Distinguishing cores from cusps in the dark matter density profile using the proper motions measurements

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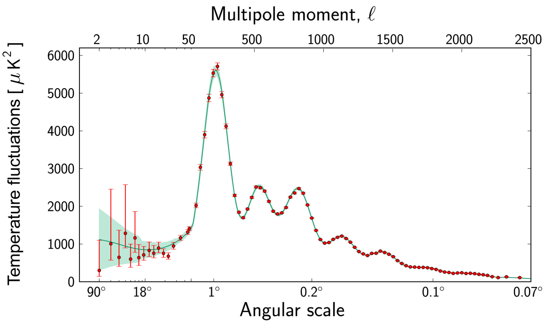
Outline

The concordance cosmological model: ΛCDM

Modern Astrophysics and Cosmology are entirely based on **General Relativity** + **DM** + Λ.



Planck 2018 results. VI. Cosmological parameters, 2020, A&A 641, A6

Parameter	TT+lowE 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022
$\Omega_c h^2$	0.1206 ± 0.0021
$100\theta_{MC}$	1.04077 ± 0.00047
τ	0.0522 ± 0.0080
$\ln(10^{10} A_s)$	3.040 ± 0.016
n_s	0.9626 ± 0.0057
H_0 [km s^{-1} Mpc^{-1}] . .	66.88 ± 0.92
Ω_Λ	0.679 ± 0.013
Ω_m	0.321 ± 0.013
$\Omega_m h^2$	0.1434 ± 0.0020
$\Omega_m h^3$	0.09589 ± 0.00046
σ_8	0.8118 ± 0.0089

Cold Dark Matter - What is it?

G. Bertone, T. M. P. Tait, Nature 562, 51 (2018)

Cold Dark Matter - The small scale crisis

This Cold Dark Matter scenario encounters some difficulties in describing structures at galactic scales. These difficulties include, for example, the *cusp/core* problem, the problem of the *missing satellites*, the *too-big-to-fail* problem, and the problem of the *planes of satellite galaxies*.

The *cusp/core* problem (CCP)

The CCP is the discrepancy between the flat dark matter density profile observed at the centres of dwarf and ultra-faint galaxies, and the cuspy profile predicted in collisionless N-body simulations. In particular, N-body simulations show cuspy density profiles of dark matter halos of galaxy size with density increasing with decreasing radius r as $r^{-\beta}$ with β in the range $\sim [1-1.5]$. These slopes do not match the cores favored by the observed rotation curves. Nevertheless, modelling the kinematics of stars in dwarf galaxies does not lead to a clear conclusion to whether these galaxies are dominated by a core or a cusp in their innermost regions.

Navarro et al. Mon. Not. R. Astron. Soc. 1996, 283, L72-L78
Navarro et al. The Astrophysical Journal, 1997, 490, 493-508
Ferrero et al. Mon. Not. R. Astron. Soc., 2012, 425, 2817-2823
Genina et al. Mon. Not. R. Astron. Soc., 2018, 474, 1398-1411

Cold Dark Matter - Possible solutions to the small scale crisis

The *cusp/core* problem (CCP)

Possible solutions to the CCP, in the context of the CDM scenario, can originate either from neglected physical processes, mostly affecting the baryonic matter, or from systematic effects and/or observational limits. The most popular solutions rely on supernova feedback and dynamical friction:

- Supernova and stellar winds produce energy feedback that can drastically modify the shape of the dwarf galaxies by forcing the gas and the dark matter particles to move outwards, change the gravitational potential well and flatten the density profile.
- Dynamical friction between gas clumps with individual mass $10^5 - 10^6 M_\odot$ on the scale of dwarf galaxies would transfer angular momentum from the gas to the dark matter particles that, on turn, would move away from the central region of the halo and flatten its density profile.

Gnedin et al. Mon. Not. R. Astron. Soc., 2002,333, 299, 306
Mashchenko et al. Science, 2008,2378319, 174-177
Ogiyaet al. Astrophys. J., 2014, 793, 46
El-Zant et al. The Astrophysical Journal, 2001, 560, 636-643

Cold Dark Matter - Possible solutions to the small scale crisis

Summary of the ability of alternative dark matter and gravity models to either solve or not display the challenges of the CDM model **I. De Martino et al., Universe, 2020, 6(8), 107**

	Rotation curves and scaling relations	Cusp/Core Problem	Missing Satellites Problem	Planes of Satellites Problem	Large Scale Structure and Cosmic Scales
Warm DM	✔	✘	✔	🔍	🔍
Self-interacting DM	✔	✔	🔍	🔍	✔
QCD axions	✔	🔍	🔍	🔍	✔
Fuzzy DM	🔍	✔	✔	🔍	🔍
MOND	✔	✔	✔	🔍	✘
MOG	✔	✘	🔍	🔍	✔
$f(R)$-gravity	✔	🔍	✘	🔍	✔

✔ = Successfully solved / **Not present** ✘ = Not solved 🔍 = Under investigation

Can we distinguish cusps from cores?

