Holographic Bound on area of Compact Binary Merger Remnant

Contributed talk at the Spanish-Portuguese Relativity Meeting EREP2021, University of Aveiro, Aveiro, Portugal. 13-16 September 2021

Parthasarathi Majumdar

School of Physical Sciences
Indian Association for the Cultivation of Science
Kolkata, India

Collaborator : Anarya Ray, University of Wisconsin-Milwaukee, USA

Ref. Phys Rev D104, 044306 (2021), arXiv 2008.13425

September 15, 2021

GW150914 : signal from BBH merger ? Or BECO merger ?

B P Abbott et. al. (LIGO-VIRGO 2016-2021) : Data consistent with BBH merger

Giudice et. al., 2016; Cardoso et. al. 2017 : Data can be explained by non-bh BECO merger : boson stars, gravastars, wormholes : predict appropriate signatures

Insufficient evidence to resolve the issue !

Addazzi et. al. 2020 : accretional instability of ECOs to BH, demonstration using Thorne's Hoop conjecture

No evidence of accretion for coalescing binary

Prediction indep of nature of coalescence or remnant !

Bekenstein's limiting laws provide prediction even if merger not BBH

Lower bound on cross-sectional area of postmerger remnant derived

Direct check on prediction for all compact binary coalescence !

First-ever direct observational hint possible on LQG effects !

Likely constraints for NS EoS's for BNS mergers without considering tidal deformabilities !

Generalized Second Law (Bekenstein 1973)

For BBH merger into compact obj + GW emission,

$$S_{CO} + S_{GW} > S_{bh1}(A_1) + S_{bh2}(A_2)$$

- **Valid for QG frmwk giving bh entropy ab initio**
- **Supercedes Hawking's Second law of bh mechanics :** $A_{rem} > A_1 + A_2$
- **Holds including LQG corr to S_{bh} (Kaul & PM 1998, 2000)**
- **Holographic character natural in LQG formulation of bh entropy**
- **Area (of horizon) natural observable for quantum bh, rather than mass**

Basics : Background-indep Loop Quantum Gravity

Canonical quantum theory of $SU(2)$ connection A_a^I , $I = 1, 2, 3$ and densitized triads $E_a^I \leftarrow SL(2, C)$ gauge theory in time gauge

$$\left[A_a^I(\mathrm{x}) \ , \ E_J^b(\mathrm{y})\right] = \delta_a^b \ \delta_J^I \ \delta^{(3)}(\mathrm{x}, \mathrm{y})$$

For nonsingular CCR, better option : global variables $\rightarrow$ Wilson lines, fluxes

$$h_A(C) \equiv \mathcal{P} \exp \int_C A \cdot dx \ ; \ \Phi_E(S, f) \ \equiv \ \int_S f_I(S) E^I \cdot ds$$

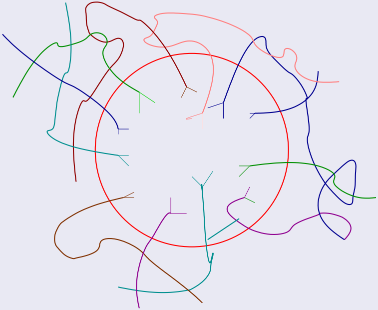
- **Quantum sptm geometry indep of classical backgrounds**
- **LQG states** $\Psi = \sum c_i \psi_i$, ψ_i $\rightarrow$ **spin network states**
- **Observables (e.g., area operator) related to fluxes, are diagonalized by spinnet basis**

Quantum Space : spin network

Horizon DoF & Dynamics

- **IH is a null inner boundary** $\Rightarrow h_{ab}dx^a dx^b = 0$, $h_{ab} \rightarrow$ **induced metric on IH**
- $\Rightarrow \det h = 0 \Rightarrow$ **theory on IH cannot be** $\int \sqrt{h}\ {}^3R(h)$**, or any theory requiring inverting** h_{ab}
- **Theory on IH must be topological** $\Rightarrow S_{IH} \neq S_{IH}[h]$
- **On IH, the boundary gauge potentials are described by an** $SU(2)$ **Chern Simons theory of** A_i^a **coupled to** $\Sigma_{ij}^{ab} \equiv E_I^{[a} E_J^{b]}$ **with coupling** $k \equiv (A_{IH}/8\pi l_P^2)$
- CS EoM : $(k/2\pi) F_{ab}^I = -\Sigma_{ab}^I$

Quantum Black Hole Entropy (Kaul, PM 1998,2000)



Bulk spin network edges (LQG) interact as point sources with $SU(2)_k$ Chern-Simons theory states describing qu bh horizon, 'depositing' their spin at punctures

Count $\mathcal{N} \equiv dim\mathcal{H}_{CS+spins}$ using Witten's formula

$\dim \mathcal{H}_{CS+spins} = \#conf - blks[SU(2)\ WZW]$ for $k \equiv \frac{A_h}{A_{Pl}} >>> 1$

$S_{bh} = \log \mathcal{N} \Rightarrow$ **holographic char of S_{bh} in LQG**

Quantum Black Hole Entropy (contd)

