Effects of a scalar field potential on primordial perturbations in Hybrid (Loop) Quantum Cosmology.

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- Introduction to the problem.
  - Motivate the problem.
  - Introduce the existing work on this issue.
- Our own work.
  - Show our solution to the problem.
  - Talk about the consequences of this work.

Introduction: Quantum Gravity?

We are living the Golden Age of Observational Cosmology. This means:

- Ability to measure cosmological parameters with great precision.
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- Tensions with Standard Cosmology (Statistically exceptional universe)
    - Lack of power at low l
    - Anomalies for multipoles at $l \sim 30$ and below.
    - Amplitude of gravitational lensing.

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- Quantum Gravity effects?

Introduction: LQG.

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- Spacetime geometry is quantized.
- It is a canonical and non-perturbative quantization of GR.
- Independent of the background structure in spacetime.

Introduction: LQC.

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And, what is LQC? Results?

- LQC is the application of techniques of LQG in cosmological systems.
- Applied succesfully to FLRW $\Rightarrow$ Big Bang singularity *replaced* by Big Bounce.
- FLRW with matter content or other spatial topology.
- Quantization of Bianchi I models (and others).
- Inhomogeneous models (Gowdy Model).

Introduction: Hybrid Approach.

Currently, attention is centered on the Hybrid Approach. In it:

- Physical DoF of perturbations are gauge invariants (constant under perturbative diffeomorphisms).

Based on the hypothesis that:

- Most important quantum effects are encoded in the zero-modes $\Rightarrow$ Quantized with Polymeric Quantization of LQC.
- Other modes may be described with QFT techniques (Fock quantization).

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It has been applied to:

- Gowdy model with a scalar field as matter content.
- Cosmological perturbations in inflationary scenarios.

Introduction: Hybrid Approach.

More specifically, perturbations of an FLRW spacetime with a scalar field (inflaton) and coupled to a potential (usually quadratic for simplicity) have recently been considered.

- Perturbations present in both the scalar field and the metric.
- An Ansatz with separation of variables is used for physical states.

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From this one obtains:

- A quantum version of GR.
- Equations for the inhomogeneities analogous to MS eq. with corrections $\Rightarrow$ Power spectra $\Rightarrow$ Comparison with data.

Kinetically Dominated Regimes.

Great interest in Kinetically Dominated (KD) Regimes. Why?

- Pre-inflationary effects may have left a trace in the Universe, for example if:
  - Slow-roll not too large.
  - Fast-roll inflation before slow-roll.
- In these epochs, Universe is KD.

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In case of hybrid quantization with potential and KD:

- Pre-Inflationary scalar perturbations $\Rightarrow$ Anisotropies of CMB.
- Perturbations modeled by modified MS equations.
- MMS eqs are dependent on effective mass.
- Effective mass is dependent on scalar potential.

Current State.

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The problem is to find the evolution of the perturbations. We need to solve MMS eqs. on an evolving quantum background. Analytic solution not known. Numerical calculations have problems.

- Geometric operators of the background $\leftrightarrow f(\phi, W)$.
- MMS equations $\leftrightarrow f(\eta, s)$.
- Inflaton (ϕ) and conformal time (η) relation $\leftrightarrow f(W)$.
- Effective mass (s) $\leftrightarrow f(W)$.
- Expected values of operators on states $\leftrightarrow f(W)$.
- **Extremely hard numerical problem for each potential**.
- **Has to be calculated for every potential**.

Current Solution and Choice of Vacuum.

Current approach is to choose a highly peaked initial state.

- But this neglects the quantum nature of background.
- Still have to calculate it for every potential.
- Hard to see the effects of slight variations in potential.

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Vacuum problems not even considered:

- Bunch-Davies state not justifiable $\Rightarrow$ Which one is natural?
- Variety of proposals, we chose the NO vacuum. Why?
    - No oscillations $\Rightarrow$ No unwanted amplifications on the Power Spectra.
    - Allows an analytical treatment.
    - Gives rise to natural variables which give $f > 0$ solutions.

