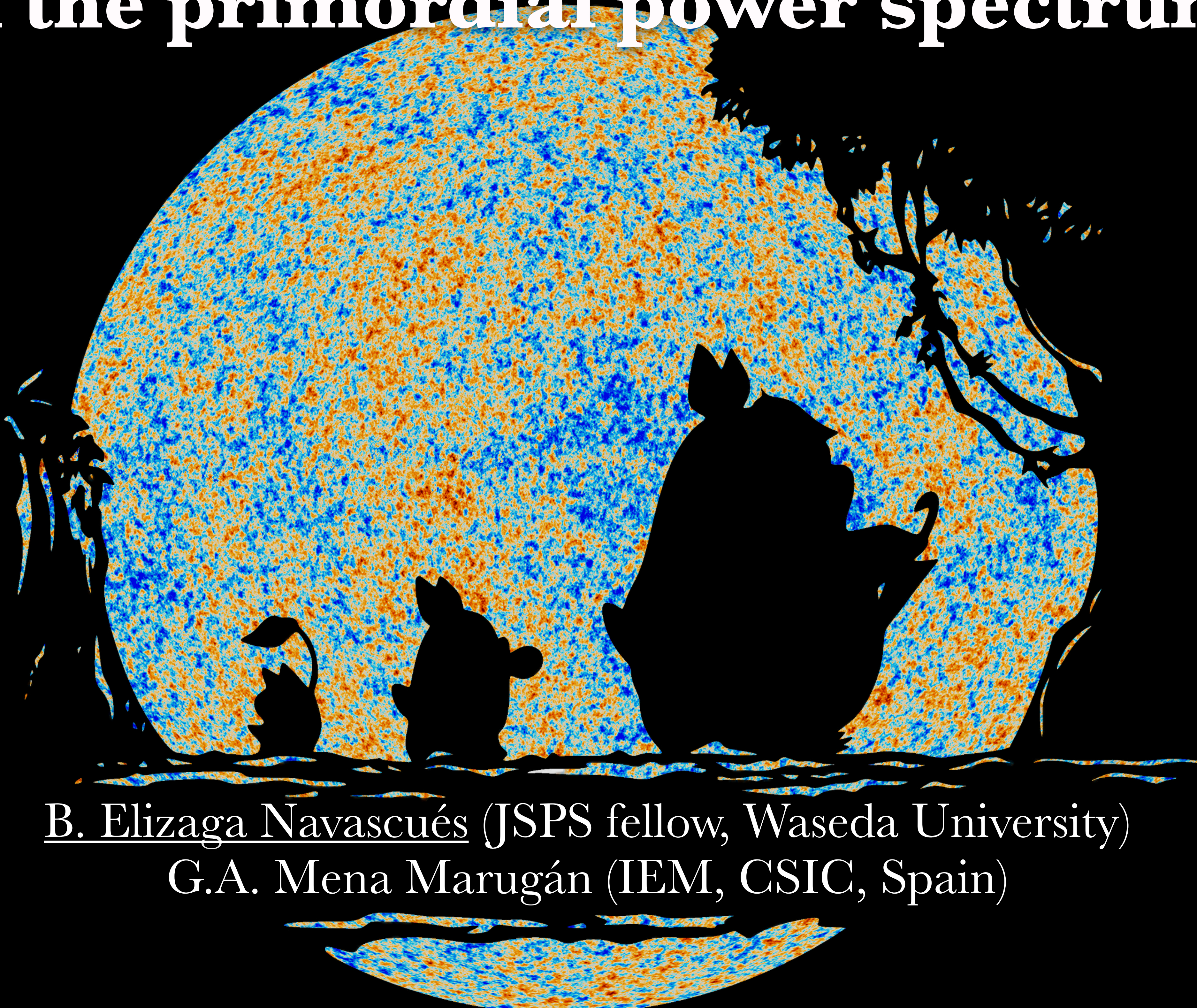
Relativistic vs. loop quantum effects in the primordial power spectrum



B. Elizaga Navascués (JSPS fellow, Waseda University)
G.A. Mena Marugán (IEM, CSIC, Spain)

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Motivation

- Standard cosmological model with inflation is extremely successful describing primordial inhomogeneities.
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- Theoretically:
    - ★ Big-Bang singularity: loss of predictability.
    - ★ Quantum gravity phenomena?
    - ★ Non-inflationary epoch: State for the perturbations?