For macroscopic bhs, $k >>> 1$

$$\mathcal{N}(j_1, \dots j_P) = \prod_{i=1}^{P} \sum_{m_i=-j_i}^{j_i} [\delta_{\sum_{n=1}^{P} m_n, 0} - \frac{1}{2}\delta_{\sum_{n=1}^{P} m_n, -1} - \frac{1}{2}\delta_{\sum_{n=1}^{P} m_n, 1}]$$

With $S_{bh} = \log \mathcal{N}$ **obtain**

$$S_{bh} = S_{BH} - \frac{3}{2}\log S_{BH} + \mathcal{O}(S_{BH}^{-1})$$

$$S_{BH} = \frac{A_{hor}}{4 l_P^2}$$

Holographic QG result indep of classical bkgd

First Entropy Bound (Bekenstein 1974)

For any compact star of cross-sectional area A_C , $S_C < S_{bh}(A_C)$

Adiabatic accretion : A_C does not change significantly

Limiting situation : $A_C \sim A_{Sch} \Rightarrow$ virtual collapse to bh

During accretion $S_C \nearrow\ S_{bh}(A_C)$

Of all compact astrophysical objects of a given cross-sectional area, a black hole whose horizon area equals the cross-sectional area, carries the largest entropy

The accretion process considered is virtual. Actual rate of accretion is irrelevant

Consider BBH merger to any Compact object with GW emission

$$S_{CO} < S_{bh}(A_{CO}) = S_{BH}(A_{CO}) - \frac{3}{2}\log S_{BH}(A_{CO})$$
$$S_{bh}(A_{CO}) + S_{GW} > S_{bh1}(A_1) + S_{bh2}(A_2)$$

For BBH merger to a bh with GW emission

$$S_{bh}(A_{bhrem}) + S_{GW} > S_{bh1}(A_1) + S_{bh2}(A_2)$$

Holographic entropy correctionsfrom LQG are systematically included

Does this generalize if binary coalescence is not BBH ? Yes !

Generalization to any compact binary coalescence

Generalized Second law + Entropy bound ⇒

$$S_{bh}(A_{CO}) + S_{GW} > S_{CO1} + S_{CO2}$$

Entropy bound by itself ⇒

$$S_{CO1} < S_{bh}(A_{CO1}) \ , \ S_{CO2} < S_{bh}(A_{CO2})$$

This ⇒

$$S_{bh}(A_{CO}) + S_{GW} > S_{bh}(A_{CO1}) + S_{bh}(A_{CO2})$$

Strictly speaking a sufficiency condition, but since we want to include BBH mergers, physically it is also a necessary requirement

Applicable version of general area bound

In general, for all compact binary coalescence into a compact remnant

$$\frac{\exp \bar{A}_{CO}}{\bar{A}_{CO}^{3/2}} > \frac{\exp(\bar{A}_{CO1} + \bar{A}_{CO2} - S_{GW}^{EQ})}{(\bar{A}_{CO1}\bar{A}_{CO2})^{3/2}}$$

where $\bar{A} \equiv A/4l_P^2$ **is expressed in units of Planck area**

Bound is indep of astrophysical nature of binary inspiral and postmerger remnant !

Define compactness ratio $C_C \equiv R_C/r_{SC}$, $C_C = \mathcal{O}(1)$ **for bh,** $C_C = \mathcal{O}(10)$ **for ns,** $C_C = \mathcal{O}(10^3)$ **for wd** $\Rightarrow$

$$A_C(R_C) = A(C_C r_{SC}) = C_C^2 A_C(r_{SC}) = C_C^2 M_C^2$$

$$\frac{\exp[C_C^2 \bar{A}_C]}{(C_C^2 \bar{A}_C)^{3/2}} > \frac{\exp[C_{C1}^2 \bar{A}_{C1} + C_{C2}^2 \bar{A}_{C2} - S_{GW}^{EQ}]}{(C_{C1}^2 \bar{A}_{C1} C_{C2}^2 \bar{A}_{C2})^{3/2}}$$

Estimating GW entropy (Ma, 1983)

Why should $S_{GW} \neq 0$? 2-bdy inspiral gives monochr sig with $S_{GW} = 0$!

Argument invalid close to merger : dynamical sptm $\Rightarrow$ particle creation (Parker 1967) !

Largest contribution to S_{GW} comes from this dynamical situation at merger

$$S^{EQ}_{GW} = \int_{\omega_1}^{\omega_2} \frac{d\omega}{2\pi} \frac{I(\omega)}{I_0} \log \left[\frac{N_0}{N_c(\omega)} \right]$$