So,
What's New?

What have we developed?.

Analytical study of leading-order corrections due to a scalar potential in KD regimes.

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Advantages of our solution:

- Potential dependent solution as function of free solution $\Rightarrow$ **Only need to calculate free case**.
- $W = 0 \Rightarrow$ Possible to calculate expected values $\Rightarrow$ **No need to choose highly peaked states**.

Theoretical Framework.

We work with:

- Flat FLRW geometry with compact sections (T^3).
- Homogeneous scalar field subject to potential $W(\phi)$.
- Background subject to constraint:

$$\frac{\hat{\pi}_\phi^2 - \hat{\mathcal{H}}_0^{(2)}}{2} = 0, \qquad \hat{\mathcal{H}}_0^{(2)} = \left[\hat{\mathcal{H}}_0^{(F)}\right]^2 - 2W(\phi)\hat{V}^2$$

  where $\mathcal{H}_0^{(F)} = 2\sqrt{3\pi G}|\pi_V|V$ is the generator of the free evolution.
- Perturbations are introduced and the action is truncated at quadratic order.
- Quantized with hybrid approach.

Modified MS equations.

- We introduce an Ansatz of separation of variables for physical states.

$$\chi(V,\phi) = \mathcal{P}\left[\exp\left(i\int_{\phi_0}^{\phi} d\tilde{\phi}\,\hat{\mathcal{H}}_0(V,\tilde{\phi})\right)\right]\chi_0(V).$$

- Hamiltonian constraint + Ansatz $\Rightarrow$ Modified MS eqs.

$$\ddot{v}_{\vec{k},\varepsilon} + \left[k^2 + \underbrace{\frac{\langle \hat{\vartheta}_e^q + (\hat{\vartheta}_o \hat{\mathcal{H}}_0)_{\mathsf{sym}}\rangle_\chi}{\langle \hat{\vartheta}_e \rangle_\chi}}_{s(\eta)}\right] v_{\vec{k},\varepsilon} = 0.$$

Modified MS equations.

The operators are ($\hbar = c = 1$):

$$
\begin{aligned}
\hat{\vartheta}_e^q &= \frac{1}{2\pi}\widehat{\left[\frac{1}{V}\right]}^{1/3}\hat{\mathcal{H}}_0^{(2)}\left(19 - 18(\hat{\mathcal{H}}_0^{(F)})^{-2}\hat{\mathcal{H}}_0^{(2)}\right)\widehat{\left[\frac{1}{V}\right]}^{1/3} + \\
&+ \frac{3}{8\pi^2 G}\hat{V}^{4/3}\left(W''(\phi) - \frac{16\pi G}{3}W(\phi)\right), \\
\hat{\vartheta}_o &= \frac{3}{\pi}\sqrt{\frac{3}{\pi G}}W'(\phi)\hat{V}^{2/3}(\hat{\mathcal{H}}_0^{(F)})^{-1}\hat{\Lambda}_0^{(F)}(\hat{\mathcal{H}}_0^{(F)})^{-1}\hat{V}^{2/3} \\
\hat{\vartheta}_e &= \frac{3}{2G}\hat{V}^{2/3}.
\end{aligned}
$$

Operators dependent on ϕ, but MMS eqs on η. Relation given by:

$$\langle\hat{\mathcal{H}}_0\rangle_\chi d\eta = \langle\hat{\vartheta}_e\rangle_\chi d\phi.$$

Vacuum State.

- Determined via diagonalization of the Hamiltonian of the perturbations $\Rightarrow$ NO vacuum.
- Given $s(\eta)$, we find positive frequency solutions:

$$\mu_k = \frac{1}{\sqrt{-2\Im(h_k)}} e^{i\int_{\eta_0}^{\eta} d\tilde{\eta}\Im(h_k)},$$

where h_k is solution of:

$$\begin{cases} \dot{h}_k = k^2 + s + h_k^2 \\ h_k(\eta_0) = ?? \end{cases}$$

- From them we can extract the Power Spectra at the end of the slow-roll:

$$\mathcal{P}(k, \eta_{\text{end}}) = \frac{k^3}{2\pi^2} \left|\mu_k(\eta_{\text{end}})\right|^2 = -\frac{k^3}{4\pi^2 \Im\left(h_k(\eta_{\text{end}})\right)}.$$

Vacuum State.

