkappa\chi_{\mathrm{H}}(u)}{4\pi}\log\left(\frac{\rho^2}{\rho_0^2}\right)$	$J = \frac{\varkappa\chi_{\mathrm{J}}(u)}{4\pi}\left(1 - e^{-\frac{\rho^2}{4\epsilon^2}}\right)$ $H = \frac{\varkappa\chi_{\mathrm{H}}(u)}{4\pi}\left[\log\left(\frac{\rho^2}{\rho_0^2}\right) - \mathrm{Ei}\left(-\frac{\rho^2}{4\epsilon^2}\right)\right]$
$\Xi_{21} = -\frac{i\chi_{\mathrm{J}}(u)}{2^3\sqrt{2}}\left[\frac{y}{\rho}\delta(x)\delta'(y) + \frac{x}{\rho}\delta'(x)\delta(y)\right]$ $\Xi_{22} = \frac{\chi_{\mathrm{H}}(u)}{2}\delta(x)\delta(y) + \frac{\varkappa\chi_{\mathrm{J}}^2(u)}{2^4}(\delta(x)\delta(y))^2$	$\Xi_{21} = \frac{i\chi_{\mathrm{J}}(u)}{2^6\sqrt{2}\pi\epsilon^4}\rho e^{-\rho^2/4\epsilon^2}$ $\Xi_{22} = \frac{\chi_{\mathrm{H}}(u)}{2^3\pi\epsilon^2}e^{-\rho^2/4\epsilon^2} + \frac{\varkappa\chi_{\mathrm{J}}^2(u)}{2^8\pi^2\epsilon^4}e^{-\rho^2/2\epsilon^2}$

distributional sources well-defined only for lin./slow-rot. $O(\chi_{\mathrm{J}}^2) \approx 0$

specializes to type [N,N] if $\chi_{\mathrm{J}} = 0$ or outside the source

Field equations:

$$\mathcal{F}_1(\Box) = \alpha + \beta/4\,, \quad \mathcal{F}_2(\Box) = \beta\,, \quad \mathcal{F}_3(\Box) = 0\,, \quad m^2 \equiv -1/\varkappa\beta > 0$$

additional spin-2 ghost and healthy spin-0; *GR limit* $m \to \infty$

$$\boxed{\big(1 - m^{-2}\triangle_1\big)\Phi_{21} = \varkappa\Xi_{21}\,, \quad \big(1 - m^{-2}\triangle_0\big)\Phi_{22} - m^{-2}\Diamond\Phi_{21} = \varkappa\Xi_{22}}$$

Homogeneous parts:

$$\big(1 - m^{-2}\triangle_1\big)\circledcirc J_{\mathrm{hom}} = 0\,, \quad \big(1 - m^{-2}\triangle_0\big)\triangle_0 H_{\mathrm{hom}} = 0$$

$$\begin{aligned} J_{\mathrm{hom}} &= c_1(u)m\rho I_1(m\rho) + c_2(u)m\rho K_1(m\rho) + c_3(u)\rho^2 + c_4(u) \\ H_{\mathrm{hom}} &= c_5(u)I_0(m\rho) + c_6(u)K_0(m\rho) + c_7(u)\log\rho + c_8(u) \end{aligned}$$

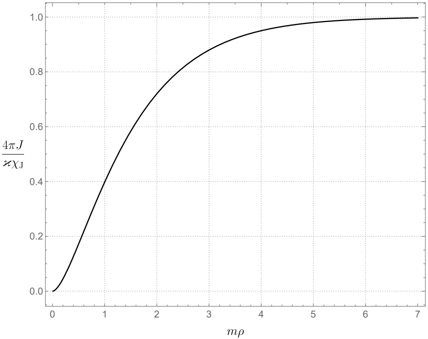
Vacuum ($\sim$GR for $\rho \to \infty$):

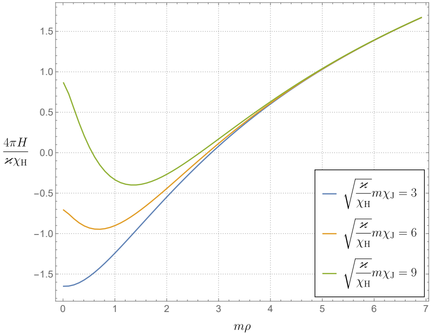
$$\boxed{J = \frac{\varkappa\chi_{\mathrm{J}}(u)}{4\pi}\big(1 - m\rho K_1(m\rho)\big)\,, \quad H = \ldots \text{(arXiv:2107.11884)}}$$

of type [III,III] even outside the source

Gyratons (spinning null matter)

Stelle gravity: Solution





Properties of J and H:

- GR limit $m \to \infty$; GR behavior for $\rho \to \infty$
- regular; $J = O(\rho^2)$ and $H = O(1)$ near $\rho = 0$
- lin./slow-rot. $O(\chi_J^2) \approx 0$:

$$H_{\text{lin}} = \frac{\varkappa \chi_H(u)}{4\pi} \left[\log\left(\frac{\rho^2}{\rho_0^2}\right) + 2K_0(m\rho) \right]$$

