General Perturbations of LRS Class II Cosmologies with Applications to Dissipative Fluids

Philip Semrén and Michael Bradley

Umeå University

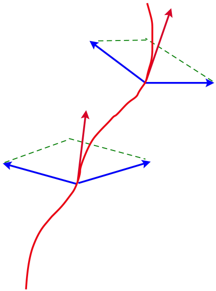
15 September 2021, Aveiro, EREP2021

1+3 covariant split of spacetime[1]

- Prefered timelike vector u^a, $u^a u_a = -1$.
  Projection operator onto 3-space:
  $h_{ab} = g_{ab} + u_a u_b$.
  Projection along time: $U_{ab} = -u_a u_b$

- Covariant "time" derivative:
  $\dot{\psi}_{a..b} \equiv u^c \nabla_c \psi_{a...b}$

- Projected "spatial" derivative:
  $D_c \psi_{a...b} \equiv h_c^f h_a^d ... h_b^e \nabla_f \psi_{d...e}$



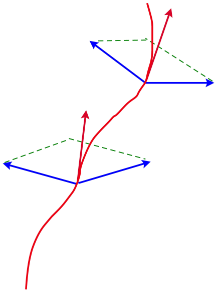
[1] G.F.R Ellis and M. Bruni, Phys. Rev. D, **40**, 1804 (1989)
G.F.R Ellis and H. van Elst, arXiv:gr-qc/9812046v5

1+3 covariant split of spacetime[1]

- Prefered timelike vector u^a, $u^a u_a = -1$.
  Projection operator onto 3-space:
  $h_{ab} = g_{ab} + u_a u_b$.
  Projection along time: $U_{ab} = -u_a u_b$

- Covariant "time" derivative:
  $\dot{\psi}_{a..b} \equiv u^c \nabla_c \psi_{a...b}$

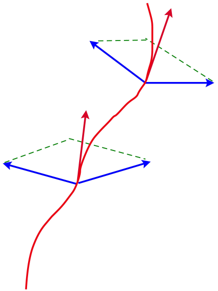
- Projected "spatial" derivative:
  $D_c \psi_{a...b} \equiv h^f_c h^d_a ... h^e_b \nabla_f \psi_{d...e}$



[1] G.F.R Ellis and M. Bruni, Phys. Rev. D, **40**, 1804 (1989)
G.F.R Ellis and H. van Elst, arXiv:gr-qc/9812046v5

1+3 covariant split of spacetime[1]

- Prefered timelike vector u^a, $u^a u_a = -1$.
  Projection operator onto 3-space:
  $h_{ab} = g_{ab} + u_a u_b$.
  Projection along time: $U_{ab} = -u_a u_b$



- Covariant "time" derivative:
  $\dot{\psi}_{a..b} \equiv u^c \nabla_c \psi_{a...b}$

- Projected "spatial" derivative:
  $D_c \psi_{a...b} \equiv h^f_c h^d_a ... h^e_b \nabla_f \psi_{d...e}$

[1] G.F.R Ellis and M. Bruni, Phys. Rev. D, **40**, 1804 (1989)
G.F.R Ellis and H. van Elst, arXiv:gr-qc/9812046v5

Covariant Variables 1+3

- Kinematic quantities of u^a: A_a, θ, σ_{ab} and ω_{ab} (acceleration, expansion, shear and vorticity) from

$$\nabla_a u_b = -u_a A_b + D_a u_b = -u_a A_b + \frac{1}{3}\theta h_{ab} + \omega_{ab} + \sigma_{ab}$$

- R_{ab} quantities: μ, p, q_a, π_{ab} and Λ (energy density, isotropic stress, energy flux, anisotropic stress and cosmological constant) from

$$T_{ab} = (\mu + p)u_a u_b + p g_{ab} + 2q_{(a}u_{b)} + \pi_{ab} = R_{ab} - \frac{1}{2}R g_{ab} + \Lambda g_{ab}$$

- Weyl tensor: $E_{ab} \equiv C_{acbd}u^c u^d$ and $H_{ab} \equiv \frac{1}{2}\eta_{ade}C^{de}{}_{bc}u^c$ (Electric and magnetic parts)

Covariant Variables 1+3

- Kinematic quantities of u^a: A_a, θ, σ_{ab} and ω_{ab} (acceleration, expansion, shear and vorticity) from

$$\nabla_a u_b = -u_a A_b + D_a u_b = -u_a A_b + \frac{1}{3}\theta h_{ab} + \omega_{ab} + \sigma_{ab}$$

- R_{ab} quantities: μ, p, q_a, π_{ab} and Λ (energy density, isotropic stress, energy flux, anisotropic stress and cosmological constant) from

$$T_{ab} = (\mu + p)u_a u_b + p g_{ab} + 2q_{(a}u_{b)} + \pi_{ab} = R_{ab} - \frac{1}{2}R g_{ab} + \Lambda g_{ab}$$

- Weyl tensor: $E_{ab} \equiv C_{acbd}u^c u^d$ and $H_{ab} \equiv \frac{1}{2}\eta_{ade}C^{de}{}_{bc}u^c$ (Electric and magnetic parts)

Covariant Variables 1+3

- Kinematic quantities of u^a: A_a, θ, σ_{ab} and ω_{ab}
(acceleration, expansion, shear and vorticity) from

$$\nabla_a u_b = -u_a A_b + D_a u_b = -u_a A_b + \frac{1}{3}\theta h_{ab} + \omega_{ab} + \sigma_{ab}$$

- R_{ab} quantities: μ, p, q_a, π_{ab} and Λ
(energy density, isotropic stress, energy flux, anisotropic stress and cosmological constant) from

$$T_{ab} = (\mu + p)u_a u_b + p g_{ab} + 2q_{(a}u_{b)} + \pi_{ab} = R_{ab} - \frac{1}{2}R g_{ab} + \Lambda g_{ab}$$

- Weyl tensor: $E_{ab} \equiv C_{acbd}u^c u^d$ and $H_{ab} \equiv \frac{1}{2}\eta_{ade}C^{de}{}_{bc}u^c$
(Electric and magnetic parts)

1+1+2 covariant split[2]

- Prefered spacelike vector n^a with $u^a n_a = 0$. Projection operator onto perpendicular 2-space with $N_{ab} = h_{ab} - n_a n_b$.
- Derivative along n^a:

$$\hat{\psi}_{a\ldots b} \equiv n^c D_c \psi_{a\ldots b} = n^c h_c^f h_a^d \ldots h_b^e \nabla_f \psi_{d\ldots e}$$

- Derivative perpendicular to n^a:

$$\delta_c \psi_{a\ldots b} \equiv N_c^f N_a^d \ldots N_b^e D_f \psi_{d\ldots e}$$

[2] C. A. Clarkson & R. Barrett, Class. Quan. Grav. **20**, 3855 (2003)
C.A. Clarkson, Phys. Rev. D, **76**, 104034 (2007)

1+1+2 covariant split[2]

- Prefered spacelike vector n^a with $u^a n_a = 0$. Projection operator onto perpendicular 2-space with $N_{ab} = h_{ab} - n_a n_b$.