- h_k has the asymptotic behaviour:

$$\frac{k}{h_k} \sim i\left[1 - \frac{1}{2k^2}\sum_{n=0}^{\infty}\left(\frac{-i}{2k}\right)^n \gamma_n\right].$$

where γ_n is given by:

$$\gamma_{n+1} = -\dot{\gamma}_n + 4s\left[\gamma_{n-1} + \sum_{m=0}^{n-3}\gamma_m\gamma_{n-(m+3)}\right] - \sum_{m=0}^{n-1}\gamma_m\gamma_{n-(m+1)},$$

and $\gamma_0 = s$ and $\gamma_{-n} = 0 \; \forall n > 0$.

Leading-Order Corrections.

We will calculate the LO corrections due to W in KD. There are corrections in:

- The operators determining s
- Evolution of the background.
- Relation between ϕ and η.
- Vacuum determination.

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We will begin with the effective mass.

Corrections on the operators.

The mass is given by:

$$\frac{\langle \hat{\vartheta}_e^q + (\hat{\vartheta}_o \hat{\mathcal{H}}_0)_{\mathsf{sym}} \rangle_\chi}{\langle \hat{\vartheta}_e \rangle_\chi}.$$

And we have that:

- $\hat{\vartheta}_e$ is W independent.
- $\hat{\vartheta}_e^q$ has a free part and a linear part.
- $\hat{\vartheta}_0$ is linear in $W \Rightarrow \hat{\mathcal{H}}_0 = \hat{\mathcal{H}}_0^{(F)}$

Corrections on the evolution.

We use an interaction picture to obtain:

$$\chi(V,\phi) = \left[1 - i\int_{\phi_0}^{\phi} d\tilde{\phi}\, \hat{K}(\phi,\tilde{\phi}) W(\tilde{\phi}) + O(W^2)\right] \chi^{(F)}(V,\phi),$$

where we have defined:

- The free evolution:

$$\chi^{(F)}(V,\phi) = \exp\left[i\hat{\mathcal{H}}_0^{(F)}(\phi-\phi_0)\right]\chi_0(V).$$

- The part due to the interaction Hamiltonian. In it we make first:

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0^{(F)} - W(\phi)(\hat{\mathcal{H}}_0^{(F)})^{-1/2}\hat{V}^2(\hat{\mathcal{H}}_0^{(F)})^{-1/2},$$

  and from there we obtain the expression for $\hat{K}$:

$$\hat{K}(\phi,\tilde{\phi}) = e^{i\hat{\mathcal{H}}_0^{(F)}(\phi-\tilde{\phi})}(\hat{\mathcal{H}}_0^{(F)})^{-1/2}\hat{V}^2(\hat{\mathcal{H}}_0^{(F)})^{-1/2}e^{-i\hat{\mathcal{H}}_0^{(F)}(\phi-\tilde{\phi})}.$$

Corrections in the time relations.

From the relation $\langle\hat{\mathcal{H}}_0\rangle_\chi d\eta = \langle\hat{\vartheta}_e\rangle_\chi d\phi$, we expand and consider $\eta = \eta^{(F)} + \eta^{(W)} + \mathcal{O}(W^2)$ to get the free relation:

$$\eta^{(F)}(\phi) = \frac{3}{2G}\int_{\phi_0}^{\phi} d\tilde{\phi}\, \frac{\langle\hat{V}^{2/3}\rangle_{\chi^{(F)}}}{\langle\hat{\mathcal{H}}_0^{(F)}\rangle_{\chi^{(F)}}}, \text{ and its inverse } \phi^{(F)}(\eta).$$

Then we make $\phi = \phi^{(F)} + \phi^{(W)} + \mathcal{O}(W^2)$ to get:

$$\phi^{(W)}(\eta) = -\phi^{(F)}\left(\eta^{(W)}\big(\phi^{(F)}(\eta)\big)\right).$$

Corrections to the effective mass.