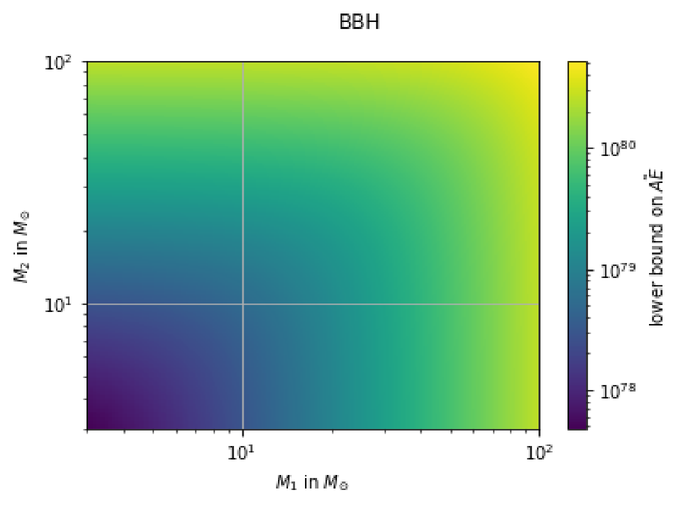
$N_c << N_0$ **for the equilibrium situation (maximal randomness)** $\Rightarrow N_c \simeq 1$ **(maximal incoherence)**

GW150914 datasheet : $N_0 \equiv \mathcal{E}_{GW} / \mathcal{E}_{grav} \simeq 10^{77}$

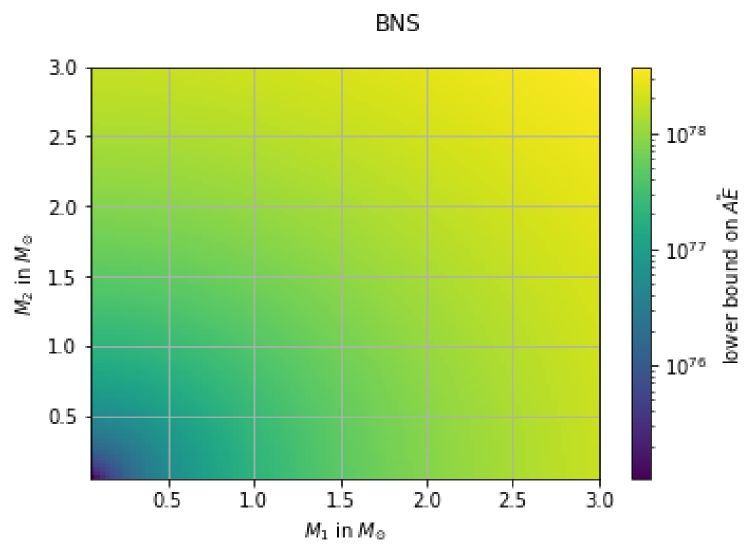
Peak $\omega = 150\ Hz \Rightarrow$ **Estimate** $S^{EQ}_{GW} \sim \mathcal{O}(10^2)$.

For BBH mergers (slow spins) :

$C_{C1} = C_{C2} = 1 \ , \ A_C \sim M_C^2$



For BNS mergers : $C_{C1} = C_{C2} \simeq 5$; chosen EoS



Bound on remnant area (BNS merger) constrains ns EoS

- **Given measured inspiral ns masses M_1 M_2, and remnant mass M_{rem}, and fiducial EoS, compute r_1, r_2, r_{rem} and hence cross-sectional areas, LIGO Arithmetic Library Simulations.**
- **If $A_{rem,meas} < A_{rem,th}$ from graph, discard chosen EoS**
- **Additional data on tidal deformabilities is not directly needed**
- **Not exhaustive, proof of principle**
- **Success depends on accuracy of mass measurement**
- **GW170718 data does not yet give remnant mass accurately !**

GW170718 data vs lower bound on remnant area

Choose inspiral component ns masses from data

Compute the component radii and lower bound remnant radius for every available posterior sample of (M_1, M_2) obtained from a parameter estimation run of GW170817, given a fiducial EoS

Histogram them and compute 90% confidence intervals by subtracting 0.05 and 0.95 quantiles.

EoS	**r_1 (km)**	**r_2 (km)**	**Min r_{rem} (km)**
ALF1	$8.96^{+0.69}_{-0.75}$	$8.2^{+1.65}_{-0.54}$	$12.28^{+1.26}_{-0.7}$
AP3	$12.1^{+0.0}_{-0.0}$	$12.07^{+0.01}_{-0.02}$	$17.09^{+0.0}_{-0.02}$
BSK20	$11.72^{+0.02}_{-0.07}$	$11.77^{+0.0}_{-0.01}$	$16.62^{+0.01}_{-0.05}$
FPS	$7.55^{+2.74}_{-0.54}$	$9.37^{+1.7}_{-2.43}$	$12.36^{+2.5}_{-2.47}$
GNH3	$14.34^{+0.16}_{-0.37}$	$14.63^{+0.12}_{-0.1}$	$20.49^{+0.03}_{-0.17}$
H5	$13.03^{+0.16}_{-0.54}$	$13.28^{+0.02}_{-0.06}$	$18.61^{+0.06}_{-0.35}$

Missing : measured $r_{rem,meas}$ for any EoS → paucity of data

Summary

- More detailed data on remnant properties for BNS mergers will enable greater applicability ofour bound to constrain NS EoS's
- LQG bh entropy corrections have been related to directly measureable quantities in compact binary mergers with GW emission. Astrophysical significance of these QG corrections, vis-a-vis observations, is likely to improve with time
- Greater rigour in establishing the Bekenstein bound, based on work of Casini (2008) is in progress