- Derivative along n^a:

$$\hat{\psi}_{a...b} \equiv n^c D_c \psi_{a...b} = n^c h_c^f h_a^d ... h_b^e \nabla_f \psi_{d...e}$$

- Derivative perpendicular to n^a:

$$\delta_c \psi_{a...b} \equiv N_c^f N_a^d ... N_b^e D_f \psi_{d...e}$$

[2] C. A. Clarkson & R. Barrett, Class. Quan. Grav. **20**, 3855 (2003)
C.A. Clarkson, Phys. Rev. D, **76**, 104034 (2007)

1+1+2 covariant split[2]

- Prefered spacelike vector n^a with $u^a n_a = 0$. Projection operator onto perpendicular 2-space with $N_{ab} = h_{ab} - n_a n_b$.

- Derivative along n^a:

$$\hat{\psi}_{a\ldots b} \equiv n^c D_c \psi_{a\ldots b} = n^c h_c^f h_a^d \ldots h_b^e \nabla_f \psi_{d\ldots e}$$

- Derivative perpendicular to n^a:

$$\delta_c \psi_{a\ldots b} \equiv N_c^f N_a^d \ldots N_b^e D_f \psi_{d\ldots e}$$

[2] C. A. Clarkson & R. Barrett, Class. Quan. Grav. **20**, 3855 (2003)
C.A. Clarkson, Phys. Rev. D, **76**, 104034 (2007)

Covariant Variables 1+1+2

- Kinematic quantities of n^a: a_a, ϕ, ζ_{ab}, ξ, $\mathcal{A}$ and α_a from

$$D_a n_b = n_a a_b + \frac{1}{2}\phi N_{ab} + \zeta_{ab} + \xi \epsilon_{ab}\,,\ \dot{n}_a = \mathcal{A} u_a + \alpha_a$$

- 1+2 split:

  Vectors: $\omega^a = \Omega n^a + \Omega^a$ etc.

  Tensors: $\sigma_{ab} = \Sigma(n_a n_b - \frac{1}{2}N_{ab}) + 2\Sigma_{(a} n_{b)} + \Sigma_{ab}$ etc.

- A_a, ω_a, q_a, σ_{ab}, E_{ab}, H_{ab} and Π_{ab} split into

  $\mathcal{A}, \mathcal{A}_a, \Omega, \Omega_a, Q, Q_a, \mathcal{E}, \mathcal{E}_a, \mathcal{E}_{ab}, \mathcal{H}, \mathcal{H}_a, \mathcal{H}_{ab}, \Pi, \Pi_a, \Pi_{ab}$ respectively

Covariant Variables 1+1+2

- Kinematic quantities of n^a: a_a, ϕ, ζ_{ab}, ξ, $\mathcal{A}$ and α_a from

$$D_a n_b = n_a a_b + \frac{1}{2}\phi N_{ab} + \zeta_{ab} + \xi\epsilon_{ab}\,,\;\; \dot{n}_a = \mathcal{A}u_a + \alpha_a$$

- 1+2 split:

  Vectors: $\omega^a = \Omega n^a + \Omega^a$ etc.

  Tensors: $\sigma_{ab} = \Sigma(n_a n_b - \frac{1}{2}N_{ab}) + 2\Sigma_{(a}n_{b)} + \Sigma_{ab}$ etc.

- A_a, ω_a, q_a, σ_{ab}, E_{ab}, H_{ab} and Π_{ab} split into

  $\mathcal{A}, \mathcal{A}_a, \Omega, \Omega_a, Q, Q_a, \mathcal{E}, \mathcal{E}_a, \mathcal{E}_{ab}, \mathcal{H}, \mathcal{H}_a, \mathcal{H}_{ab}, \Pi, \Pi_a, \Pi_{ab}$
  respectively

Covariant Variables 1+1+2

- Kinematic quantities of n^a: a_a, ϕ, ζ_{ab}, ξ, $\mathcal{A}$ and α_a from

$$D_a n_b = n_a a_b + \frac{1}{2}\phi N_{ab} + \zeta_{ab} + \xi\epsilon_{ab}\,,\ \dot{n}_a = \mathcal{A}u_a + \alpha_a$$

- 1+2 split:

  Vectors: $\omega^a = \Omega n^a + \Omega^a$ etc.

  Tensors: $\sigma_{ab} = \Sigma(n_a n_b - \frac{1}{2}N_{ab}) + 2\Sigma_{(a}n_{b)} + \Sigma_{ab}$ etc.

- A_a, ω_a, q_a, σ_{ab}, E_{ab}, H_{ab} and Π_{ab} split into

  $\mathcal{A}, \mathcal{A}_a, \Omega, \Omega_a, Q, Q_a, \mathcal{E}, \mathcal{E}_a, \mathcal{E}_{ab}, \mathcal{H}, \mathcal{H}_a, \mathcal{H}_{ab}, \Pi, \Pi_a, \Pi_{ab}$ respectively

Field equations, Integrability conditions, commutators

- Einstein's equations: $T_{ab} = R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab}$ (So far general energy-momentum tensor)

  Integrability conditons:

- Ricci identities for u^a and n^a:

$$u_{a;bc} - u_{a;cb} = R^d_{abc}u_d \ , \quad n_{a;bc} - n_{a;cb} = R^d_{abc}n_d$$

- Bianchi identities $R_{ab[cd;e]} = 0$, $R_{a[bcd]} = 0$

- Commutators between the differential operators:

  $\dot{} \equiv u^a\nabla_a$, $\hat{} \equiv n^a D_a$ and δ_a.

Field equations, Integrability conditions, commutators

- Einstein's equations: $T_{ab} = R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab}$ (So far general energy-momentum tensor)

  Integrability conditons:

- Ricci identities for u^a and n^a:

$$u_{a;bc} - u_{a;cb} = R^d_{abc} u_d \ , \quad n_{a;bc} - n_{a;cb} = R^d_{abc} n_d$$

- Bianchi identities $R_{ab[cd;e]} = 0$, $R_{a[bcd]} = 0$

- Commutators between the differential operators:

  $\dot{} \equiv u^a \nabla_a$, $\hat{} \equiv n^a D_a$ and δ_a.

Field equations, Integrability conditions, commutators

- Einstein's equations: $T_{ab} = R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab}$ (So far general energy-momentum tensor)

  Integrability conditons:

- Ricci identities for u^a and n^a:

$$u_{a;bc} - u_{a;cb} = R^d_{abc}u_d \ , \quad n_{a;bc} - n_{a;cb} = R^d_{abc}n_d$$

- Bianchi identities $R_{ab[cd;e]} = 0$, $R_{a[bcd]} = 0$

- Commutators between the differential operators:

  $\dot{} \equiv u^a\nabla_a$, $\hat{} \equiv n^a D_a$ and δ_a.

Field equations, Integrability conditions, commutators

- Einstein's equations: $T_{ab} = R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab}$ (So far general energy-momentum tensor)

  Integrability conditons:

- Ricci identities for u^a and n^a:

$$u_{a;bc} - u_{a;cb} = R^d_{abc}u_d \ , \quad n_{a;bc} - n_{a;cb} = R^d_{abc}n_d$$

- Bianchi identities $R_{ab[cd;e]} = 0$, $R_{a[bcd]} = 0$

- Commutators between the differential operators:

  $\dot{} \equiv u^a\nabla_a$, $\hat{} \equiv n^a D_a$ and δ_a.

