Effective Quantum Black Hole Collapse via Surface Matching

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based on

Class. Quant. Grav. 38 175015 or arXiv:2010.13480 [gr-qc]

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Motivation

- Classical gravitational collapse leads to singularity
  [Penrose '65; Hawking '66]
- Quantum Gravity (QG) might resolve this issue
- "Effective Approximation": cl. spacetime + QG modified eq.

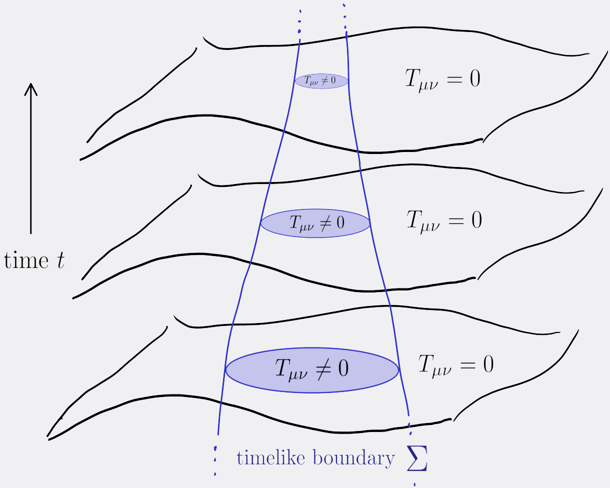
- Many regular BH models available (no formation)
  - LQG: [Gambini, Olmedo, Pullin '20; '21; Bodendorfer, Mele, JM '19; '21a; '21b; Ashtekar, Olmedo, Singh '18a; '18b; Kelly, Santacruz, Wilson-Ewing '20;...]
  - String Theory: [Nicolini, Spallucci, Wondrak '19; Easson, Keeler, Manton '20], Asymptotically safe gravity: [Adéìféoba, Eichhorn, Platania '18; Platania '19; Moti, Shojai '20], ...
  - Other [Bardeen '68; Hayward '06; Dymnikova '92; '96; Frolov '16; '18], ...
- No clear global picture for collapse (review: [Malafarina '17])
  [Kelly, Santacruz, Wilson-Ewing '20; Kiefer, Schmitz '19; Schmitz '20, Modesto '08,...]

Collapse problem complicated: Field Theory
Effective quantisation hard in LQG

Spherically Symmetric Collapse

Assume:

QG modified $(G_{\mu\nu}) = 8\pi G T_{\mu\nu}$



- Solve for $T_{\mu\nu} \neq 0$ (matter) and $T_{\mu\nu} = 0$ (vacuum) separately
- Consistency via boundary conditions at Σ

Assumptions

(1) Birkhoff Theorem

Stationarity for vacuum $T_{\mu\nu} = 0$

$$\mathrm{d}s^2 = -a(r)\mathrm{d}t^2 + N(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2 \quad \text{for} \quad r > R(t)$$

$$\Sigma = \{(t, r = R(t)) \mid t \in \mathbb{R}\} \times \mathbb{S}^2$$

(2) Homogeneous Collapse

Matter $(T_{\mu\nu})$ is homogeneous + spherically symmetric

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \frac{S(\tau)^2}{1 - k\rho^2}\mathrm{d}\rho^2 + S(\tau)^2\rho^2\mathrm{d}\Omega_2^2 \quad \text{for} \quad \rho < \rho_o(\tau)$$

$$\Sigma = \{(\tau, \rho = \rho_o(\tau)) \mid \tau \in \mathbb{R}\} \times \mathbb{S}^2$$

(3) Junction conditions

Spacetime is $C^1(M)$ across Σ

$$q^i\Big|_\Sigma = q^e|_\Sigma \quad , \quad K^i\Big|_\Sigma = K^e|_\Sigma$$

Interpretation

(1) Birkhoff Theorem

- Vacuum region remains stationary: No grav. waves, evaporation,...
- Classically: result / not clear if true in QG regime $R(t) \sim \ell_p$
- Use eternal BH model

(2) Homogeneous Collapse

- Simplest possible scenario
- FLRW-metric / cosmology

(3) Junction conditions

- $K^i\big|_\Sigma = K^e\big|_\Sigma$ enforces stress-energy of Σ vanishes
  Israel-Darmois-junction conditions [Israel '66; Darmois '27]
- Homogeneity: Pressure $P = 0$

$\Longrightarrow$ Pressureless dust collapse / Oppenheimer-Synder model [Oppenheimer, Snyder '39; Datt '38]

Vacuum region determines full spacetime

$$R(t(\tau)) = \rho_o(\tau)S(\tau) \ , \tag{1a}$$

$$\mathrm{d}t^2 = \frac{1 - \frac{S^2\dot{\rho_o}^2}{1-k\rho_o^2} + N\left(\rho_o S\right)^{\cdot\,2}}{a}\mathrm{d}\tau^2 \tag{1b}$$

$$\rho_o^2\dot{S}^2 = 1 - k\rho_o^2 - \frac{1}{N} \ , \tag{1c}$$

$$\frac{\dot{\rho}_o}{\rho_o} = \frac{(1-k\rho_o^2)\frac{\rho_o\dot{S}}{2}\left(\frac{a'}{a} + \frac{N'}{N}\right)}{1 - \frac{1}{N} + \frac{a'}{2Na}R - (1-k\rho_o^2)\,\frac{R}{2}\left(\frac{N'}{N} + \frac{a'}{a}\right)} \ , \tag{1d}$$

