Non–comoving CDM in a ΛCDM background

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Outline

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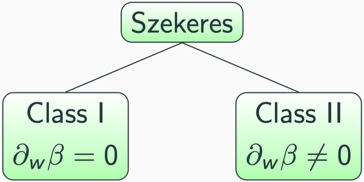
Szekeres–II models

Szekeres-Szafron family

The Szekeres-Szafron metrics are described by the line element

$$ds^2 = -dt^2 + e^{2\alpha} dw^2 + e^{2\beta}(dx^2 + dy^2)$$



Each class is further subdivided in quasi-spherical, quasi-plane and quasi-hyperbolic, corresponding to the limiting cases.

We will focus on Class II models.

Szekeres–II models

The line element characterizing Szekeeres–II solutions is

$$ds^2 = -dt^2 + S^2(t)\left[X^2 dw^2 + \frac{dx^2+dy^2}{f^2}\right], \quad f = 1 + \frac{k[x^2+y^2]}{4},$$

where $X = X(t, x^i)$ with $x^i = w, x, y$.

The metric identifies a canonical orthonormal tetrad $e^a_{(\alpha)}$ such that $g_{ab}\, e^a_{(\alpha)} e^b_{(\beta)} = \eta_{(\alpha)(\beta)}$:

$$e^a_{(0)} = \delta^a_0, \quad e^a_{(w)} = \frac{1}{SX}\delta^a_w, \quad e^a_{(x)} = \frac{f}{S}\delta^a_x, \quad e^a_{(y)} = \frac{S}{f}\delta^a_y, \qquad (1)$$

Szekeres–II models

These models are compatible with a quite general energy-momentum tensor in a comoving frame ($u^a = e^a_{(0)} = \delta^a_0$):

$$T^{ab} = (\rho + \Lambda)u^a u^b + (p - \Lambda)h^{ab} + \pi^{ab} + 2q^{(a}u^{b)}.$$

In a comoving frame the 4–acceleration and vorticity tensor vanish, the nonzero kinematic parameters are then the expansion scalar $\Theta = \bar{\nabla}_a u^a$ and shear tensor $\sigma_{ab} = \tilde{\nabla}_{(a}u_{;b)} - (\Theta/3)h_{ab}$ given by

$$\Theta = \frac{\dot{X}}{X} + \frac{3\dot{S}}{S}, \qquad \sigma^a{}_b = \sigma\,\xi^a{}_b, \qquad \sigma = -\frac{\dot{X}}{3X}, \tag{2}$$

where $\xi^a{}_b = h^a{}_b - 3\delta^a{}_w\delta^w{}_b = \mathrm{diag}[0, -2, 1, 1]$

Szekeres–II models

We consider dust with non–zero energy flux,

$$T_{ab}(\rho+\Lambda)u_a u_b - \Lambda h_{ab} + 2q_{(a}u_{b)}. \qquad (3)$$

The field equations yield

$$\kappa(\rho+\Lambda) = \frac{3\dot{S}^2}{S^2} - \frac{X_{,rr} + rX_{,r} - r^2 X_{,\phi\phi}}{r^2 S^2 X} + \frac{2\dot{S}\dot{X}}{SX},$$
$$\kappa q_r = \frac{\dot{X}_{,r}}{S^2 X}, \qquad q_\phi = \frac{\dot{X}_{,\phi}}{r^2 S^2 X},$$

Szekeres–II models

Where the metric functions satisfy

$$
\begin{aligned}
X &= A(r,\phi,w) + B(r,\phi,w)Q(t,w) + F(t,w), \\
Q &= c_1(w) + c_0(w)\int \frac{dt}{S^3}, \\
A &= \alpha_3(w)r^2 + \alpha_2(w)r\cos\phi + \alpha_1(w)r\sin\phi + \alpha_0(w), \\
B &= \beta_3(w)r^2 + \beta_2(w)r\cos\phi + \beta_1(w)r\sin\phi + \beta_0(w).
\end{aligned}
$$

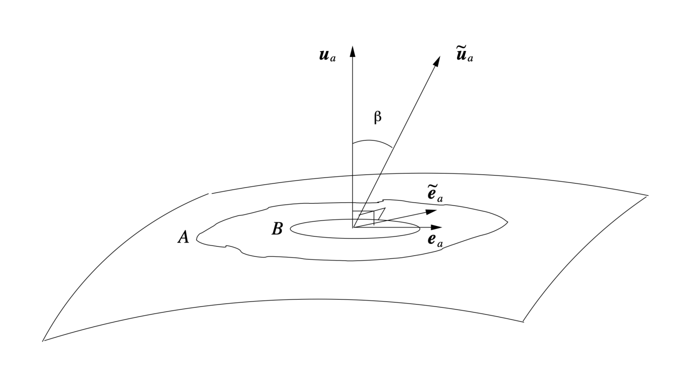
$$
\begin{aligned}
\frac{2\ddot{S}}{S} + \left(\frac{\dot{S}}{S}\right)^2 &= \Lambda, \\
\ddot{F} + 3\left(\frac{\dot{S}}{S}\right)\dot{F} &= \frac{2}{S^2}[Q\beta_3 + \alpha_3].
\end{aligned}
$$

Peculiar velocities

Peculiar velocities

Considering two non-comoving families of geodesics u^a and $\hat{u}^a$, where they are related by a boost

$$\hat{u}^a = \gamma(u^a + v^a), \quad \gamma = \frac{1}{\sqrt{1 - v^a v_a}}. \qquad (4)$$



Credit: Tsagas et al. arXiv:2105.09267.