Now we have all we need to find $s = s^{(F)} + s^{(W)} + \mathcal{O}(W^2)$. If we define the freely evolved background state in conformal time as:

$$\tilde{\chi}^{(F)}(V, \eta) = \exp\left[i\hat{\mathcal{H}}_0^{(F)}\big(\phi^{(F)}(\eta) - \phi_0\big)\right]\chi_0(V),$$

we have:

$$\chi\big(V, \phi(\eta)\big) = \left[1 + i\hat{J}^{(W)}(\eta) + O(W^2)\right]\tilde{\chi}^{(F)}(V, \eta),$$

where we have defined for ease of notation

$$\hat{J}^{(W)}(\eta) = \underbrace{\hat{\mathcal{H}}_0^{(F)}\phi^{(W)}(\eta)}_{\phi\neq\phi^{(F)}} - \underbrace{\int_{\phi_0}^{\phi^{(F)}(\eta)} d\tilde{\phi}\,\hat{K}\big(\phi^{(F)}(\eta), \tilde{\phi}\big)W(\tilde{\phi})}_{\text{From Interaction Hamiltonian}}.$$

Corrections to the effective mass.

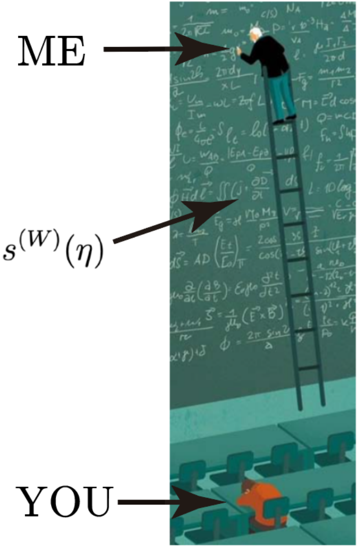
The final result for the free part is:

$$s^{(F)}(\eta) = \frac{G}{3\pi} \frac{\langle \widehat{\left[\frac{1}{V}\right]}^{1/3} (\hat{\mathcal{H}}_0^{(F)})^2 \widehat{\left[\frac{1}{V}\right]}^{1/3} \rangle_{\tilde{\chi}^{(F)}}}{\langle \hat{V}^{2/3} \rangle_{\tilde{\chi}^{(F)}}},$$

and for the leading-order correction:

$$\begin{aligned} s^{(W)}(\eta) &= \frac{2G}{3\pi \langle \hat{V}^{2/3} \rangle_{\tilde{\chi}^{(F)}}} \left\{ \left(\frac{3}{8\pi G} W'' - 2W \right) \langle \hat{V}^{4/3} \rangle_{\tilde{\chi}^{(F)}} + 17W \langle \widehat{\left[\frac{1}{V}\right]}^{1/3} \hat{V}^2 \widehat{\left[\frac{1}{V}\right]}^{1/3} \rangle_{\tilde{\chi}^{(F)}} \right\} \\ &+ \frac{2\sqrt{3G}}{\pi\sqrt{\pi} \langle \hat{V}^{2/3} \rangle_{\tilde{\chi}^{(F)}}} W' \langle (\hat{V}^{2/3} (\hat{\mathcal{H}}_0^{(F)})^{-1} \hat{\Lambda}_0^{(F)} (\hat{\mathcal{H}}_0^{(F)})^{-1} \hat{V}^{2/3} \hat{\mathcal{H}}_0^{(F)})_{\text{sym}} \rangle_{\tilde{\chi}^{(F)}} \\ &+ \frac{i}{\langle \hat{V}^{2/3} \rangle_{\tilde{\chi}^{(F)}}} \left\{ -s^{(F)} \langle [\hat{V}^{2/3}, \hat{J}^{(W)}] \rangle_{\tilde{\chi}^{(F)}} + \frac{G}{3\pi} \langle \left[\widehat{\left[\frac{1}{V}\right]}^{1/3} (\hat{\mathcal{H}}_0^{(F)})^2 \widehat{\left[\frac{1}{V}\right]}^{1/3}, \hat{J}^{(W)} \right] \rangle_{\tilde{\chi}^{(F)}} \right\}. \end{aligned}$$

I know.