1+1+2 Evolution and propagation equations, constraints

1+1+2 split:

- Evolution equations: $\dot{\phi} = \ldots$ etc
- Propagation equations: $\hat{\phi} = \ldots$ etc
- Mixture: $\hat{\mathcal{A}} - \dot{\theta} = \ldots$ etc
- Constraints: $\delta_a \Omega^a + \epsilon_{ab} \delta^a \Sigma^b = \ldots$ etc

1+1+2 Evolution and propagation equations, constraints

1+1+2 split:

- Evolution equations: $\dot{\phi} = ...$ etc
- Propagation equations: $\hat{\phi} = ...$ etc
- Mixture: $\hat{\mathcal{A}} - \dot{\theta} = ...$ etc
- Constraints: $\delta_a \Omega^a + \epsilon_{ab} \delta^a \Sigma^b = ...$ etc

1+1+2 Evolution and propagation equations, constraints

1+1+2 split:

- Evolution equations: $\dot{\phi} = \ldots$ etc
- Propagation equations: $\hat{\phi} = \ldots$ etc
- Mixture: $\hat{\mathcal{A}} - \dot{\theta} = \ldots$ etc
- Constraints: $\delta_a \Omega^a + \epsilon_{ab} \delta^a \Sigma^b = \ldots$ etc

1+1+2 Evolution and propagation equations, constraints

1+1+2 split:

- Evolution equations: $\dot{\phi} = \ldots$ etc
- Propagation equations: $\hat{\phi} = \ldots$ etc
- Mixture: $\hat{\mathcal{A}} - \dot{\theta} = \ldots$ etc
- Constraints: $\delta_a \Omega^a + \epsilon_{ab} \delta^a \Sigma^b = \ldots$ etc

1+1+2 Evolution and propagation equations, constraints

1+1+2 split:

- Evolution equations: $\dot{\phi} = ...$ etc
- Propagation equations: $\hat{\phi} = ...$ etc
- Mixture: $\hat{\mathcal{A}} - \dot{\theta} = ...$ etc
- Constraints: $\delta_a \Omega^a + \epsilon_{ab} \delta^a \Sigma^b = ...$ etc

Background: LRS II

Homogeneous LRS II cosmologies with cosmological constant Λ.

- Locally Rotationally Symmetric (LRS).
  LRS II: $H_{ab} = \omega_{ab} = \xi = 0$. Also $\phi = 0$.
- $ds^2 = -dt^2 + a_1^2(t)dz^2 + a_2^2(t)\left(d\theta^2 + f_{\mathcal{K}}(\theta)d\varphi^2\right)$

  where $f_1(\theta) = \sin^2\theta$, $f_{-1}(\theta) = \sinh^2\theta$ or $f_0(\theta) = 1$, depending on the curvature of the 2-sheets.
- The expansion and shear are given by

$$\theta = \frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}\,, \quad \Sigma \equiv \sigma_{11} = -2\sigma_{22} = -2\sigma_{33} = \frac{2}{3}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)$$

Background: LRS II

Homogeneous LRS II cosmologies with cosmological constant Λ.

- Locally Rotationally Symmetric (LRS).
  LRS II: $H_{ab} = \omega_{ab} = \xi = 0$. Also $\phi = 0$.
- $ds^2 = -dt^2 + a_1^2(t)dz^2 + a_2^2(t)\left(d\theta^2 + f_{\mathcal{K}}(\theta)d\varphi^2\right)$
  where $f_1(\theta) = \sin^2\theta$, $f_{-1}(\theta) = \sinh^2\theta$ or $f_0(\theta) = 1$, depending on the curvature of the 2-sheets.
- The expansion and shear are given by

$$\theta = \frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}, \quad \Sigma \equiv \sigma_{11} = -2\sigma_{22} = -2\sigma_{33} = \frac{2}{3}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)$$

Background: LRS II

Homogeneous LRS II cosmologies with cosmological constant Λ.

- Locally Rotationally Symmetric (LRS).
  LRS II: $H_{ab} = \omega_{ab} = \xi = 0$. Also $\phi = 0$.
- $ds^2 = -dt^2 + a_1^2(t)dz^2 + a_2^2(t)\left(d\theta^2 + f_{\mathcal{K}}(\theta)d\varphi^2\right)$

  where $f_1(\theta) = \sin^2\theta$, $f_{-1}(\theta) = \sinh^2\theta$ or $f_0(\theta) = 1$, depending on the curvature of the 2-sheets.
- The expansion and shear are given by

$$\theta = \frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}, \quad \Sigma \equiv \sigma_{11} = -2\sigma_{22} = -2\sigma_{33} = \frac{2}{3}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)$$

Background: LRS II

Homogeneous LRS II cosmologies with cosmological constant Λ.

- Locally Rotationally Symmetric (LRS).
  LRS II: $H_{ab} = \omega_{ab} = \xi = 0$. Also $\phi = 0$.
- $ds^2 = -dt^2 + a_1^2(t)dz^2 + a_2^2(t)\left(d\theta^2 + f_{\mathcal{K}}(\theta)d\varphi^2\right)$
  where $f_1(\theta) = \sin^2\theta$, $f_{-1}(\theta) = \sinh^2\theta$ or $f_0(\theta) = 1$, depending on the curvature of the 2-sheets.
- The expansion and shear are given by

$$\theta = \frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}\,, \quad \Sigma \equiv \sigma_{11} = -2\sigma_{22} = -2\sigma_{33} = \frac{2}{3}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)$$

Evolution equations

- The electric part of the Weyl tensor is given algebraically as

$$\mathcal{E} = -\frac{2}{3}\mu - \frac{2}{3}\Lambda - \Sigma^2 + \frac{2}{9}\theta^2 + \frac{1}{3}\Sigma\theta - \frac{1}{2}\Pi\,,$$

- Evolution equations

$$\begin{aligned}\dot{\Sigma} &= \frac{2}{3}\mu + \frac{2}{3}\Lambda + \frac{1}{2}\Sigma^2 - \Sigma\theta - \frac{2}{9}\theta^2 + \Pi\\ \dot{\mu} &= -\theta(\mu + p) - \frac{3}{2}\Sigma\Pi\\ \dot{\theta} &= -\frac{1}{3}\theta^2 - \frac{1}{2}(\mu + 3p - 2\Lambda) - \frac{3}{2}\Sigma^2\,.\end{aligned}$$

- p and Π freely specifiable.

Evolution equations

- The electric part of the Weyl tensor is given algebraically as

$$\mathcal{E} \quad = \quad -\frac{2}{3}\mu - \frac{2}{3}\Lambda - \Sigma^2 + \frac{2}{9}\theta^2 + \frac{1}{3}\Sigma\theta - \frac{1}{2}\Pi\,,$$

- Evolution equations

$$\begin{aligned} \dot{\Sigma} &= \frac{2}{3}\mu + \frac{2}{3}\Lambda + \frac{1}{2}\Sigma^2 - \Sigma\theta - \frac{2}{9}\theta^2 + \Pi \\ \dot{\mu} &= -\theta(\mu + p) - \frac{3}{2}\Sigma\Pi \\ \dot{\theta} &= -\frac{1}{3}\theta^2 - \frac{1}{2}(\mu + 3p - 2\Lambda) - \frac{3}{2}\Sigma^2\,. \end{aligned}$$

- p and Π freely specifiable.

