Peeling in Generalized Harmonic Gauge

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based on work done with J. Feng, E. Gasperín and D. Hilditch

CAMGSD

grit
gravitation in técnico

Peeling

Smoothness of null infinity implies certain decay of Weyl tensor components

$$\Psi_N \sim \frac{1}{R^{5-N}}$$

$$\Psi_4 = O(R^{-1})\,, \Psi_3 = O(R^{-2})\,, \Psi_2 = O(R^{-3})\,,$$
$$\Psi_1 = O(R^{-4})\,, \Psi_0 = O(R^{-5})\,.$$

Formalism or choosing variables

$$\psi^a = \partial_T^a + \mathcal{C}_+^R \partial_R^a$$
$$\underline{\psi}^a = \partial_T^a + \mathcal{C}_-^R \partial_R^a$$

$$g^{ab} = -2\tau^{-1} e^{-\varphi}\, \psi^{(a} \underline{\psi}^{b)} + \not{g}^{ab}$$
$$g_{ab} = -2\tau^{-1} e^{\varphi}\, \sigma_{(a} \underline{\sigma}_{b)} + \not{g}_{ab}$$

$$\sigma_a = e^{-\varphi} \psi_a$$
$$\underline{\sigma}_a = e^{-\varphi} \underline{\psi}_a$$

$$\sigma_a = -\mathcal{C}_+^R \nabla_a T + \nabla_a R + \mathcal{C}_A^+ \nabla_a \theta^A$$
$$\underline{\sigma}_a = \mathcal{C}_-^R \nabla_a T - \nabla_a R + \mathcal{C}_A^- \nabla_a \theta^A \,.$$

Formalism or choosing variables

$$\psi^a = \partial_T^a + \mathcal{C}_+^R \partial_R^a$$
$$\underline{\psi}^a = \partial_T^a + \mathcal{C}_-^R \partial_R^a$$

Outgoing and incoming null vectors

$$\sigma_a = e^{-\varphi} \psi_a$$
$$\underline{\sigma}_a = e^{-\varphi} \underline{\psi}_a$$

$$\sigma_a = -\mathcal{C}_+^R \nabla_a T + \nabla_a R + \mathcal{C}_A^+ \nabla_a \theta^A$$
$$\underline{\sigma}_a = \mathcal{C}_-^R \nabla_a T - \nabla_a R + \mathcal{C}_A^- \nabla_a \theta^A \, .$$

$$g^{ab} = -2\tau^{-1} e^{-\varphi} \, \psi^{(a} \underline{\psi}^{b)} + \not{g}^{ab}$$
$$g_{ab} = -2\tau^{-1} e^{\varphi} \, \sigma_{(a} \underline{\sigma}_{b)} + \not{g}_{ab}$$

angular metric

Formalism or choosing variables

$$g^{ab} = -2\tau^{-1}e^{-\varphi}\,\psi^{(a}\underline{\psi}^{b)} + \not{g}^{ab}$$

$$g_{ab} = -2\tau^{-1}e^{\varphi}\,\sigma_{(a}\underline{\sigma}_{b)} + \not{g}_{ab}$$

$$(q^{-1})^{ab} = e^{\epsilon}R^2\not{g}^{ab}$$

$$\epsilon = (\ln|\not{g}| - \ln|\mathring{\not{g}}|)/2$$

$$(q^{-1})^{AB} = \begin{bmatrix} e^{-h_+}\cosh h_\times & \frac{\sinh h_\times}{\sin\theta} \\ \frac{\sinh h_\times}{\sin\theta} & \frac{e^{h_+}\cosh h_\times}{\sin\theta^2} \end{bmatrix}$$

We end up with 10 independent variables:

$$\varphi\,,\quad \mathcal{C}^R_\pm\,,\quad \mathcal{C}^\pm_A\,,\quad \epsilon\,,\quad h_+\,,\quad h_\times$$

The good, the bad and the ugly

$$X^{\underline{\alpha}} = (T, X^{\underline{i}})$$

$$\Gamma[\mathring{\nabla}]_a{}^b{}_c = (\mathring{\nabla}_a \partial^b_{\underline{\alpha}})(dX^{\underline{\alpha}})_c$$

$$\mathring{\Box}\phi = g^{ab}\mathring{\nabla}_a\mathring{\nabla}_b\phi$$

$$\mathring{\Box} g = 0\,,$$

$$\mathring{\Box} b = (\nabla_T g)^2$$

$$\mathring{\Box} u = \tfrac{2}{R}\nabla_T u$$

The good, the bad and the ugly

$$\mathring{\Box} g = 0\,,$$
$$\mathring{\Box} b = (\nabla_T g)^2$$
$$\mathring{\Box} u = \tfrac{2}{R} \nabla_T u$$

$$g = \sum_{n=1}^{\infty} \frac{G_n(\psi^*)}{R^n}\,,$$

$$b = \sum_{n=1}^{\infty} \frac{B_n}{R^n}\,,$$

$$u = \frac{m_{u,1}}{R} + \sum_{n=2}^{\infty} \frac{U_n}{R^n}\,,$$

$$B_n = B_{n,0}(\psi^*) + B_{n,1}(\psi^*) \log R$$
$$U_n = U_{n,0}(\psi^*) + U_{n,1}(\psi^*) \log R$$