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- Observationally:

    - ⋆ Angular power spectrum in CMB: Anomalies.

    - ⋆ Power suppression $\ell \lesssim 30$, lensing amplitude > 1, …

    - ⋆ Strongly affected by cosmic variance, but could point to new physics $\longrightarrow$ Planck regime of the Universe?

Motivation

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- Is it completely satisfactory, considering the primeval stages of the Universe with increasingly high curvature?
- Theoretical and observational concerns.
- Promising candidate: Loop Quantum Cosmology (LQC).
- Typically includes a classical pre-inflationary epoch.
- Robust predictions **require** disentangling LQC from GR effects on the evolution of the perturbations.

Loop Quantum Cosmology: Mukhanov-Sasaki equations

Why LQC?

- Canonical quantization program for spacetimes with high degree of symmetry: e.g. cosmological spacetimes.
- Techniques from the non-perturbative theory of LQG.
- Widely studied in e.g. FLRW-type cosmologies.
- Provides robust quantum mechanisms to resolve the cosmological singularity ⟶ Big Bounce.
- Effective bouncing regimes with modified Friedmann eqs.
- Can be combined with standard quantum field theory techniques to include inhomogeneities.

Perturbations in LQC

- Hybrid quantization of perturbed cosmology with inflaton:
  - ⋆ Background cosmology: LQC techniques.
  - ⋆ Gauge-invariant perturbations: Fock representation.

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- Mean-field approximation on quantum constraint equation

  ⟶ Effective constraint for the perturbations, depends on background geometry via expectation values.

- Effective Mukhanov-Sasaki equations:

$$v''_{\vec{k}} + [k^2 + s_{\text{eff}}]v_{\vec{k}} = 0, \qquad s_{\text{eff}} = s_{\text{eff}}(\eta)$$

Mass codifies LQC effects on the background.

Mukhanov-Sasaki equations

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- Restrict to background states with effective LQC behavior.

- Phenomenologically interesting solutions: Large observable scales a/k today were ~ order of curvature at the bounce.

- They are all such that the kinetic energy of the inflaton greatly dominate over its potential.

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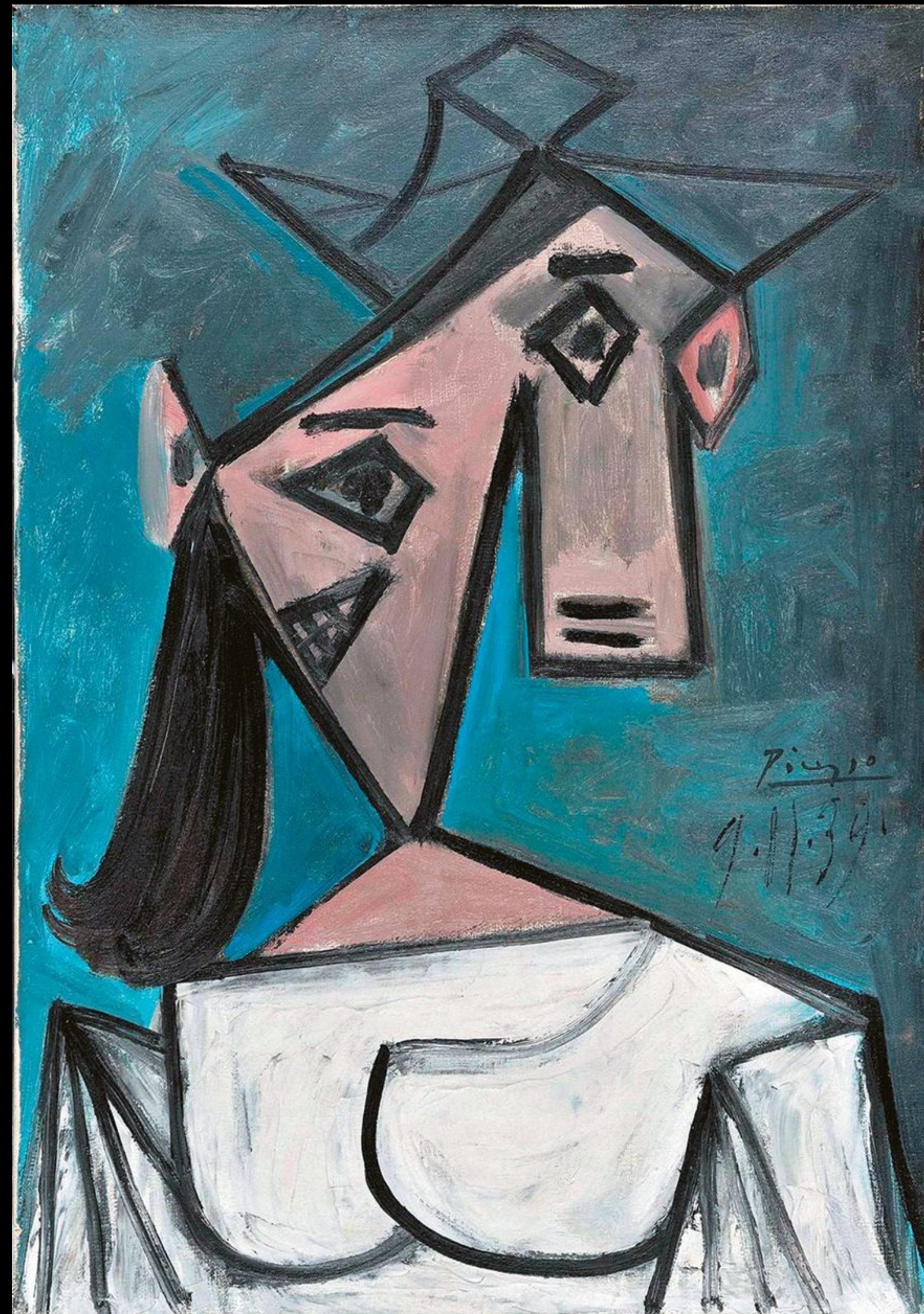
- Restrict to background states with effective LQC behavior.
- Phenomenologically interesting solutions: Large observable scales a/k today were ~ order of curvature at the bounce.
- They are all kinetically dominated at the bounce.
- Quantum effects tightly narrowed around the bounce.
- They imply a short-lived inflation ($\gtrsim$ 65 e-folds), and a classical deccelerated preinflationary expansion.