Not Yet!

Can we distinguish cusps from cores?

Not Yet!

Observational issues

- Mapping the distribution of DM depends on the type of system under investigation: inferring the DM distribution in a rotationally-supported galaxy generally derives from the fit to the rotation curve, whereas, for a pressure supported dwarf spheroidal galaxy, we rely on the profile of the line-of-sight velocity dispersion, if no other data set, like for instance multiple stellar populations, proper motions, or three dimensional positions , is available.

- The standard method is to assume a general functional form for the DM halo density profile and determine its parameters from a fit to the data.

- Unfortunately, this approach generally suffers from degenerancies among the parameters of the DM density profile and even among them and other unknown parameters, for instance between the velocity anisotropy parameter and the total halo mass in the Jeans equations. This drawback may inhibit the distinction between models with a cusp and with a core, as it happens when fitting the line-of-sight velocity dispersion profiles of the Milky Way satellites.

- This degeneracy can be lifted by adding information from multiple stellar populations or higher velocity moments.

Observational issues

- However, N-body simulations show that multiple stellar populations only partially lift the degeneracy, whereas higher velocity moments combined with proper motions appear to be more effective.

- Here, we quantify how **proper motions** can lift the mass-anisotropy degeneracy and shed light on the CCP in dwarf galaxies. The proper motions of stars of the dwarf combined with their line-of-sight velocities from their spectra provide the three-dimensional velocity field within the dwarf.

- Strigari et al. (2007) pointed out that adding the proper motion of 200 stars to their line-of-sight velocity would make it possible to constrain the log-slope of the DM density profile at twice the King radius with 20% statistical uncertainty. While using only line-of-sight velocities leaves the log-slope parameter unconstrained.

Walker M. G., Mateo M., Olszewski E. W., Peñarrubia J., Evans N. W.,Gilmore G., 2009, ApJ, 704, 1274
Walker M. G., Peñarrubia J., 2011, ApJ, 742, 20

Strigari L. E., Bullock J. S., Kaplinghat M., 2007, ApJ, 657, L1

Objectives of the work

Recently, *Theia*, a space-based mission for high-precision astrometric measures was proposed to address a number of astrophysical problems.

- *Theia* is conceived to be able to measure the proper motions of stars in nearby dwarf galaxies. In principle, these data can accurately determine the DM density profiles of the dwarfs and solve the CCP.
- Here, we create mock data sets mirroring the generic expected observational limitations of *Theia* to determine the minimum number of stars and the maximum uncertainty on the proper motion measures that are required to actually make these measures effective at solving the CCP.

Assumptions

To create an astrometric mock catalogue of stars in a dwarf galaxy, we need to define the distribution of both the stellar and the DM components. We adopt two assumptions:
(1) both the DM and the star distributions are spherically symmetric;
(2) the anisotropy velocity parameter β is independent of radius.

The Theia Collaboration et al., 2017, arXiv e-prints, p. arXiv:1707.01348

Modelling the star proper motion

Stellar distribution

For the stellar density distribution, we adopt the Plummer model:

$$\nu(r) \propto \left(1 + \frac{r^2}{a^2}\right)^{-5/2}, \tag{1}$$

where a is a scale length. The stellar mass density is $\rho_*(r) = M_* \nu(r)$, where M_* is the total stellar mass of the system, if we assume a constant stellar mass-to-light ratio.

Dark Matter distribution

We model the DM density distribution as,

$$\rho(r) = \rho_0 \left(\frac{r}{r_s}\right)^{-\gamma} \left[1 + \left(\frac{r}{r_s}\right)^{\alpha}\right]^{\frac{\gamma-\delta}{\alpha}}. \tag{2}$$

In our models, we always set $\alpha = 1$ and $\delta = 3$.

Modelling the star proper motion

Sampling the proper motion

We consider a sample of N stars, and, to each star, we assign the spherical coordinates (r, θ, ϕ): radial distribution follows the Plummer density profile, whereas the angular coordinates are sampled from the uniform distributions in the ranges $\cos\theta = [-1, 1]$ and $\phi = [0, 2\pi]$.