Remarks to the effective mass calculation.

There are important remarks to be made about this:

- It is particularizable to (hybrid) LQC and Quantum Geometrodynamics.
- Passing to classical functions, $[,] \to i\{,\}$ and evaluating on classical solutions, **General Relativity is recovered**:

$$
\begin{aligned}
s_{GR}^{(F)} &= \frac{G}{3\pi}\frac{(\mathcal{H}_0^{(F)})^2}{V^{4/3}} = 4G^2V^{2/3}\pi_V^2, \\
s_{GR}^{(W)} &= \frac{2G}{3\pi}V^{2/3}\left(\frac{3}{8\pi G}W'' + 15W - 3\sqrt{\frac{3}{\pi G}}\,sgn(\pi_V)W'\right) + \\
&- 4G^2\{V^{2/3}\pi_V^2, J^{(W)}\}.
\end{aligned}
$$

- In particular, $W = 0 \Rightarrow s^{(W)} = 0$, and $s_{GR}^{(F)}$ turns out to reproduce the standard expression $-\ddot{z}/z = -\ddot{a}/a$ in General Relativity, with $z = a^2\dot{\phi}/\dot{a}$.

Corrections to the vacuum.

For the last part, we make $h_k = h_k^{(F)} + h_k^{(W)} + \mathcal{O}(W^2)$ as well as the analogous for γ_n to get, first for h_k:

$$h_k^{(W)}(\eta) = \left(C_k^{(W)} + \int_{\eta_0}^{\eta} d\tilde{\eta}\, s^{(W)}(\tilde{\eta}) e^{-I_{h_k}^{(F)}(\tilde{\eta})} \right) e^{I_{h_k}^{(F)}(\eta)},$$

where we have defined:

$$I_{h_k}^{(F)}(\eta) = 2 \int_{\eta_0}^{\eta} d\tilde{\eta}\, h_k^{(F)}(\tilde{\eta}).$$

Integrating by parts and defining a convenient time $d\tau_k = 2h_k^{(F)}(\eta) d\eta$:

$$h_k^{(W)}(\eta) = -\sum_{n=0}^{\infty} \frac{d^n}{d\tau_k^n} \left(\frac{s^{(W)}(\eta)}{2h_k^{(F)}(\eta)} \right) + D_k^{(W)}(\eta_0) e^{I_{h_k}^{(F)}(\eta)},$$

Corrections to the vacuum.

Imposing now the asymptotic behaviour we obtain:

$$h_k^{(W)} \sim \frac{\displaystyle\sum_{n=0}^{\infty} (2ik)^{-n-1} \gamma_n^{(W)}}{\left[1 + 2\displaystyle\sum_{n=0}^{\infty} (2ik)^{-n-2} \gamma_n^{(F)}\right]^2},$$

where $\gamma_n^{(W)}$ comes from substracting γ_n and $\gamma_n^{(F)}$. The asymptotic behaviour fixes the constant to zero and finally we get:

$$h_k^{(W)}(\eta) \sim -\sum_{n=0}^{\infty} \frac{d^n}{d\tau_k^n} \left(\frac{s^{(W)}(\eta)}{2h_k^{(F)}(\eta)} \right).$$

Conclusions.

We have developed an analytical study of LO corrections of $W(\phi)$ in KD. A study which:

- May be generalized to higher orders.
- May be generalized to other prescriptions in quantum cosmology.
- Shows the advantages of the Non-Oscillating Vacuum.
- Gives analytical insight into quantum effects.
- Greatly helps in the simulation of the effects of potentials on quantum effects and power spectra.
- Can help discern certain potentials which may relax the tensions with Standard Cosmology.

The End.
Thanks for your time.