GR with KD and LQC: Approximations

Inflation with KD epoch in GR

- Deep in the pre-inflationary epoch, the potential is completely negligible compared with kinetic energy.

- Approximate this classical epoch (η_0, η_i) as a Friedmann universe with a massless scalar field.

Inflation with KD epoch in GR

- The evolution of the Universe during slow-roll inflation is of quasi-de Sitter type.

- For our purposes here, we ignore transition effects and deviations from an exact de Sitter phase.

Inflation with KD epoch in GR

- Approximate pre-inflationary epoch (η_0, η_i) as a Friedmann universe with a massless scalar field.
- Approximate the inflationary period $[\eta_i, \eta_{\text{end}}]$ as de Sitter.
- Instantaneous transition between both periods.
- Approximate Mukhanov-Sasaki equations:

$$v''_{\vec{k}} + \left(k^2 + \tilde{s}_{\text{GR}}\right) v_{\vec{k}} = 0,$$

$$\tilde{s}_{\text{GR}} = \begin{cases} \frac{1}{4}\left(\eta - \eta_0 + \frac{1}{2H_0 a_0}\right)^{-2}, & \eta \in (\eta_0, \eta_i) \\ -2H_\Lambda^2 \left[a_i^{-1} - H_\Lambda(\eta - \eta_i)\right]^{-2} & \eta \in [\eta_i, \eta_{\text{end}}] \end{cases}$$

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Discontinuity!

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Inflation with KD epoch in LQC

- Approximate pre-inflationary epoch (η_0, η_i) as a Friedmann universe with a massless scalar field.

- Approximate the inflationary period $[\eta_i, \eta_{\text{end}}]$ as de Sitter.

- For the interval $[\eta_B, \eta_0]$ with strong loop quantum effects, we approximate the mass by a Pöschl–Teller potential.

- The potential is fixed to match the exact values of the (KD) LQC and GR masses at, respectively, the bounce and η_0.

- The goodness of the approximation depends on the choice of η_0. Relative error can be made to grow at most to 15%, and quickly become negligible afterwards.