Evolution equations

- The electric part of the Weyl tensor is given algebraically as

$$\mathcal{E} = -\frac{2}{3}\mu - \frac{2}{3}\Lambda - \Sigma^2 + \frac{2}{9}\theta^2 + \frac{1}{3}\Sigma\theta - \frac{1}{2}\Pi\,,$$

- Evolution equations

$$\begin{aligned}
\dot{\Sigma} &= \frac{2}{3}\mu + \frac{2}{3}\Lambda + \frac{1}{2}\Sigma^2 - \Sigma\theta - \frac{2}{9}\theta^2 + \Pi \\
\dot{\mu} &= -\theta(\mu + p) - \frac{3}{2}\Sigma\Pi \\
\dot{\theta} &= -\frac{1}{3}\theta^2 - \frac{1}{2}(\mu + 3p - 2\Lambda) - \frac{3}{2}\Sigma^2\,.
\end{aligned}$$

- p and Π freely specifiable.

Gauge problem in perturbation theory

- How to define the background metric $\bar{g}_{ab}$ of the physical lumpy universe with metric $g_{ab} = \bar{g}_{ab} + \delta g_{ab}$? No unique way of identifying points on background and real universe.

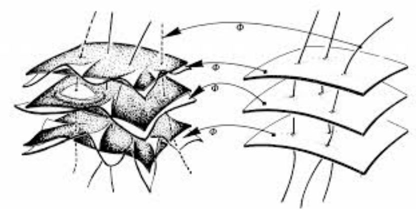


Figure: Ellis and Bruni, Phys. Rev. D, 40, 1804 (1989)

- GIC - Gauge invariant and covariant: Choose as the "small" quantities variables that are covariant and zero on background. Gauge invariant: Stewart-Walker lemma[3].

[3] Proc. R. Soc. London **A341** 49 (1974)

Gauge problem in perturbation theory

- How to define the background metric $\bar{g}_{ab}$ of the physical lumpy universe with metric $g_{ab} = \bar{g}_{ab} + \delta g_{ab}$? No unique way of identifying points on background and real universe.

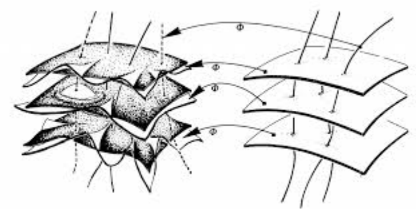


Figure: Ellis and Bruni, Phys. Rev. D, 40, 1804 (1989)

- GIC - Gauge invariant and covariant: Choose as the "small" quantities variables that are covariant and zero on background. Gauge invariant: Stewart-Walker lemma[3].

[3] Proc. R. Soc. London **A341** 49 (1974)

Perturbations

- Variables that are nonzero on the background: $\{\theta, \Sigma, \mathcal{E}, \mu, p, \Pi\}$
- Variables that are zero on the background:

$$\{a_a, \phi, \xi, \zeta_{ab}, \alpha_a, \mathcal{A}, \mathcal{A}_a, \Omega, \Omega_a, \Sigma_a, \Sigma_{ab}, \mathcal{E}_a, \mathcal{E}_{ab}, \mathcal{H}, \mathcal{H}_a, \mathcal{H}_{ab}, \Pi_a, \Pi_{ab}\}$$

  Freedom in first order choice of n_a can be used to put $a_a = 0$.
- Gradients of background variables (zero on background):

$$\{W_a \equiv \delta_a\theta,\ V_a \equiv \delta_a\Sigma,\ X_a \equiv \delta_a\mathcal{E},\ \mu_a \equiv \delta_a\mu,\ p_a \equiv \delta_a p,\ Y_a \equiv \delta_a\Pi\}$$

- (Why not $\hat{\theta} \equiv n^a D_a\theta$ etc.?: Can be expressed in terms of $\delta_a\theta$ etc. when doing a harmonic decomposition.)
- 0:th order + 1:st order system

Perturbations

- Variables that are nonzero on the background: $\{\theta, \Sigma, \mathcal{E}, \mu, p, \Pi\}$
- Variables that are zero on the background:

  $$\{a_a, \phi, \xi, \zeta_{ab}, \alpha_a, \mathcal{A}, \mathcal{A}_a, \Omega, \Omega_a, \Sigma_a, \Sigma_{ab}, \mathcal{E}_a, \mathcal{E}_{ab}, \mathcal{H}, \mathcal{H}_a, \mathcal{H}_{ab}, \Pi_a, \Pi_{ab}\}$$

  Freedom in first order choice of n_a can be used to put $a_a = 0$.
- Gradients of background variables (zero on background):

  $$\{W_a \equiv \delta_a\theta,\ V_a \equiv \delta_a\Sigma,\ X_a \equiv \delta_a\mathcal{E},\ \mu_a \equiv \delta_a\mu,\ p_a \equiv \delta_a p,\ Y_a \equiv \delta_a\Pi\}$$

- (Why not $\hat{\theta} \equiv n^a D_a\theta$ etc.?: Can be expressed in terms of $\delta_a\theta$ etc. when doing a harmonic decomposition.)
- 0:th order + 1:st order system

Perturbations

- Variables that are nonzero on the background: $\{\theta, \Sigma, \mathcal{E}, \mu, p, \Pi\}$
- Variables that are zero on the background:

  $$\{a_a, \phi, \xi, \zeta_{ab}, \alpha_a, \mathcal{A}, \mathcal{A}_a, \Omega, \Omega_a, \Sigma_a, \Sigma_{ab}, \mathcal{E}_a, \mathcal{E}_{ab}, \mathcal{H}, \mathcal{H}_a, \mathcal{H}_{ab}, \Pi_a, \Pi_{ab}\}$$

  Freedom in first order choice of n_a can be used to put $a_a = 0$.
- Gradients of background variables (zero on background):

  $$\{W_a \equiv \delta_a\theta,\ V_a \equiv \delta_a\Sigma,\ X_a \equiv \delta_a\mathcal{E},\ \mu_a \equiv \delta_a\mu,\ p_a \equiv \delta_a p,\ Y_a \equiv \delta_a\Pi\}$$
- (Why not $\hat{\theta} \equiv n^a D_a\theta$ etc.?: Can be expressed in terms of $\delta_a\theta$ etc. when doing a harmonic decomposition.)
- 0:th order + 1:st order system

Perturbations

- Variables that are nonzero on the background:
  $\{\theta, \Sigma, \mathcal{E}, \mu, p, \Pi\}$
- Variables that are zero on the background:

  $$\{a_a, \phi, \xi, \zeta_{ab}, \alpha_a, \mathcal{A}, \mathcal{A}_a, \Omega, \Omega_a, \Sigma_a, \Sigma_{ab}, \mathcal{E}_a, \mathcal{E}_{ab}, \mathcal{H}, \mathcal{H}_a, \mathcal{H}_{ab}, \Pi_a, \Pi_{ab}\}$$