Gyratons (spinning null matter)

Infinite derivative gravity

Field equations:

$$\varkappa \mathcal{F}_2(\Box) = -4\varkappa \mathcal{F}_1(\Box) = \frac{e^{-\ell^2 \Box} - 1}{\Box}\ , \quad \mathcal{F}_3(\Box) = 0$$

no ghosts or extra d.o.f.; *GR limit* $\ell \to 0$; integral repr. of double-sums

$$\boxed{e^{-\ell^2 \triangle_1} \Phi_{21} = \varkappa \Xi_{21}\ , \quad e^{-\ell^2 \triangle_0} \Phi_{22} - \ell^2 \int_0^1 dt\, e^{-t\ell^2 \triangle_0} \diamondsuit e^{-(1-t)\ell^2 \triangle_1} \Phi_{21} = \varkappa \Xi_{22}}$$

Homogeneous parts:

$$e^{-\ell^2 \triangle_1} \circledcirc J_{\mathrm{hom}} = 0\ , \quad e^{-\ell^2 \triangle_0} \triangle_0 H_{\mathrm{hom}} = 0$$

$$J_{\mathrm{hom}} = c_1(u) \rho^2\ , \quad H_{\mathrm{hom}} = c_4(u)$$

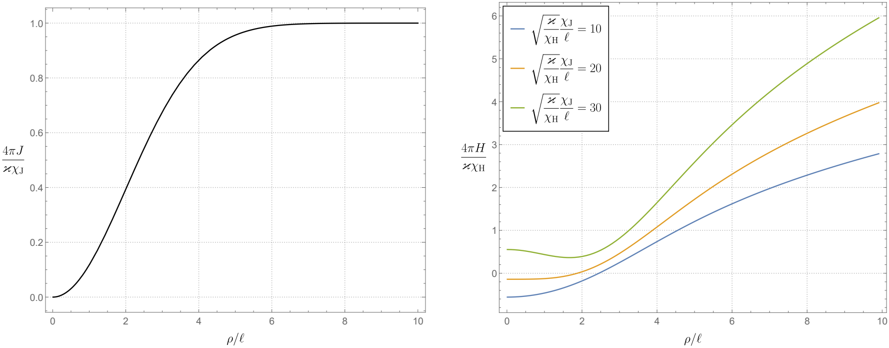
$e^{\ell^2 s^2}$ has no zeros; $c_2(u)$ and $c_3(u) \log \rho$ are excluded (GR: $\rho > 0$)

Gaussian beam ($\sim$GR-like for $\rho \to \infty$):

$$\boxed{J = \frac{\varkappa \chi_{\mathrm{J}}(u)}{4\pi} \left(1 - e^{-\frac{\rho^2}{4(\ell^2 + \epsilon^2)}}\right)\ , \quad H = \ldots \text{(arXiv:2107.11884)}}$$

Gyratons (spinning null matter)

Infinite derivative gravity: Solution



Properties of J and H:

- GR limit $\ell \to 0$; GR-like behavior for $\rho \to \infty$;
- regular; $J = O(\rho^2)$ and $H = O(1)$ near $\rho = 0$
- slow-rot/lin. $O(\chi_J^2) \approx 0$ (GR with $\epsilon^2 \to \epsilon^2 + \ell^2$):

$$H_{\text{lin}} = \frac{\varkappa \chi_H(u)}{4\pi}\left[\log\left(\frac{\rho^2}{\rho_0^2}\right) - \text{Ei}\left(-\frac{\rho^2}{4\left(\ell^2+\epsilon^2\right)}\right)\right]$$

- lin. solution for particles $H_{\text{lin}}(\epsilon \to 0)$

Hotta–Tanaka within Kundt [N,N] ($\Lambda \neq 0$):

$$\boldsymbol{g} = -\mathbf{d}u \vee \left[\mathbf{d}r + H\mathbf{d}u + \left(W\mathbf{d}\zeta + \text{c.c.}\right)\right] + P^{-2}\mathbf{d}\zeta \vee \mathbf{d}\bar{\zeta}$$

$$W = \tfrac{2\bar{\tau}}{P}r\,, \quad H = -\left(\tau\bar{\tau} + \tfrac{\Lambda}{6}\right)r^2 + H^\circ(u,\zeta,\bar{\zeta})\,, \quad P = 1 + \tfrac{\Lambda}{6}\zeta\bar{\zeta}$$

$$\tau = \left(-b + \tfrac{\Lambda}{3}a\zeta + \tfrac{\Lambda}{6}\bar{b}\zeta^2\right)/Q\,, \quad Q = a + \bar{b}\zeta + b\bar{\zeta} - \tfrac{\Lambda}{6}a\zeta\bar{\zeta}\,,$$