Peculiar velocities

The relation of the physical quantities between frames become

$$
\begin{aligned}
\rho &= \hat{\rho} + \Lambda + 2\gamma \hat{q}^a v_a + \left\{ \gamma^2 v^a v_a (\hat{\rho} + \hat{p}) + \hat{\Pi}^{ab} v_a v_b \right\}, \\
p &= \hat{p} - \Lambda + \frac{2}{3}\gamma \hat{q}^a v_a + \frac{1}{3}\left\{ \gamma^2 v^a v_a (\hat{\rho} + \hat{p}) + \hat{\Pi}^{ab} v_a v_b \right\} \\
q^a &= \hat{q}^a + (\hat{\rho} + \hat{p}) v^a + \left\{ (\gamma - 1)\hat{q}^a - \gamma \hat{q}^b v_b \hat{u}^a + \gamma^2 v^b v_b (\hat{\rho} + \hat{p}) v^a \right. \\
&\quad \left. + \hat{\Pi}^{ab} v_b - \hat{\Pi}^{ab} v_b v_c \hat{u}^a \right\} \\
\Pi^{ab} &= \hat{\Pi}^{ab} + \left\{ \gamma^2 (\hat{\rho} + \hat{p}) v^{\langle a} v^{b \rangle} - 2u^{(a}\hat{\Pi}^{b)c} v_c + \hat{\Pi}^{cd} v_c v_d \hat{u}^a \hat{u}^b \right. \\
&\quad \left. - \frac{1}{3}\hat{\Pi}^{cd} v_c v_d \hat{h}^{ab} - 2\gamma \hat{q}^c v_c u^{(a} v^{b)} + 2\gamma v^{\langle a} \hat{q}^{b \rangle} \right\},
\end{aligned}
$$

Notice energy–flux is first order on v^a while pressure is a second order quantity.

Pancake models

A FLRW limit?

FLRW models

$$ds^2 = -dt^2 + \frac{a^2(t)\,(dx^2 + dy^2 + dz^2)}{\tilde{f}^2}, \qquad \tilde{f} = 1 + \frac{\tilde{k}[x^2 + y^2 + z^2]}{4},$$

$$T^{ab} = (\tilde{\rho} + \Lambda)\tilde{u}^a\tilde{u}^b + (\tilde{p} - \Lambda)\tilde{h}^{ab}$$

Field equations read

$$\frac{\kappa}{3}\tilde{\rho} = \frac{\dot{a}^2}{a^2} + \frac{\tilde{k}}{a^2} - \Lambda, \qquad \kappa\tilde{p} = -\frac{2\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{\tilde{k}}{3a^2} + \Lambda.$$

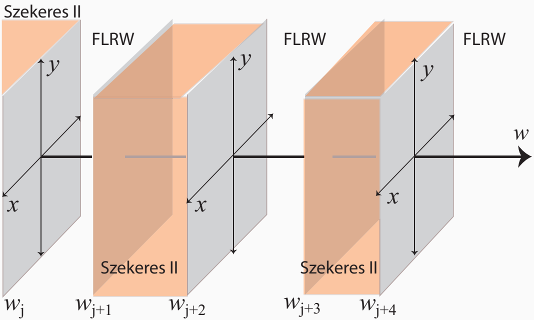
These equations are formally identical to

$$\frac{\kappa}{3}\bar{\rho} = \frac{\dot{S}^2}{S^2} + \frac{k}{S^2} - \Lambda, \quad \kappa\bar{p} = -\frac{2\ddot{S}}{S} - \frac{\dot{S}^2}{S^2} - \frac{k}{3S^2} + \Lambda.$$

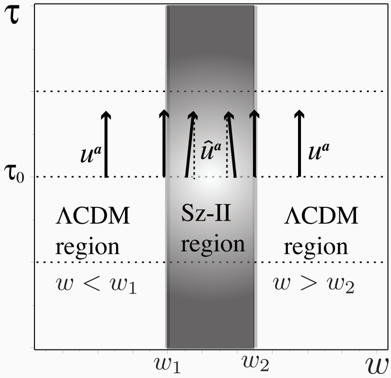
Pancake models

Matching conditions

$$X^{(+)} = 1, \quad (X_{,x})^{(+)} = (X_{,y})^{(+)} = (\dot{X})^{(+)} = (\ddot{X})^{(+)} = 0,$$
$$S(t) = a(t), \qquad k = \tilde{k} = 0 \quad \Rightarrow \quad f = \tilde{f} = 1,$$



Pancake models



Non–comoving CDM in a Λ-CDM background

Non–comoving CDM in a Λ-CDM background

Using the "pancake models" we can establish a toy model whose free parameters allow for a description of CDM structures which are non–comoving in the CMB frame, with peculiar velocities consistent with observed values.