  Freedom in first order choice of n_a can be used to put $a_a = 0$.
- Gradients of background variables (zero on background):

  $$\{W_a \equiv \delta_a\theta,\ V_a \equiv \delta_a\Sigma,\ X_a \equiv \delta_a\mathcal{E},\ \mu_a \equiv \delta_a\mu,\ p_a \equiv \delta_a p,\ Y_a \equiv \delta_a\Pi\}$$
- (Why not $\hat{\theta} \equiv n^a D_a\theta$ etc.?: Can be expressed in terms of $\delta_a\theta$ etc. when doing a harmonic decomposition.)
- 0:th order + 1:st order system

Perturbations

- Variables that are nonzero on the background:
  $\{\theta, \Sigma, \mathcal{E}, \mu, p, \Pi\}$
- Variables that are zero on the background:

  $$\{a_a, \phi, \xi, \zeta_{ab}, \alpha_a, \mathcal{A}, \mathcal{A}_a, \Omega, \Omega_a, \Sigma_a, \Sigma_{ab}, \mathcal{E}_a, \mathcal{E}_{ab}, \mathcal{H}, \mathcal{H}_a, \mathcal{H}_{ab}, \Pi_a, \Pi_{ab}\}$$

  Freedom in first order choice of n_a can be used to put $a_a = 0$.
- Gradients of background variables (zero on background):

  $$\{W_a \equiv \delta_a\theta,\ V_a \equiv \delta_a\Sigma,\ X_a \equiv \delta_a\mathcal{E},\ \mu_a \equiv \delta_a\mu,\ p_a \equiv \delta_a p,\ Y_a \equiv \delta_a\Pi\}$$

- (Why not $\hat{\theta} \equiv n^a D_a\theta$ etc.?: Can be expressed in terms of $\delta_a\theta$ etc. when doing a harmonic decomposition.)
- 0:th order + 1:st order system

Perturbations

- Variables that are nonzero on the background: $\{\theta, \Sigma, \mathcal{E}, \mu, p, \Pi\}$
- Variables that are zero on the background:

$$\{a_a, \phi, \xi, \zeta_{ab}, \alpha_a, \mathcal{A}, \mathcal{A}_a, \Omega, \Omega_a, \Sigma_a, \Sigma_{ab}, \mathcal{E}_a, \mathcal{E}_{ab}, \mathcal{H}, \mathcal{H}_a, \mathcal{H}_{ab}, \Pi_a, \Pi_{ab}\}$$

  Freedom in first order choice of n_a can be used to put $a_a = 0$.
- Gradients of background variables (zero on background):

$$\{W_a \equiv \delta_a\theta,\ V_a \equiv \delta_a\Sigma,\ X_a \equiv \delta_a\mathcal{E},\ \mu_a \equiv \delta_a\mu,\ p_a \equiv \delta_a p,\ Y_a \equiv \delta_a\Pi\}$$

- (Why not $\hat{\theta} \equiv n^a D_a\theta$ etc.?: Can be expressed in terms of $\delta_a\theta$ etc. when doing a harmonic decomposition.)
- 0:th order + 1:st order system

Harmonic decomposition[4]

Harmonic decomposition in terms of comoving wavenumbers $k_\parallel$ and $k_\perp$. Turns system into first order ODEs and constraints.

- Scalars: $\Psi = \sum_{k_\parallel, k_\perp} \Psi^S_{k_\parallel k_\perp} P_{k_\parallel} Q_{k_\perp}$,

  $\hat{\hat{P}} = -\frac{k_\parallel^2}{a_1^2} P, \quad \delta^2 Q_\perp = -\frac{k_\perp^2}{a_2^2} Q_\perp$

- Vectors: $\Psi_a = \sum_{k_\parallel, k_\perp} P_{k_\parallel} \left(\Psi^V_{k_\parallel k_\perp} Q_a^{k_\perp} + \overline{\Psi}^V_{k_\parallel k_\perp} \overline{Q}_a^{k_\perp} \right)$,

  $Q_a^{k_\perp} = a_2 \delta_a Q^{k_\perp} \ , \quad \overline{Q}_a^{k_\perp} = a_2 \varepsilon_{ab} \delta^b Q^{k_\perp}$

- Tensors: $\Psi_{ab} = \sum_{k_\parallel, k_\perp} P_{k_\parallel} \left(\Psi^T_{k_\parallel, k_\perp} Q_{ab}^{k_\perp} + \overline{\Psi}^T_{k_\parallel, k_\perp} \overline{Q}_{ab}^{k_\perp} \right)$,

  $Q_{ab}^{k_\perp} = a_2^2 \delta_{\{a} \delta_{b\}} Q^{k_\perp} \ , \ \overline{Q}_{ab}^{k_\perp} = a_2^2 \varepsilon_{c\{a} \delta^c \delta_{b\}} Q^{k_\perp}$

[4] C.A. Clarkson, Phys. Rev. D, **76**, 104034 (2007)

Harmonic decomposition[4]

Harmonic decomposition in terms of comoving wavenumbers $k_\parallel$ and $k_\perp$. Turns system into first order ODEs and constraints.

- Scalars: $\Psi = \sum_{k_\parallel, k_\perp} \Psi^S_{k_\parallel k_\perp} P_{k_\parallel} Q_{k_\perp}$,

  $\hat{\hat{P}} = -\frac{k_\parallel^2}{a_1^2} P, \quad \delta^2 Q_\perp = -\frac{k_\perp^2}{a_2^2} Q_\perp$

- Vectors: $\Psi_a = \sum_{k_\parallel, k_\perp} P_{k_\parallel} \left(\Psi^V_{k_\parallel k_\perp} Q_a^{k_\perp} + \overline{\Psi}^V_{k_\parallel k_\perp} \overline{Q}_a^{k_\perp} \right)$,

  $Q_a^{k_\perp} = a_2 \delta_a Q^{k_\perp} \ , \quad \overline{Q}_a^{k_\perp} = a_2 \varepsilon_{ab} \delta^b Q^{k_\perp}$

- Tensors: $\Psi_{ab} = \sum_{k_\parallel, k_\perp} P_{k_\parallel} \left(\Psi^T_{k_\parallel, k_\perp} Q_{ab}^{k_\perp} + \overline{\Psi}^T_{k_\parallel, k_\perp} \overline{Q}_{ab}^{k_\perp} \right)$,

  $Q_{ab}^{k_\perp} = a_2^2 \delta_{\{a} \delta_{b\}} Q^{k_\perp} \ , \quad \overline{Q}_{ab}^{k_\perp} = a_2^2 \varepsilon_{c\{a} \delta^c \delta_{b\}} Q^{k_\perp}$

[4] C.A. Clarkson, Phys. Rev. D, **76**, 104034 (2007)

Harmonic decomposition[4]

Harmonic decomposition in terms of comoving wavenumbers $k_{\parallel}$ and $k_{\perp}$. Turns system into first order ODEs and constraints.