Finally, we sample the three velocity components from the velocity distribution function:

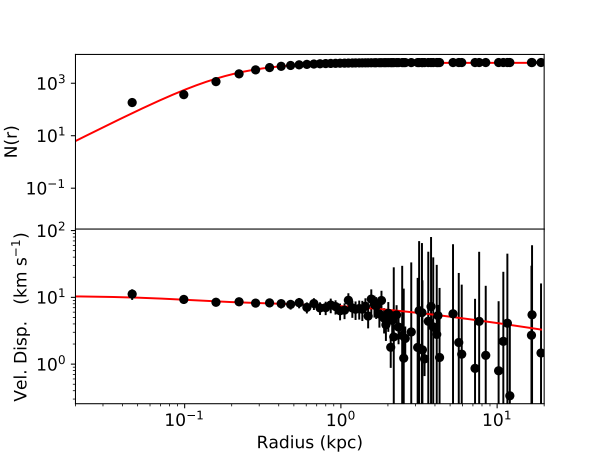
$$p(\boldsymbol{v}|\boldsymbol{r}) = \frac{\exp\left[-\frac{1}{2}\left(\mathbf{v} - \boldsymbol{\mu}(\boldsymbol{x})\right)^T \mathbf{C}^{-1}(\boldsymbol{r})\left(\mathbf{v} - \boldsymbol{\mu}(\boldsymbol{r})\right)\right]}{\sqrt{(2\pi)^3 \left|\mathbf{C}(\mathbf{r})\right|}}, \tag{3}$$

The mean velocity and the covariance matrix simplify in spherical symmetry, one must only solve the radial component of the Jeans equation

$$\frac{1}{\nu(r)} \frac{\mathrm{d}[\nu(r)\overline{\mathrm{v}_r^2}(r)]}{\mathrm{d}r} + 2\beta(r)\frac{\overline{\mathrm{v}_r^2}(r)}{r} = -\frac{\mathrm{d}\Phi(r)}{\mathrm{d}r}. \tag{4}$$

Modelling the star proper motion

Testing the mock catalogues



Modelling the star proper motion

Monte-Carlo-Markov-Chain analysis

We adopt a Bayesian approach to determine the minimum size of the sample of stars and the minimum uncertainty on the proper motions that are required to properly recover the parameters of the DM distribution.

We use the Monte-Carlo-Markov-Chain (MCMC) varying four parameters $\mathbf{f} = (\rho_0, r_s, \gamma, \beta)$. The likelihood function is

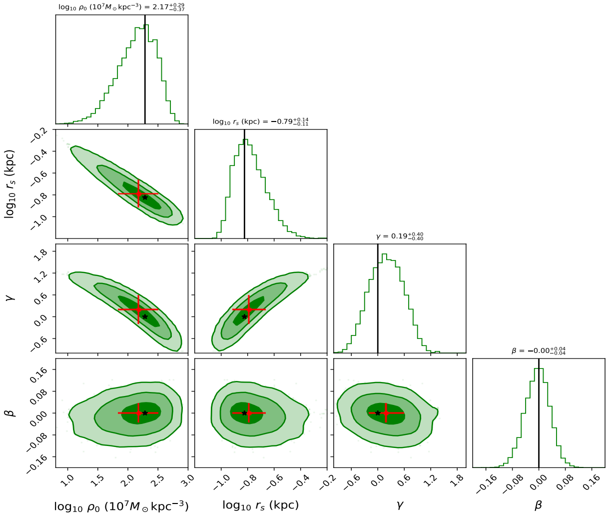
$$\mathcal{L} = \prod_{i=1}^{n} p(\boldsymbol{v}_i|\boldsymbol{r}_i) \tag{5}$$