GBU with stratified null forms

$$\mathring{\Box} g = \mathcal{N}_g \,,$$
$$\mathring{\Box} b = (\nabla_T g)^2 + \mathcal{N}_b$$
$$\mathring{\Box} u = \frac{2}{R}\nabla_T u + \mathcal{N}_u$$

$$g = \frac{G_{1,0}(\psi^*)}{R} + \sum_{n=2}^{\infty}\sum_{k=0}^{N_n^g} \frac{(\log R)^k G_{n,k}(\psi^*)}{R^n}$$

$$b = \frac{B_1}{R} + \sum_{n=2}^{\infty}\sum_{k=0}^{N_n^b} \frac{(\log R)^k B_{n,k}(\psi^*)}{R^n}$$

$$u = \frac{m_{u,1}}{R} + \sum_{n=2}^{\infty}\sum_{k=0}^{N_n^u} \frac{(\log R)^k U_{n,k}(\psi^*)}{R^n}\,,$$

Reduced Einstein field equations

Reduced Ricci tensor:

$$\mathcal{R}_{ab} := R_{ab} - \nabla_{(a}Z_{b)} + W_{ab}$$

$$X^{\underline{\alpha}'} = (T', X^{\underline{i}'}) = (T, R, \theta^A)$$

$$\Gamma[\mathring{\nabla}]_b{}^a{}_c = (\mathring{\nabla}_b \partial^a_{\underline{\alpha}'})(dX^{\underline{\alpha}'})_c$$

$$Z^a := \mathring{\Gamma}^a + F^a$$

Reduced Einstein field equations

Reduced Ricci tensor:

$$\mathcal{R}_{ab} := R_{ab} - \nabla_{(a}Z_{b)} + W_{ab}$$

Ricci

Constraint addition

$$X^{\underline{\alpha}'} = (T', X^{\underline{i}'}) = (T, R, \theta^A)$$

$$\Gamma[\dot{\nabla}]_b{}^a{}_c = (\dot{\nabla}_b \partial^a_{\underline{\alpha}'})(dX^{\underline{\alpha}'})_c$$

$$Z^a := \dot{\Gamma}^a + F^a$$

Constraints

Gauge source functions

Gauge choice and constraint addition

Cartesian harmonic gauge

$$F^a = g^{bc}\Gamma[\mathring{\nabla}, \dot{\nabla}]^a{}_{bc}$$

Coordinates are harmonic:

$$g^{bc}\Gamma[\nabla, \mathring{\nabla}]_b{}^a{}_c = 0$$

Constraint addition

Each of the 4 constraints is a time derivative of a variable to leading order

We can turn 4 of the equations into uglies

Asymptotic system

$$\mathring{\Box}\varphi = \mathcal{N}_\varphi ,$$

$$\mathring{\Box}\mathcal{C}^R_+ = \frac{2}{R}\nabla_T \mathcal{C}^R_+ + \mathcal{N}_{\mathcal{C}^R_+} ,$$

$$\mathring{\Box}\mathcal{C}^R_- = -\frac{1}{2}(\nabla_T h_+)^2 - \frac{1}{2}(\nabla_T h_\times)^2 + \mathcal{N}_{\mathcal{C}^R_-} ,$$

$$\mathring{\Box}\hat{\mathcal{C}}^+_A = \frac{2}{R}\nabla_T \hat{\mathcal{C}}^+_A + \mathcal{N}_{\mathcal{C}^+_A} ,$$

$$\mathring{\Box}\hat{\mathcal{C}}^-_A = \frac{4}{R}\nabla_T \hat{\mathcal{C}}^-_A + \mathcal{N}_{\mathcal{C}^-_A} ,$$

$$\mathring{\Box}\epsilon = \frac{2}{R}\nabla_T \epsilon + \mathcal{N}_\epsilon ,$$

$$\mathring{\Box}h_+ = \mathcal{N}_{h_+} ,$$

$$\mathring{\Box}h_\times = \mathcal{N}_{h_\times} .$$

$$\mathring{\Box}g = \mathcal{N}_g ,$$

$$\mathring{\Box}b = (\nabla_T g)^2 + \mathcal{N}_b$$

$$\mathring{\Box}u = \tfrac{2}{R}\nabla_T u + \mathcal{N}_u$$

With this gauge and constraint addition:

- 3 goods
- 1 bad
- 6 uglies

GBU with stratified null forms

$$\mathring{\Box} g = \mathcal{N}_g \,,$$
$$\mathring{\Box} b = (\nabla_T g)^2 + \mathcal{N}_b$$
$$\mathring{\Box} u = \tfrac{2}{R}\nabla_T u + \mathcal{N}_u$$

$$g = \frac{G_{1,0}(\psi^*)}{R} + \sum_{n=2}^{\infty}\sum_{k=0}^{N_n^g} \frac{(\log R)^k G_{n,k}(\psi^*)}{R^n}$$

$$b = \frac{B_1}{R} + \sum_{n=2}^{\infty}\sum_{k=0}^{N_n^b} \frac{(\log R)^k B_{n,k}(\psi^*)}{R^n}$$

$$u = \frac{m_{u,1}}{R} + \sum_{n=2}^{\infty}\sum_{k=0}^{N_n^u} \frac{(\log R)^k U_{n,k}(\psi^*)}{R^n},$$

GBU with stratified null forms

$$\mathring{\Box} g = \mathcal{N}_g \,,$$
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$$\mathring{\Box} u = \frac{2}{R} \nabla_T u + \mathcal{N}_u$$

$$g = \frac{G_{1,0}(\psi^*)}{R} + \sum_{n=2}^{\infty} \sum_{k=0}^{N_n^g} \frac{(\log R)^k G_{n,k}(\psi^*)}{R^n}$$
$$b = \frac{B_1}{R} + \sum_{n=2}^{\infty} \sum_{k=0}^{N_n^b} \frac{(\log R)^k B_{n,k}(\psi^*)}{R^n}$$
$$u = \frac{m_{u,1}}{R} + \sum_{n=2}^{\infty} \sum_{k=0}^{N_n^u} \frac{(\log R)^k U_{n,k}(\psi^*)}{R^n} ,$$

Violation of peeling

With cartesian harmonic gauge and our particular constraint addition, we get:

$$\Psi_4 = O(R^{-1})\,, \Psi_3 = O(R^{-2})\,, \Psi_2 = O(\log R/R^3)$$

Peeling is violated

Recovering peeling

Cartesian harmonic gauge + subheading

$$F^a = \mathring{F}^a + \check{F}^a$$

Constraint addition

Each of the 4 constraints is a time derivative of a variable to leading order

We can turn 4 of the equations into uglies of a special kind:

$$\mathring{\square} u = \frac{2p}{R} \nabla_T u + \mathcal{N}_u$$

Recovering peeling

$$
\begin{aligned}
\mathring{\Box}\varphi &= \nabla_T \check{F}^{\sigma} + N_{\varphi}\,, \\
\mathring{\Box}\mathcal{C}^R_+ &= \frac{2p}{R}\nabla_T \mathcal{C}^R_+ + N_{\mathcal{C}^R_+}\,, \\
\mathring{\Box}\mathcal{C}^R_- &= -\frac{1}{2}(\nabla_T h_+)^2 - \frac{1}{2}(\nabla_T h_\times)^2 - 2\nabla_T \check{F}^{\underline{\sigma}} + N_{\mathcal{C}^R_-}\,, \\
\mathring{\Box}\hat{\mathcal{C}}^+_A &= \frac{2p}{R}\nabla_T \hat{\mathcal{C}}^+_A + N_{\hat{\mathcal{C}}^+_A}\,, \\
\mathring{\Box}\hat{\mathcal{C}}^-_A &= \frac{4}{R}\nabla_T \hat{\mathcal{C}}^-_A - 2R\nabla_T \check{F}^A + N_{\hat{\mathcal{C}}^-_A}\,, \\
\mathring{\Box}\epsilon &= \frac{2p}{R}\nabla_T \epsilon + N_{\epsilon}\,, \\
\mathring{\Box}h_+ &= N_{h_+}\,, \\
\mathring{\Box}h_\times &= N_{h_\times}\,.
\end{aligned}
$$

$$
\begin{aligned}
\mathring{\Box}\varphi &= \frac{2p}{R}\nabla_T \varphi + N_{\varphi}\,, \\
\mathring{\Box}\mathcal{C}^R_+ &= \frac{2p}{R}\nabla_T \mathcal{C}^R_+ + N_{\mathcal{C}^R_+}\,, \\
\mathring{\Box}\mathcal{C}^R_- &= \frac{2p}{R}\nabla_T \mathcal{C}^R_- + N_{\mathcal{C}^R_-}\,, \\
\mathring{\Box}\hat{\mathcal{C}}^+_A &= \frac{2p}{R}\nabla_T \hat{\mathcal{C}}^+_A + N_{\hat{\mathcal{C}}^+_A}\,, \\
\mathring{\Box}\hat{\mathcal{C}}^-_A &= \frac{2p}{R}\nabla_T \hat{\mathcal{C}}^-_A + N_{\hat{\mathcal{C}}^-_A}\,, \\
\mathring{\Box}\epsilon &= \frac{2p}{R}\nabla_T \epsilon + N_{\epsilon}\,, \\
\mathring{\Box}h_+ &= N_{h_+}\,, \\
\mathring{\Box}h_\times &= N_{h_\times}\,.
\end{aligned}
$$

- 2 goods
- 8 uglies

Regularization?