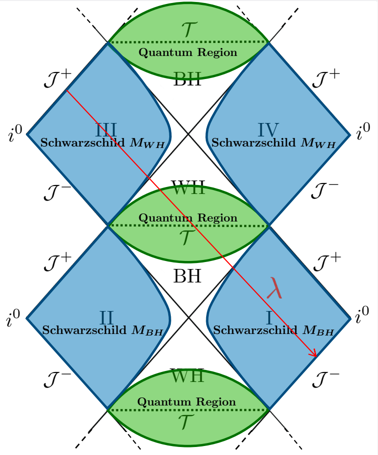
Assume: $a(r)$, $N(r)$ (vacuum region metric) known
Unknown:

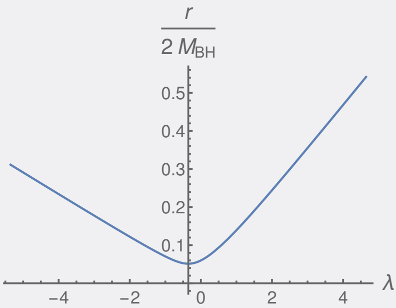
- $t(\tau)$,
- Boundary dynamics: $R(t)$, $\rho_o(\tau)$
- Matter region metric: $S(\tau)$

$\longrightarrow$ 4 equations for 4 unknowns

Application: LQG-inspired bouncing models

Many models available: [Gambini, Olmedo, Pullin '20;'21; Bodendorfer, Mele, JM '19; '21a; '21b; Ashtekar, Olmedo, Singh '18a; '18b; Kelly, Santacruz, Wilson-Ewing '20;...]



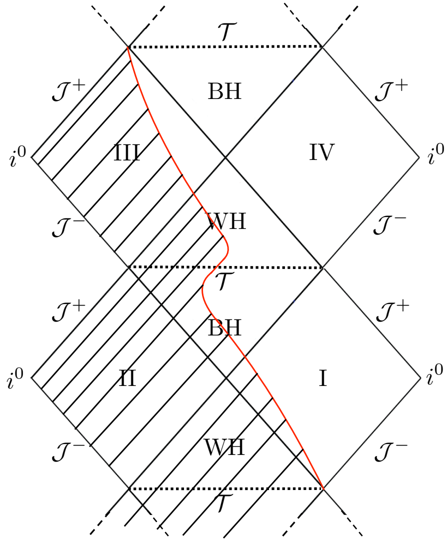


- Two integration constants M_{BH}, M_{WH} (+ two quant. param.)
- Singularity replaced by a bounce

Application: [Bodendorfer, Mele, JM CQG '19 or arXiv:1902.04542]

- Here: Marginally free collapse $(k = 0)$
- Plug in known $a(r)$, $N(r)$
- solve for $t(\tau)$, $R(t)$, $S(\tau)$, $\rho_o(\tau)$
- Exterior point of view: plot $R(t)$ in Penrose-diagram

- Vacuum ST for $r > R(t)$
- Replace with matter ST for $r < R(t)$
- Matter reaches speed of light at bounce
- Vacuum causal structure is still infinite
- Constraint on M_{BH} and M_{WH}



Conclusions

Summary

- General strategy: eternal BH $\to$ dust collapse
- global picture of the collapse
- no concrete matter model required

Results

- infinite tower of Penrose diagram not regularised
- matter becomes light-like at the bounce

Conditions too strong?

- Birkhoff theorem not valid?
- ...

Something essential missing?

- Quantum spacetime effects
- BH evaporation

Future Directions

- Other regular BH models
- From matter region (LQC) deduce vacuum [Ben Achour, Brahma, Uzan '20; Ben Achour, Uzan '20; Ben Achour, Brahma, Mukohyama, Uzan '20]
- Include BH evaporation, structure of matter region, qu. effects,...

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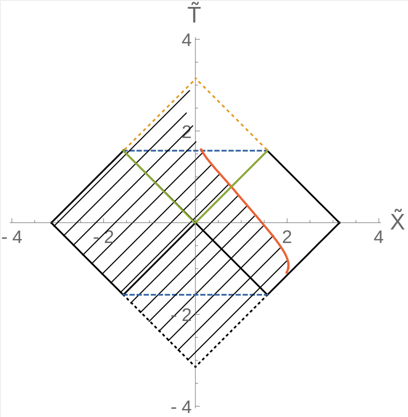
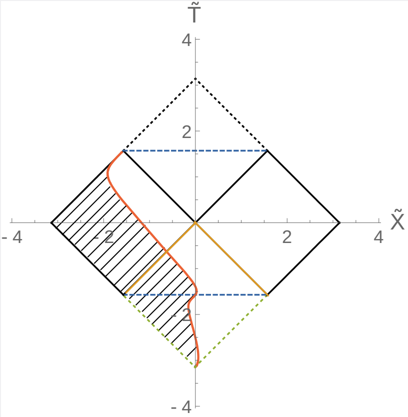
Thank you for your attention!

Exact Solutions Vacuum Point of View

Exact computed Penrose diagram for vacuum observer point of view for
[Bodendorfer, Mele, JM '19]





Exact Solutions Matter Point of View

Solution for $S(\tau)$ and $\rho_o(\tau)$ for [Bodendorfer, Mele, JM '19]