Vacuum state and power spectra

Power spectrum in de Sitter

- In de Sitter, solutions to Mukhanov-Sasaki equations:

$$\mu_k = A_k \frac{e^{ik(\eta - \eta_i - a_i^{-1} H_\Lambda^{-1})}}{\sqrt{2k}} \left[1 + \frac{i}{k(\eta - \eta_i - a_i^{-1} H_\Lambda^{-1})} \right]$$

$$+ B_k \frac{e^{-ik(\eta - \eta_i - a_i^{-1} H_\Lambda^{-1})}}{\sqrt{2k}} \left[1 - \frac{i}{k(\eta - \eta_i - a_i^{-1} H_\Lambda^{-1})} \right]$$

- Primordial power spectrum is well-approximated by:

$$\mathscr{P}(k) = \frac{H_\Lambda^2}{4\pi^2} |B_k - A_k|^2, \qquad |B_k|^2 - |A_k|^2 = 1$$

- Dephasing between constants typically leads to oscillations.

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→ If no interference in previous epoch(s), origin can be traced to instantaneous changes of the mass function.

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- <u>Dephasing</u> between constants: Oscillations.

- For well-behaved initial state, <u>we remove it</u> in the end.

Choice of vacuum state

- Vacuum state: Initial conditions for the perturbations.
- In de Sitter, natural choice is Bunch-Davies: $A_k = 0$, $B_k = 1$.
- What if there are observable scales k that are sensitive to the spacetime curvature in the pre-inflationary epoch?

Choice of vacuum becomes an open question

Choice of vacuum state

- Vacuum state: Initial conditions for the perturbations.
- In de Sitter, natural choice is Bunch-Davies: $A_k = 0$, $B_k = 1$.
- What if there are observable scales k that are sensitive to the spacetime curvature in the pre-inflationary epoch?
- For a robust comparative study: Criterion of choice should be applicable to different types of cosmological dynamics.
- Ideally, it should also be motivated by fundamental considerations, and lead to positive-frequency solutions that do not present rapid oscillations in time and/or k.

Choice of vacuum state

- Vacuum state: Initial conditions for the perturbations.
- Here, criterion is fixed based on previous investigations:
    - ⋆ Originates from an ultraviolet diagonalization of the Hamiltonian in quantum cosmology.
    - ⋆ In the ultraviolet regime, it is the unique one that does not display rapid time oscillations of frequency k.
    - ⋆ Applied to Minkowski and de Sitter spacetimes, leads to Poincaré and Bunch-Davies vacua.

Choice of vacuum state

- Vacuum state: Initial conditions for the perturbations.

- Here, criterion is fixed based on previous investigations:

$$\mu_k = \sqrt{-\frac{1}{2\mathrm{Im}(h_k)}} e^{i\int_{\eta_0}^{\eta} \mathrm{Im}(h_k)}, \qquad k h_k^{-1} \sim i\left[1 - \frac{1}{2k^2}\sum_{n=0}^{\infty}\left(\frac{-i}{2k}\right)^n \gamma_n\right],$$

$$\gamma_0 = s, \quad \gamma_{n+1} = -\gamma_n' + 4s\left[\gamma_{n-1} + \sum_{m=0}^{n-3}\gamma_m\gamma_{n-(m+3)}\right] - \sum_{m=0}^{n-1}\gamma_m\gamma_{n-(m+1)}$$

- The smooth mass s can be evaluated on GR or effLQC.

- Formula not applicable with our approximations (due to discontinuities), but can be used to fix initial conditions in the earliest smooth epoch (KD or bouncing regime).

Power spectrum in GR

- In the case of GR with KD, our criterion fixes the following positive-frequency solutions in the epoch (η_0, η_i):

$$\mu_k = \sqrt{\frac{\pi y}{4}} H_0^{(2)}(ky), \qquad y = \eta - \eta_0 + \frac{1}{2H_0 a_0}$$

- By continuity, fixes positive-frequency solutions in de Sitter.