$\longrightarrow$ eqns. reduce to a linear PDE with $\triangle$ on $\mathbb{S}^2$ or $\mathbb{H}^2$

$$\boxed{\mathcal{G}(\triangle - 2\Lambda/3)\,(\triangle + 2\Lambda/3)\,\hat{H}^\circ = \hat{\Xi}\,, \quad \hat{H}^\circ \equiv \tfrac{P}{Q}H^\circ\,, \quad \hat{\Xi} \equiv \tfrac{P}{Q}\Xi}$$

Infinite derivative gravity with $\Lambda \neq 0$:

$$\mathcal{F}_1(\Box) = \mathcal{F}_2(\Box) = 0\,, \quad \mathcal{F}_3(\Box) = \frac{1}{2}\,\frac{e^{-\ell^2(\Box - 8\Lambda/3)} - 1}{\Box - 8\Lambda/3} \implies \boxed{\mathcal{G}(\triangle - 2\Lambda/3) = e^{-\ell^2\triangle}}$$

no ghosts or extra d.o.f.; *GR limit* $\ell \to 0$

Particular solution:

$$\hat{H}^\circ_{\text{part}} = \sum\!\!\!\!\!\!\int_\alpha \frac{e^{-\ell^2\mu_\alpha^2}}{-\mu_\alpha^2 + 2\Lambda/3}\hat{\Xi}_\alpha\psi_\alpha = -e^{-2\ell^2\Lambda/3}\int_{\ell^2}^{\infty} ds\, e^{2s\Lambda/3}\int_{\mathrm{x}'\in\mathbb{M}} \mathfrak{q}^{\frac{1}{2}}(\mathrm{x}')K(s,\mathrm{x},\mathrm{x}')\hat{\Xi}(u,\mathrm{x}')$$

eigenfunctions ψ_α, heat kernel K

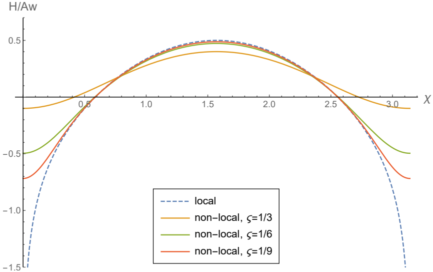
Hotta–Tanaka (null matter in (A)dS)

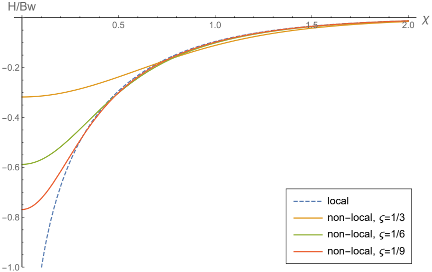
Infinite derivative gravity: Solutions

Vacuum (sources: poles of $\mathbb{S}^2$, origin of $\mathbb{H}^2$):

$$\hat{H}^\circ \propto \sum_{k=0}^{\infty} \frac{(4k+1)e^{-2k(2k+1)\varsigma_+^2}}{(2k+1)k-1} P_{2k}(\cos\chi)\,, \qquad \hat{H}^\circ \propto \int_\chi^\infty d\tilde{\chi}\, \frac{e^{\frac{3\tilde{\chi}}{2}}\left[\operatorname{erf}\left(\frac{\tilde{\chi}+3\varsigma_-^2}{2\varsigma_-}\right)-1\right]+e^{-\frac{3\tilde{\chi}}{2}}\left[\operatorname{erf}\left(\frac{\tilde{\chi}-3\varsigma_-^2}{2\varsigma_-}\right)+1\right]}{\sqrt{\cosh\tilde{\chi}-\cosh\chi}}$$

$\Lambda > 0$: expansion into Y_l^m; $\Lambda < 0$: explicit expression for K; $\varsigma_\pm \equiv \sqrt{|\Lambda|/3}\,\ell$





Properties of $\hat{H}^\circ$:

- GR limits $\varsigma_\pm \to 0$: $\hat{H}^\circ_{\text{loc}} \propto Q_1(\cos\chi), Q_1(\cosh\chi)$, where $Q_1(x) \equiv \frac{x}{2}\log\left|\frac{1+x}{1-x}\right| - 1$
- complete smearing $\varsigma_\pm \to \infty$: non-zero constant on $\mathbb{S}^2$ (finite volume)
- regular; $H = O(1)$ near the sources

Open problems

non-scalar curvature singularities? all spacetimes are of VSI/CSI; non-scalar curvature singularities require study of curvature components in PP frame along time-like and null geodesics

higher-order curvature? non-vanishing contribution: quadratic terms ($b = -2$) arising from cubic terms in the action; J remains unchanged; equation for H is still exactly solvable (additional terms quadratic in J)

gyratons propagating in (A)dS? $R =$ const. $\neq 0$; requires type [III,II], however, $b = 0$ components are constant