- Scalars: $\Psi = \sum_{k_{\parallel},k_{\perp}} \Psi^{S}_{k_{\parallel} k_{\perp}} P_{k_{\parallel}} Q_{k_{\perp}}$,

  $\hat{\hat{P}} = -\frac{k_{\parallel}^2}{a_1^2} P, \quad \delta^2 Q_{\perp} = -\frac{k_{\perp}^2}{a_2^2} Q_{\perp}$

- Vectors: $\Psi_a = \sum_{k_{\parallel},k_{\perp}} P_{k_{\parallel}} \left(\Psi^{V}_{k_{\parallel} k_{\perp}} Q^{k_{\perp}}_a + \overline{\Psi}^{V}_{k_{\parallel} k_{\perp}} \overline{Q}^{k_{\perp}}_a \right)$,

  $Q^{k_{\perp}}_a = a_2 \delta_a Q^{k_{\perp}} \;, \quad \overline{Q}^{k_{\perp}}_a = a_2 \varepsilon_{ab} \delta^b Q^{k_{\perp}}$

- Tensors: $\Psi_{ab} = \sum_{k_{\parallel},k_{\perp}} P_{k_{\parallel}} \left(\Psi^{T}_{k_{\parallel},k_{\perp}} Q^{k_{\perp}}_{ab} + \overline{\Psi}^{T}_{k_{\parallel},k_{\perp}} \overline{Q}^{k_{\perp}}_{ab} \right)$,

  $Q^{k_{\perp}}_{ab} = a_2^2 \delta_{\{a} \delta_{b\}} Q^{k_{\perp}} \;, \; \overline{Q}^{k_{\perp}}_{ab} = a_2^2 \varepsilon_{c\{a} \delta^c \delta_{b\}} Q^{k_{\perp}}$

[4] C.A. Clarkson, Phys. Rev. D, **76**, 104034 (2007)

Evolution equations

The 1:st order system can be reduced into two subsystems for the even and odd harmonic coefficients respectively [5].

- Even sector:
  Evolution equations for: $\overline{\Omega}^V$, μ^V, $\overline{\mathcal{H}}^T$, $\mathcal{E}^T$, Σ^T, Q^V and Q^S

  Freely specifiable: p^V, $\mathcal{A}^V$, $\mathcal{A}^S$, Π^V, Π^T and Y^V

  Algebraically given: ζ^T, $\mathcal{E}^V$, $\overline{\mathcal{H}}^V$, Σ^V, α^V, W^V, V^V, X^V and ϕ^S
- Odd sector:
  Evolution equations for: Ω^S, $\mathcal{H}^T$, $\overline{\mathcal{E}}^T$, $\overline{Q}^V$

  Freely specifiable: $\overline{\mathcal{A}}^V$, $\overline{\Pi}^V$ and $\overline{\Pi}^T$

  Algebraically given: $\overline{\zeta}^T$, $\overline{\mathcal{E}}^V$, $\mathcal{H}^S$, $\mathcal{H}^V$, $\overline{\Sigma}^V$, $\overline{\Sigma}^T$, $\overline{\alpha}^V$, $\overline{V}^V$, $\overline{W}^V$, $\overline{X}^V$, $\overline{Y}^V$, ξ^S, Ω^V, $\overline{p}^V$ and $\overline{\mu}^V$

[5] In the following the indices $k_\parallel k_\perp$ on the harmonic coefficients are suppressed

Evolution equations

The 1:st order system can be reduced into two subsystems for the even and odd harmonic coefficients respectively [5].

- Even sector:
  Evolution equations for: $\overline{\Omega}^V$, μ^V, $\overline{\mathcal{H}}^T$, $\mathcal{E}^T$, Σ^T, Q^V and Q^S

  Freely specifiable: p^V, $\mathcal{A}^V$, $\mathcal{A}^S$, Π^V, Π^T and Y^V

  Algebraically given: ζ^T, $\mathcal{E}^V$, $\overline{\mathcal{H}}^V$, Σ^V, α^V, W^V, V^V, X^V and ϕ^S
- Odd sector:
  Evolution equations for: Ω^S, $\mathcal{H}^T$, $\overline{\mathcal{E}}^T$, $\overline{Q}^V$

  Freely specifiable: $\overline{\mathcal{A}}^V$, $\overline{\Pi}^V$ and $\overline{\Pi}^T$

  Algebraically given: $\overline{\zeta}^T$, $\overline{\mathcal{E}}^V$, $\mathcal{H}^S$, $\mathcal{H}^V$, $\overline{\Sigma}^V$, $\overline{\Sigma}^T$, $\overline{\alpha}^V$, $\overline{V}^V$, $\overline{W}^V$, $\overline{X}^V$, $\overline{Y}^V$, ξ^S, Ω^V, $\overline{p}^V$ and $\overline{\mu}^V$

[5] In the following the indices $k_{\|} k_{\perp}$ on the harmonic coefficients are suppressed

Evolution equations for odd sector

$$\dot{\overline{Q}}^V = -\left(\tfrac{4}{3}\Theta - \tfrac{1}{2}\Sigma\right)\overline{Q}^V - \left(\mu + p - \tfrac{1}{2}\Pi\right)\overline{\mathcal{A}}^V + \frac{a_2\left(\dot{\Pi} - 2\dot{p}\right)}{k_\perp^2}\Omega^S + \frac{\mathcal{R}a_2^2 - k_\perp^2}{2a_2}\overline{\Pi}^T - \frac{ik_\parallel}{a_1}\overline{\Pi}^V,$$

$$\dot{\Omega}^S = \left(\Sigma - \tfrac{2\Theta}{3}\right)\Omega^S + \tfrac{1}{2a_2}\overline{\mathcal{A}}^V,$$

$$\dot{\mathcal{H}}^T = \frac{ia_1}{k_\parallel a_2 B}\left(\left(\Sigma + \tfrac{\Theta}{3}\right)\left(\tilde{k}^2 + 3\Pi\right) - 3\Sigma\frac{k_\parallel^2}{a_1^2}\right)Q^V + \frac{a_1}{2ik_\parallel B}\left(\left(\mathcal{R} - \tilde{k}^2\right)\left(\tilde{k}^2 + 3\Pi\right) - 9\Sigma^2\frac{k_\parallel^2}{a_1^2}\right)\left(\overline{\mathcal{E}}^T + \tfrac{1}{2}\overline{\Pi}^T\right) - \frac{ik_\parallel}{a_1}\overline{\Pi}^T - \frac{1}{a_2}\overline{\Pi}^V - \tfrac{3}{2}\left(2E + F - 2\tfrac{\Pi}{B}\left(\Sigma + \tfrac{\Theta}{3}\right)\right)\mathcal{H}^T + (S + U)\,\Omega^S,$$

$$\dot{\overline{\mathcal{E}}}^T + \tfrac{1}{2}\dot{\overline{\Pi}}^T = \frac{ik_\parallel}{a_1}\left(1 - D + \tfrac{2\Pi}{B}\right)\mathcal{H}^T + P\Omega^S + \left(\Sigma + \tfrac{\Theta}{3}\right)\overline{\Pi}^T + \frac{Pk_\perp^2}{2a_2\left(\mu + p - \frac{\Pi}{2}\right)}\overline{Q}^V - \tfrac{3}{2}\left(F + \Sigma\left(D - \tfrac{2\Pi}{B}\right)\right)\left(\overline{\mathcal{E}}^T + \frac{\overline{\Pi}^T}{2}\right)$$

Dissipative fluids

1-component fluid: Let u^a be 4-velocity of matter

- q_a heatflow, π_{ab} shear viscosity, $p = \tilde{p} + p_{\zeta_B}$ where $\tilde{p}$ is equilbrium pressure and p_{ζ_B} bulk viscosity.
- $\mathcal{N}$ particle density, satisfies $\dot{\mathcal{N}} + \Theta\mathcal{N} = 0$ (particle conservation). $Z_a \equiv \delta_a \mathcal{N}$
- Assume $\tilde{p} = \tilde{p}(\mu, \mathcal{N})$ and $T = T(\mu, \mathcal{N})$ (temperature).
- Eckart theory: Acausal.