- Resulting power spectrum displays artificial oscillations around $k_I = a_i H_\Lambda$, which we remove with the transformation:

$$A_k \to A_k^{\text{kin}} = |A_k|, \qquad B_k \to B_k^{\text{kin}} = |B_k|$$

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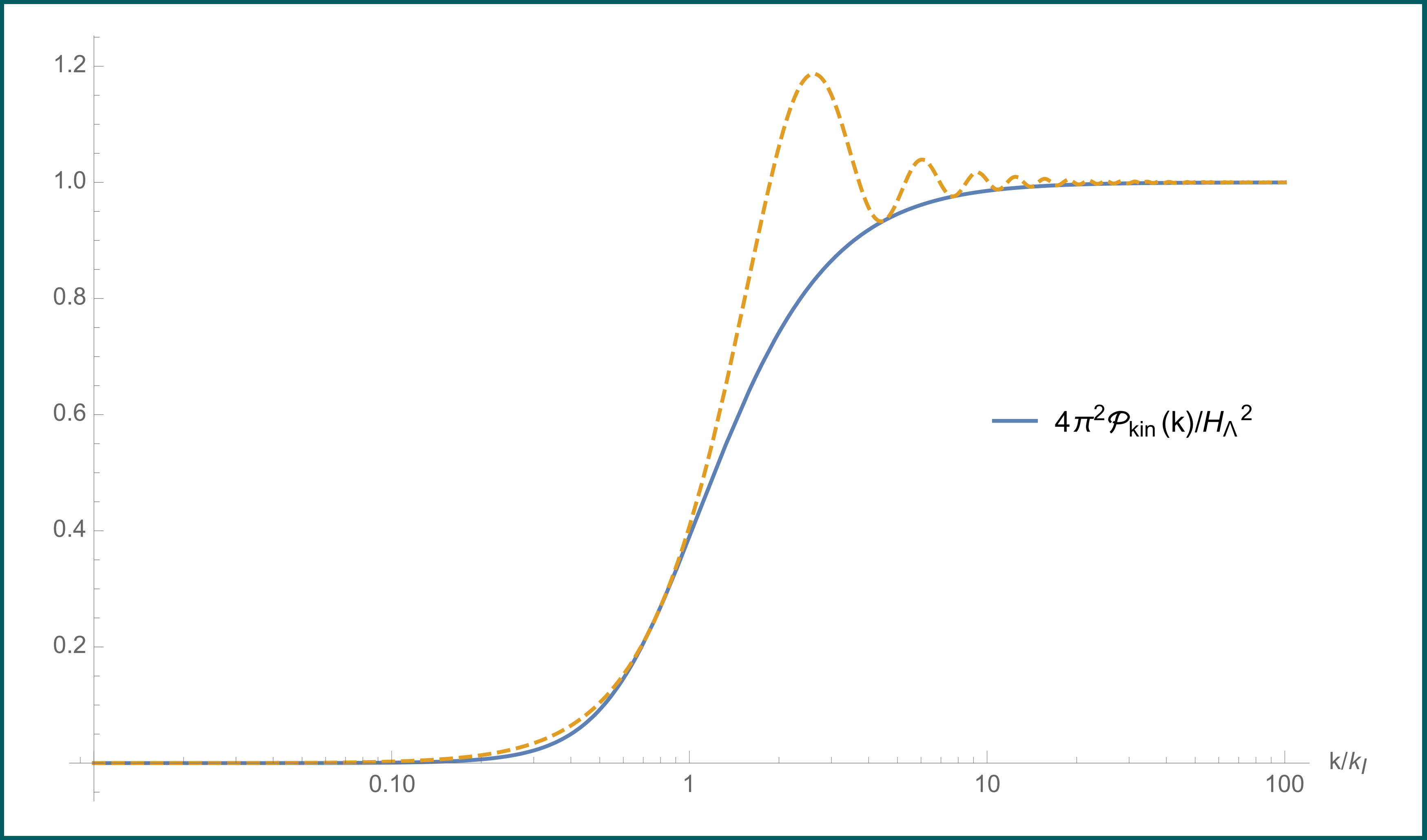

Power spectrum in (hybrid) LQC

- In the case of hybrid LQC, our criterion fixes positive-frequency solutions in the epoch $[\eta_B, \eta_0]$ by means of:

$$h_k = -i\alpha\tilde{k} - 2\alpha x(1-x)\frac{cd}{1+i\tilde{k}}\frac{{}_2F_1\left(c+1, d+1; 2+i\tilde{k}; x\right)}{{}_2F_1\left(c, d; 1+i\tilde{k}; x\right)}, \quad \tilde{k} = k/\alpha$$

- By continuity, fixes positive-frequency solutions in the KD classical epoch and these, in turn, in the de Sitter regime.