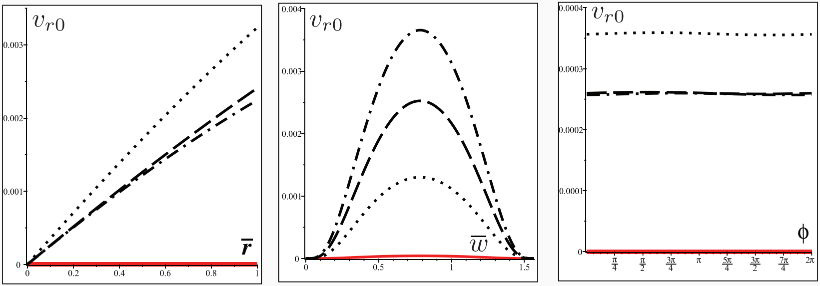
$$
\begin{aligned}
\Omega^{\rho} &\equiv \frac{8\pi\rho}{3H_0^2} = \bar{\Omega}^{\rho} + \delta^{\Omega}, \\
&\quad \bar{\Omega}^{\rho} = \frac{\Omega_0^m}{S^3} + \Omega_0^{\Lambda}, \qquad \Omega_0^{\Lambda} = \frac{8\pi\Lambda}{3H_0^2}, \\
&\quad \delta^{\Omega} = \frac{8\pi\left[X_{,\phi\phi} - rX_{,r} - r^2 X_{,rr}\right]}{3H_0^2 r^2 S^2 X} - \frac{2S_{,\tau}X_{,\tau}}{3SX}, \\
\Omega^{q_r} &\equiv \frac{8\pi q_r}{3H_0^2} = \frac{X_{r,\tau}}{3H_0^2 S^2 X} \quad \Rightarrow \quad v_r = \frac{\Omega^{q_r}}{\Omega^{\rho}}
\end{aligned}
$$

Non-comoving cold dark matter in a ΛCDM background

$$\Theta = \frac{\dot{X}}{X} + \frac{3\dot{S}}{S}, \qquad \sigma^a_b = \sigma\,\xi^a_b, \qquad \sigma = -\frac{\dot{X}}{3X},$$
$$\mathcal{H} = \frac{\Theta}{3H_0}, \qquad \bar{\mathcal{H}} = \frac{S_{,\tau}}{S}, \qquad \delta^{\mathcal{H}} = \frac{X_{,\tau}}{3X},$$

- $\alpha_0(w) = \cos^2(\nu_0 w)$, $\alpha_i(w) = \mathcal{C}_i \sin^2(\nu_i w)$,
- $0 < \mathcal{C}_i < 1$, $\nu_i \geq 2H_0/c$,
- $v_a v^a \ll 1$ requier $\beta_i^2 \ll 1$,
- $\beta_i(w) = \epsilon_i \varphi_i(w)$, $\epsilon_i \epsilon_j \ll 1$, $\varphi_i = \sin^2(\nu_i w)$,
- $S(\tau) = \left(\frac{\Omega_0^m}{\Omega_0^\Lambda}\right)^{1/3} \sinh^{\frac{2}{3}}\left(\frac{3}{2}\sqrt{\Omega_0^\Lambda}\tau\right)$,
- $S(\tau_0) = 1 \quad \Rightarrow \tau_0 = 0,9662$,
- $\Omega_0^m = 0,3$, $\Omega_0^\Lambda = 0,7$,
- B.C. $F_{,\tau}(\tau_0, w) = 0$ y $F(\tau_0, w) = \epsilon_0 \varphi_0(w)$.

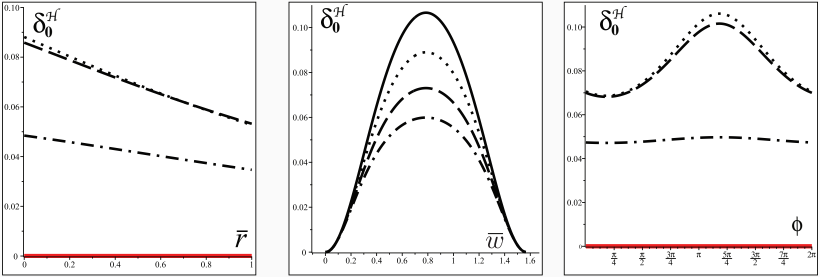
Non-comoving cold dark matter in a ΛCDM background



(a) **(b)** **(c)**

Non-comoving cold dark matter in a ΛCDM background

The Hubble scalar's contrast $\delta^{\mathcal{H}}$ with respect to the background value H_0 given by the ΛCDM model, is entirely determined by the shear tensor, producing H_0 fluctuations of the same order of magnitude, $\sim 10\,\%$ difference, present in the "H_0 tension".



(a) **(b)** **(c)**

References

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