  Bulk viscosity: $p_{\zeta_B} = -\zeta_B \Theta$,

  Shear viscosity: $\pi_{ab} = -2\eta\sigma_{ab}$,

  Heat flow: $q_a = -\kappa \left(D_a T + T A_a \right)$

  where
  $\zeta_B = \zeta_B(\mu, \mathcal{N}), \eta = \eta(\mu, \mathcal{N}), \kappa = \kappa(\mu, \mathcal{N})$

Dissipative fluids

1-component fluid: Let u^a be 4-velocity of matter

- q_a heatflow, π_{ab} shear viscosity, $p = \tilde{p} + p_{\zeta_B}$ where $\tilde{p}$ is equilbrium pressure and p_{ζ_B} bulk viscosity.
- $\mathcal{N}$ particle density, satisfies $\dot{\mathcal{N}} + \Theta\mathcal{N} = 0$ (particle conservation). $Z_a \equiv \delta_a \mathcal{N}$
- Assume $\tilde{p} = \tilde{p}(\mu, \mathcal{N})$ and $T = T(\mu, \mathcal{N})$ (temperature).
- Eckart theory: Acausal.

  Bulk viscosity: $p_{\zeta_B} = -\zeta_B \Theta$,

  Shear viscosity: $\pi_{ab} = -2\eta\sigma_{ab}$,

  Heat flow: $q_a = -\kappa \left(D_a T + T A_a \right)$

  where
  $\zeta_B = \zeta_B(\mu, \mathcal{N}), \eta = \eta(\mu, \mathcal{N}), \kappa = \kappa(\mu, \mathcal{N})$

Dissipative fluids

1-component fluid: Let u^a be 4-velocity of matter

- q_a heatflow, π_{ab} shear viscosity, $p = \tilde{p} + p_{\zeta_B}$ where $\tilde{p}$ is equilbrium pressure and p_{ζ_B} bulk viscosity.
- $\mathcal{N}$ particle density, satisfies $\dot{\mathcal{N}} + \Theta\mathcal{N} = 0$ (particle conservation). $Z_a \equiv \delta_a \mathcal{N}$
- Assume $\tilde{p} = \tilde{p}(\mu, \mathcal{N})$ and $T = T(\mu, \mathcal{N})$ (temperature).
- Eckart theory: Acausal.

  Bulk viscosity: $p_{\zeta_B} = -\zeta_B \Theta$,

  Shear viscosity: $\pi_{ab} = -2\eta\sigma_{ab}$,

  Heat flow: $q_a = -\kappa \left(D_a T + T A_a \right)$

  where
  $\zeta_B = \zeta_B(\mu, \mathcal{N}), \eta = \eta(\mu, \mathcal{N}), \kappa = \kappa(\mu, \mathcal{N})$

Dissipative fluids

1-component fluid: Let u^a be 4-velocity of matter

- q_a heatflow, π_{ab} shear viscosity, $p = \tilde{p} + p_{\zeta_B}$ where $\tilde{p}$ is equilbrium pressure and p_{ζ_B} bulk viscosity.
- $\mathcal{N}$ particle density, satisfies $\dot{\mathcal{N}} + \Theta\mathcal{N} = 0$ (particle conservation). $Z_a \equiv \delta_a \mathcal{N}$
- Assume $\tilde{p} = \tilde{p}(\mu, \mathcal{N})$ and $T = T(\mu, \mathcal{N})$ (temperature).
- Eckart theory: Acausal.

  Bulk viscosity: $p_{\zeta_B} = -\zeta_B \Theta$,

  Shear viscosity: $\pi_{ab} = -2\eta\sigma_{ab}$,

  Heat flow: $q_a = -\kappa \left(D_a T + T A_a \right)$

  where
  $\zeta_B = \zeta_B(\mu, \mathcal{N}), \eta = \eta(\mu, \mathcal{N}), \kappa = \kappa(\mu, \mathcal{N})$

Closed system for Eckart theory

The two systems now close

- Even sector:

  Evolution equations for: $Z^V, \overline{\Omega}^V, \mu^V, \overline{\mathcal{H}}^T, \mathcal{E}^T, \Sigma^T, Q^V$ and Q^S

  Algebraically given: $p^V, \mathcal{A}^V, \mathcal{A}^S, \Pi^V, \Pi^T, Y^V, \zeta^T, \mathcal{E}^V, \overline{\mathcal{H}}^V, \Sigma^V, \alpha^V, W^V, V^V, X^V$ and ϕ^S

- Odd sector:

  Evolution equations for: $\Omega^S, \mathcal{H}^T, \overline{\mathcal{E}}^T, \overline{Q}^V$

  Algebraically given: $\overline{\mathcal{A}}^V, \overline{\Pi}^V, \overline{\Pi}^T, \overline{\zeta}^T, \overline{\mathcal{E}}^V, \mathcal{H}^S, \mathcal{H}^V, \overline{\Sigma}^V, \overline{\Sigma}^T, \overline{\alpha}^V, \overline{V}^V, \overline{W}^V, \overline{X}^V, \overline{Y}^V, \xi^S, \Omega^V, \overline{p}^V$ and $\overline{\mu}^V$

Closed system for Eckart theory

The two systems now close

- Even sector:

  Evolution equations for: $Z^V, \overline{\Omega}^V$, μ^V, $\overline{\mathcal{H}}^T$, $\mathcal{E}^T$, Σ^T, Q^V and Q^S

  Algebraically given: p^V, $\mathcal{A}^V$, $\mathcal{A}^S$, Π^V, Π^T , Y^V, ζ^T, $\mathcal{E}^V$, $\overline{\mathcal{H}}^V$, Σ^V, α^V, W^V, V^V, X^V and ϕ^S

- Odd sector:

  Evolution equations for: Ω^S, $\mathcal{H}^T$, $\overline{\mathcal{E}}^T$, $\overline{Q}^V$

  Algebraically given: $\overline{\mathcal{A}}^V$, $\overline{\Pi}^V$, $\overline{\Pi}^T$, $\overline{\zeta}^T$, $\overline{\mathcal{E}}^V$, $\mathcal{H}^S$, $\mathcal{H}^V$, $\overline{\Sigma}^V$, $\overline{\Sigma}^T$, $\overline{\alpha}^V$, $\overline{V}^V$, $\overline{W}^V$, $\overline{X}^V$, $\overline{Y}^V$, ξ^S, Ω^V, $\overline{p}^V$ and $\overline{\mu}^V$

Causal theory

A simplified causal theory is given by [6]

- $$\begin{aligned} \tau_1 \dot{p}_{\zeta_B} + p_{\zeta_B} &= -\zeta_B \Theta \\ \tau_2 \dot{\pi}_{<ab>} + \pi_{ab} &= -2\eta\sigma_{ab} \\ \tau_3 \dot{q}^{<a>} + q^a &= -\kappa \left(D^a T + T A^a \right) , \end{aligned}$$

  where τ_i are relaxation times.
- The two systems again close but with evolution equations added for Π^V, Π^T, Z^V, Y^V , p^V_ζ and $\overline{\Pi}^V$, $\overline{\Pi}^T$, respectively.

