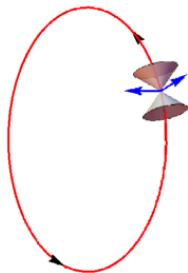


Motion of a gyroscope on a closed timelike curve



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Overview

- Consistency in spacetimes with closed timelike curves (CTCs).
- Motion of a gyroscope: Fermi-Walker transport.
- The solution space.
- Examples
- Conclusions and a conjecture.

Consistency and CTCs

David Lewis (1941-2001)

“Time travel, I maintain, is possible.” (The paradoxes of time travel, Univ. of Adelaide, 1971.) Key point: time travel is not to be ruled out *a priori*, and is possible only if it does not lead to any contradictions.



- Time travel in GR is identified with the presence of closed timelike curves.
- Chronology (by definition) is violated: we're more or less at the bottom of the ladder of causality conditions.
- Early 1990's: interest in physics in the presence of CTCs.

Consistency and CTCs

- (Novikov) Principle of Self-Consistency: “the only solutions to the laws of physics that can occur locally in the real Universe are those which are globally self-consistent” (Friedman et al. 1990)
- Probed in a number of papers, with perhaps surprising results:
 - ▶ Cauchy problem for a scalar field in class of spacetimes with CTCs generated by wormholes: data corresponding to consistent solutions dominates. (Friedman et al. 1990).
 - ▶ Classical billiards that may collide with their earlier selves (and thereby prevent the motion leading to that collision...): consistent evolutions overwhelm inconsistent ones. (Echeverria et al. 1991).
 - ▶ Consistent evolution of self-interacting mechanical systems e.g. pistons (Novikov 1991).

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- What about extended bodies?

Extended bodies

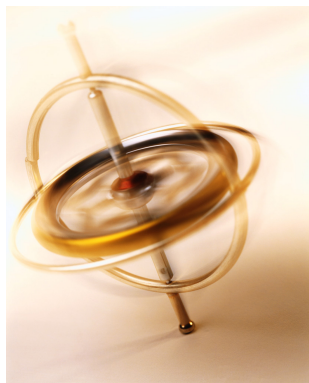
- Framework due to Dixon (1970's) - see also Harte (2015).
 - ▶ Linear & angular momentum; multipole moments; centre of mass worldline; self-interaction.
- *Can extended bodies undergo time travel in the same way that point particles can?*
- First step: consider **gyroscopes** which offer a simple way of considering extended structure on CTCs.
- *Is the motion of a gyroscope carried by a CTC consistent?*

Motion of a gyroscope...

- Gyroscope: maintains direction and angular momentum.
- Curved spacetime: gyroscope carried by a worldline γ is identified with a **spin vector** s^a - a spacelike, unit length vector, Fermi-Walker transported along γ (tangent velocity u^a , acceleration a^a):

$$u^a \nabla_a s^b = (u^b a_a - a^b u_a) s^a.$$

- **Lemma:** *Inner products (and hence norms) of spin-vectors are conserved along γ .*

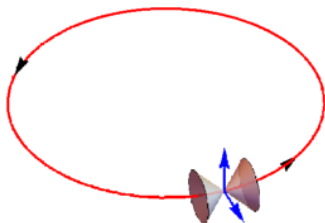


...on a CTC

- Consider initial and terminal values $t = 0, t = T$ of proper time along γ , where $p = \gamma(0) = \gamma(T)$ (γ is T -periodic).
- Define

$$T_p^{\perp,1}(M) := \{\vec{s} \in T_p(M) : g(\vec{s}, \vec{u}) = 0, g(\vec{s}, \vec{s}) = 1\} \simeq \mathbb{S}^2.$$

- F-W transport maps initial data $s^a(0) \in T_{\gamma(0)}^{\perp,1}$ to $s^a(T) \in T_{\gamma(T)}^{\perp,1}$.
- This mapping is a **rotation** of $\mathbb{S}^2 \simeq T_{\gamma(0)}^{\perp,1} = T_{\gamma(T)}^{\perp,1}$.



The solution space

Proposition

Every T -periodic closed time-like curve admits a T -periodic spin-vector.

Proposition

Let γ be a T -periodic CTC. Then either

- (i) every spin-vector along γ is T -periodic, or*
- (ii) in the set of initial data for spin-vectors along γ , initial data which yield a T -periodic spin-vector along γ form a set of measure zero.*

- (i) Gyroscopic motion on the CTC is consistent.
- (ii) Gyroscopic motion on the CTC is generically inconsistent.

Examples: Stationary, cylindrical symmetry

- $ds^2 = -F(r)d\tau^2 + 2M(r)d\tau d\phi + L(r)d\phi^2 + H(r)(dr^2 + d\zeta^2)$
- $\phi \in [0, 2\pi)$, periodic; $r \geq 0$ with regular axis at $r = 0$.
- Includes Gödel, Som-Raychaudhuri, Van Stockum (and so Tipler machines).
- Circular CTCs at constant τ, r, ζ provided $L(r) < 0$.

Proposition

Every spin-vector carried by a circular CTC γ with radius r is T_γ -periodic if and only if

$$\lambda(r) := \frac{(ML' - LM')^2}{4H|L|(FL + M^2)} = n^2 \quad \text{for some } n \in \mathbb{N}. \quad (1)$$

If this condition is not met, then there is exactly one spin-vector along γ which is T_γ -periodic.

The Gödel Profile

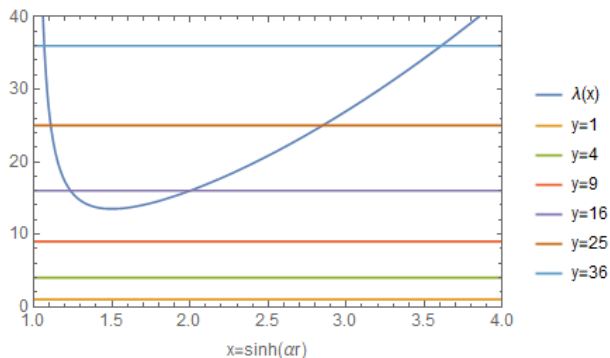
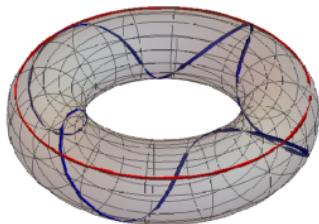


Figure: Plot of the function $\lambda(r)$ of Proposition 3 for Gödel's spacetime. The values of $r > r_{\min}$ which correspond to a circular CTC with consistent gyroscopic motion form a (countably infinite) set of measure zero. Similar results for other spacetimes in this class.

Other examples

- Kerr: CTCs beyond the inner horizon. Gödel profile holds for circular CTCs.
- Taub-NUT: The number of circular CTCs with consistent gyroscopic motion is *finite*.
- Ori's asymptotically flat time-machine (2007). Family of circular CTCs for which gyroscopic motion is *always* consistent...perturb to a family traversing a torus in both the toroidal and poloidal directions to find that the Gödel profile applies again.



Conclusions and a conjecture

- Examples: consistent gyroscopic motion occurs only on sets of measure zero.
- In general, consistency only occurs when the *transition matrix* has 1 as a triple eigenvalue. Non-unity eigenvalues $e^{\pm i\theta}$ occur on *open subsets* of the configuration space (congruence of CTCs).
- We conjecture that inconsistency is generic: these open subsets are dense.

Conclusions and a conjecture

- As well as generating 'practical' difficulties for a time-traveller's navigational system, this gives rise to *paradoxes of identity* for the gyroscope that are *enforced* by the laws of physics: inconsistency is unavoidable.