where i denotes the i-th star in the data set, and

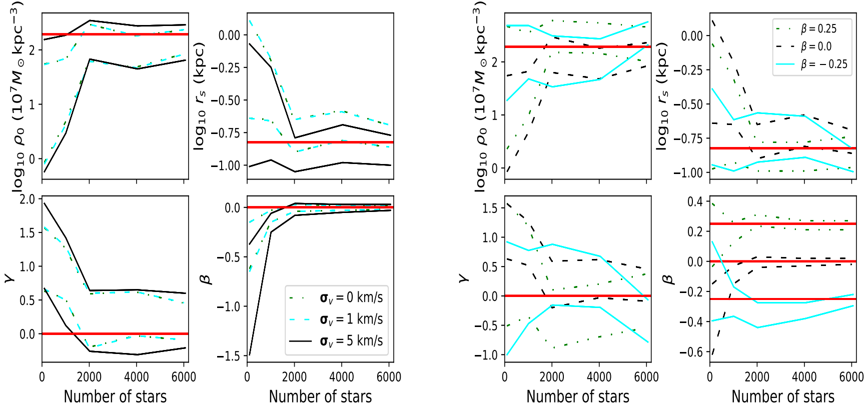
$$p(\boldsymbol{v}|\boldsymbol{r}) = \frac{\exp\left\{-\frac{1}{2}\left[\mathbf{v}-\boldsymbol{\mu}(\boldsymbol{r})\right]^T\left[\mathbf{C}(\boldsymbol{r})+\mathbf{S}(\boldsymbol{r})\right]^{-1}\left[\mathbf{v}-\boldsymbol{\mu}(\boldsymbol{r})\right]\right\}}{\sqrt{(2\pi)^n\left|\mathbf{C}(\boldsymbol{r})+\mathbf{S}(\boldsymbol{r})\right|}}, \tag{6}$$

is the convolution between the Gaussian distribution with covariance matrix $\mathbf{S}(\mathbf{r})$, representing the instrumental errors, and $\mathbf{C}(\mathbf{r})$ representing the probability distribution of the velocity components.

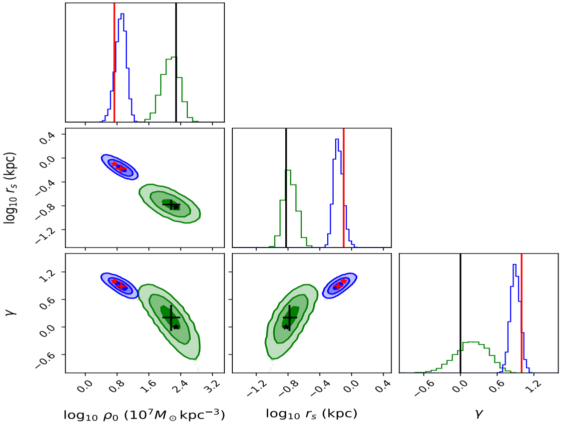
Forecast from the next generation astrometric satellite: *Theia*



Forecast from the next generation astrometric satellite: *Theia*



Cuspy/core problem: insights from the the next generation astrometric satellite



Conclusions and future perspectives

✓ We built and used mock catalogues of proper motion data to determine the minimum number of stars and the minimum uncertainty on the data needed to distinguish between a cusp and a cored dark matter density profile using the velocity information alone.
✓ We created a set of astrometric mock catalogues based on the DM parameters and the Plummer scale length corresponding to Draco galaxy which is designed as a possible target of *Theia* satellite.
✓ Our mock catalogues have different number of stars, ranging from 100 to 6000, different uncertainties on the velocity field, from 0 km/s to 5 km/s, and different anisotropy parameter, namely $\beta = [-0.25; 0; 0.25]$.
✓ Furthermore, we built catalogues for a core and a cuspy dark matter density profile.
✓Then, we used a MCMC algorithm to verify whether the dark matter distribution is recovered.
✓ Our MCMC algorithm returns the parameter estimates within 1-σ from the input value for $N \geq 2000$ stars. While, the velocity uncertainty has only a moderate impact.
✓ We also demonstrated that our methodology works for arbitrary values of β, and for core and cusp dark matter profiles.

Conclusions and future perspectives

✓ Finally, we found that the measure of the proper motions of at least 6000 stars with an accuracy of 1 km s^{-1} at most can distinguish between a cusp and a core in the dark matter density profile.

Nevertheless, our modelling is based on some simplifying assumptions that deserves further discussion and investigation.

⋆ We assumed a constant anisotropy parameter, this can be avoided by using a radial dependent anisotropy parameter.
⋆ We built the galaxy as a spherically symmetric system. However, it is well known that dwarf spheroidals are not spherically symmetric. Therefore, the axis-symmetric Jeans' equations would be required for a more rigorous modelling of these systems.
⋆ Finally, we have also assumed no errors on position of the dwarf galaxy with respect of the observe, and of the stars within the galaxy with respect to the galaxy centre. Therefore, our results only hold for accurate 3D position information, while the impact of the uncertainties on the position will be the subject of future studies.