[6] R. Maartens, Lecture notes, Natal University (1996)

Causal theory

A simplified causal theory is given by [6]

- $$\begin{aligned} \tau_1 \dot{p}_{\zeta_B} + p_{\zeta_B} &= -\zeta_B \Theta \\ \tau_2 \dot{\pi}_{<ab>} + \pi_{ab} &= -2\eta\sigma_{ab} \\ \tau_3 \dot{q}^{<a>} + q^a &= -\kappa \left(D^a T + T A^a\right) , \end{aligned}$$

  where τ_i are relaxation times.
- The two systems again close but with evolution equations added for Π^V, Π^T, Z^V, Y^V , p_ζ^V and $\overline{\Pi}^V$, $\overline{\Pi}^T$, respectively.

[6] R. Maartens, Lecture notes, Natal University (1996)

Generation of vorticity

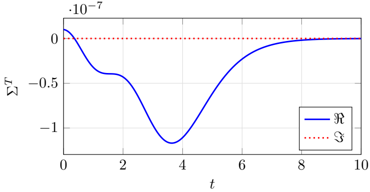
- Vorticy cannot be generated in a barotropic perfect fluid.
- Even sector Eckart theory $(Z^V, \overline{\Omega}^V, \mu^V, \overline{\mathcal{H}}^T, \mathcal{E}^T, \Sigma^T, Q^V, Q^S)$

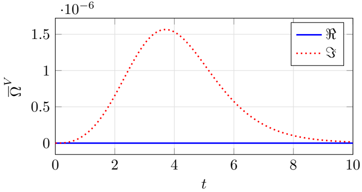
$$\dot{\overline{\Omega}}^V = \frac{ik_{\parallel}}{2a_1\kappa T}Q^V - \frac{1}{2a_2\kappa T}Q^S - \left(\frac{2\Theta}{3} + \frac{\Sigma}{2} + \frac{\dot{T}}{T}\right)\overline{\Omega}^V$$

$$\dot{\Sigma}^T = \left(\Sigma - \frac{2\Theta}{3} - \eta\right)\Sigma^T - \frac{1}{a_2\kappa T}\left(Q^V + \kappa\left(\tau_0\mu^V + \tau_1 Z^V\right)\right) - \mathcal{E}^T$$

$$\dot{Q}^V = ..., \; \dot{Q}^S = ..., \; \dot{\overline{\mathcal{H}}}^T = ..., \; \dot{\mu}^V = ..., \; \dot{\mathcal{E}}^T = ..., \; \dot{Z}^V = ...$$

- In the graphs below it can be seen how an intial nonzero shear and zero heatflow produces a vorticity due to shear viscosity.





Generation of vorticity

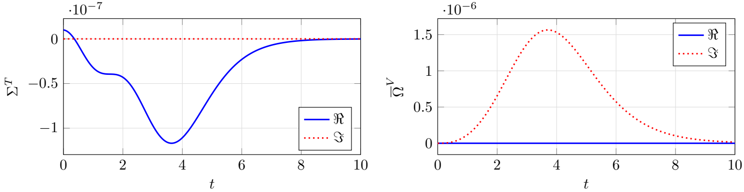
- Vorticy cannot be generated in a barotropic perfect fluid.
- Even sector Eckart theory ($Z^V, \overline{\Omega}^V, \mu^V, \overline{\mathcal{H}}^T, \mathcal{E}^T, \Sigma^T, Q^V, Q^S$)

$$\dot{\overline{\Omega}}^V = \frac{ik_\parallel}{2a_1\kappa T}Q^V - \frac{1}{2a_2\kappa T}Q^S - \left(\frac{2\Theta}{3} + \frac{\Sigma}{2} + \frac{\dot{T}}{T}\right)\overline{\Omega}^V$$

$$\dot{\Sigma}^T = \left(\Sigma - \frac{2\Theta}{3} - \eta\right)\Sigma^T - \frac{1}{a_2\kappa T}\left(Q^V + \kappa\left(\tau_0\mu^V + \tau_1 Z^V\right)\right) - \mathcal{E}^T$$

$$\dot{Q}^V = ...,\ \dot{Q}^S = ...,\ \dot{\overline{\mathcal{H}}}^T = ...,\ \dot{\mu}^V = ...,\ \dot{\mathcal{E}}^T = ...,\ \dot{Z}^V = ...$$

- In the graphs below it can be seen how an intial nonzero shear and zero heatflow produces a vorticity due to shear viscosity.



Generation of vorticity

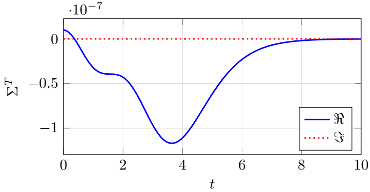
- Vorticy cannot be generated in a barotropic perfect fluid.
- Even sector Eckart theory ($Z^V, \overline{\Omega}^V, \mu^V, \overline{\mathcal{H}}^T, \mathcal{E}^T, \Sigma^T, Q^V, Q^S$)

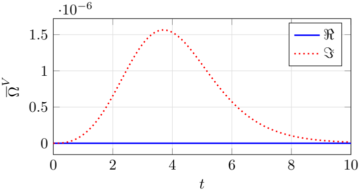
$$\dot{\overline{\Omega}}^V = \frac{ik_\parallel}{2a_1\kappa T}Q^V - \frac{1}{2a_2\kappa T}Q^S - \left(\frac{2\Theta}{3} + \frac{\Sigma}{2} + \frac{\dot{T}}{T}\right)\overline{\Omega}^V$$

$$\dot{\Sigma}^T = \left(\Sigma - \frac{2\Theta}{3} - \eta\right)\Sigma^T - \frac{1}{a_2\kappa T}\left(Q^V + \kappa\left(\tau_0\mu^V + \tau_1 Z^V\right)\right) - \mathcal{E}^T$$

$$\dot{Q}^V = ..., \ \dot{Q}^S = ..., \ \dot{\overline{\mathcal{H}}}^T = ..., \ \dot{\mu}^V = ..., \ \dot{\mathcal{E}}^T = ..., \ \dot{Z}^V = ...$$

- In the graphs below it can be seen how an intial nonzero shear and zero heatflow produces a vorticity due to shear viscosity.





Summary

- Evolution equations for general first order covariant perturbations on homogeneous LRS II cosmologies determined.
- Closed systems obtained for 1-component dissipative fluids, both in Eckart´s acausal theory and in a simplified causal theory.
- Seen how vorticity can be generated from shear viscosity

Summary

- Evolution equations for general first order covariant perturbations on homogeneous LRS II cosmologies determined.
- Closed systems obtained for 1-component dissipative fluids, both in Eckart´s acausal theory and in a simplified causal theory.
- Seen how vorticity can be generated from shear viscosity

Summary

- Evolution equations for general first order covariant perturbations on homogeneous LRS II cosmologies determined.
- Closed systems obtained for 1-component dissipative fluids, both in Eckart´s acausal theory and in a simplified causal theory.
- Seen how vorticity can be generated from shear viscosity