- Resulting power spectrum displays artificial oscillations for $k \lesssim k_{\rm LQC} = \alpha\,(\sim 3)$, which we remove in analogous way:

$$A_k \to A_k^{\rm LQC} = |A_k|, \qquad B_k \to B_k^{\rm LQC} = |B_k|$$

Power spectrum in (hybrid) LQC

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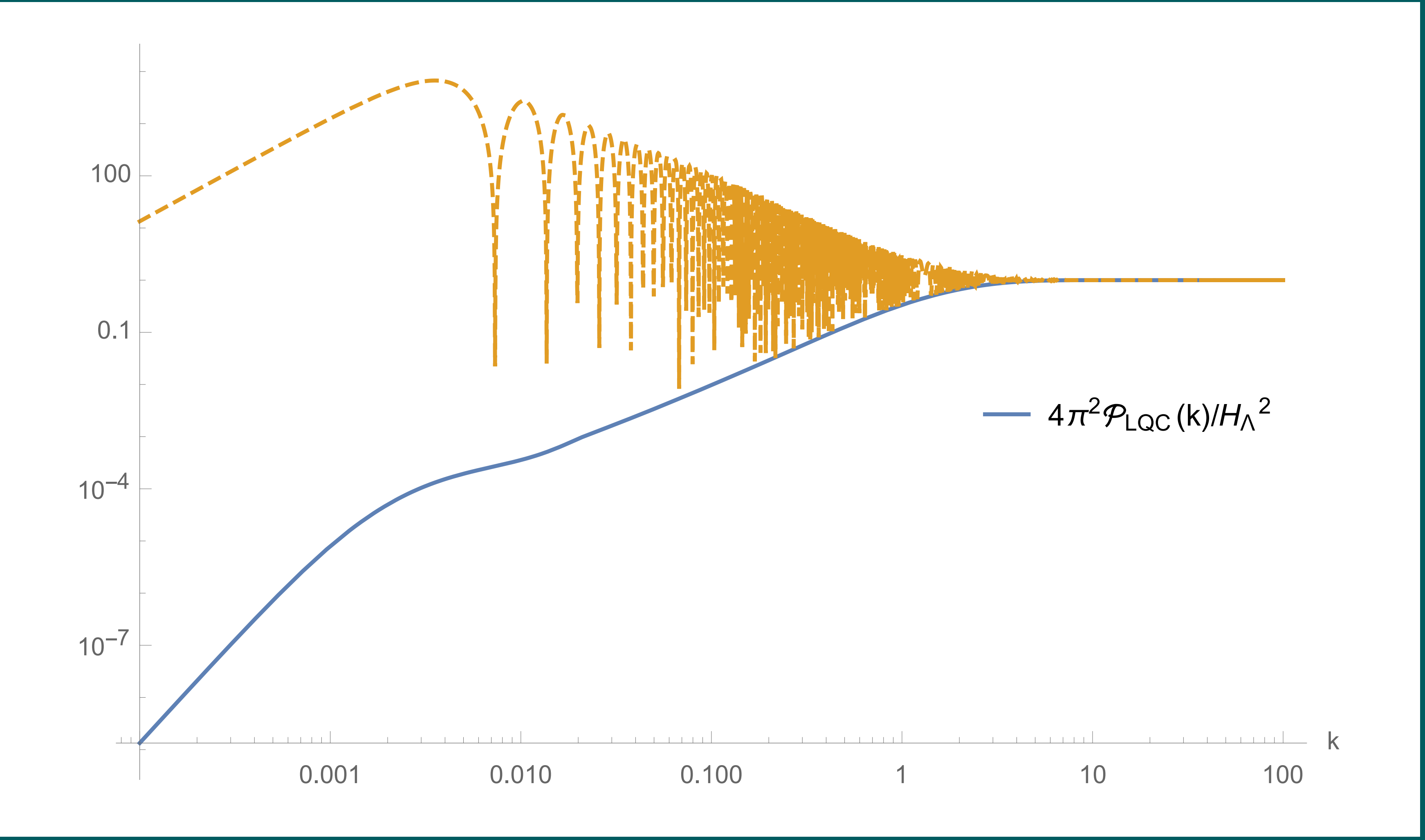
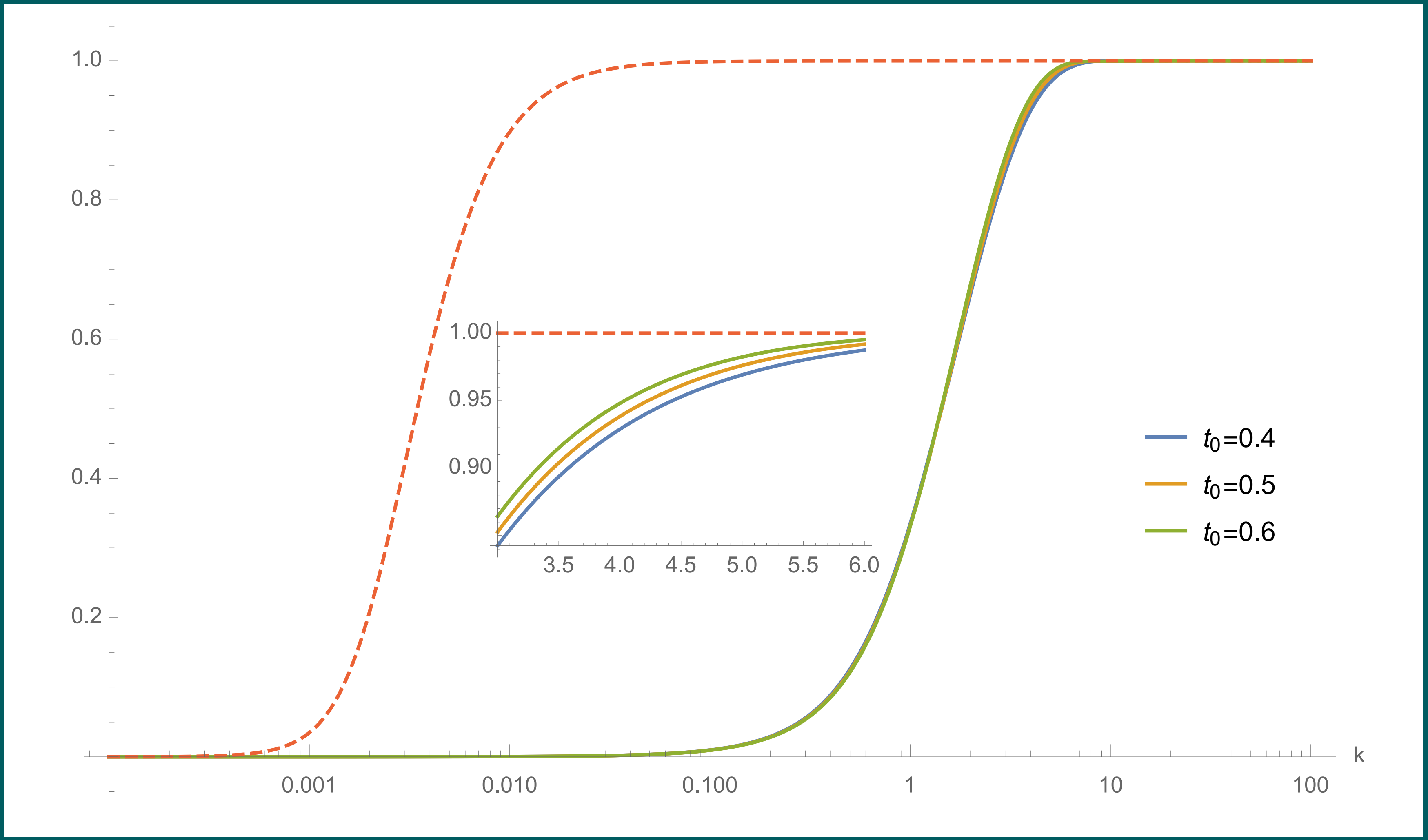


Power spectrum in (hybrid) LQC

- Resulting power spectrum displays artificial oscillations for $k \lesssim k_{\text{LQC}} = \alpha\,(\sim 3)$, which we remove.
- We compare it with the one in the GR with KD model for which inflation starts at the same scale as in LQC: $k_I \sim 10^{-3}$.



----- GR

— LQC

Conclusions

- Approximative methods to understand **analytically** the main differences between (classical) KD preinflationary and LQC effects leading to suppression in power spectra.
- Differences traceable to existence of two distinct scales:
    - ⋆ Curvature at onset of inflation (both models).
    - ⋆ Curvature around the bounce (only in LQC).
- They always differ in 3 orders of magnitude for interesting LQC solutions (phenomenologically speaking).
- Study can be used to compare other preinflationary models.
- Approximations yet rough: Call for further developing the studies about the dynamical behavior of the chosen vacua.