Entanglement Entropy at Critical Points in the Multiverse

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Based on:

A. Balcerzak, S.B.B., M. P. Dąbrowski,
S. Robles-Pérez. Phys. Rev. D 103, 043507.

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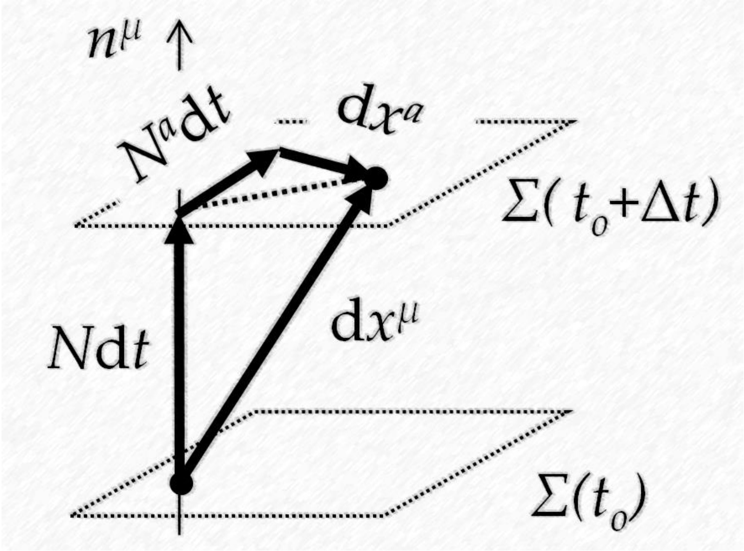
1. Canonical Quantum Gravity

- ADM Formalism – Foliation:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (N_a N^a - N^2)dt^2 + 2N_a dx^a dt + h_{ab}dx^a dx^b$$



$$g_{\mu\nu} \longrightarrow \{N^a, N, h_{ab}\}$$

$$S = \int d^4x \left(p_{ab}\dot{h}_{ab} - NH - N^a H_a\right)$$

N *is a Lagrange Multiplier*

$\Longrightarrow$ $H = 0$

1. Canonical Quantum Gravity

- Idea of Quantization à la Dirac:

$$h_{ab} \longrightarrow \hat{h}_{ab} \qquad p^{ab} \longrightarrow \hat{p}^{ab} = -i\,\frac{\delta}{\delta h_{ab}}$$

$$\Longrightarrow \quad \boxed{\hat{H}\psi = 0}$$

· Wheeler-DeWitt Equation ·

$\psi =$ Wave Function of the Universe

· Superspace ·

Infinite Degrees of Freedom !!

1. Canonical Quantum Gravity

- Solution: Minisuperspace
  - FLRW: $h_{ab} \to a$
  - Scalar Field: ϕ

2 d.o.f.

$\Longrightarrow$ <u>Solvable</u>

- Wheeler-DeWitt Equation $(\alpha = \ln a)$

$$\left[\frac{\partial^2}{\partial \alpha^2} - \frac{\partial^2}{\partial \phi^2} + \omega^2(\alpha, K, \Lambda)\right]\psi = 0$$

K is the curvature index

Λ is the cosmological constant

1. Canonical Quantum Gravity

(and non self-interacting field)

- Our Case: Massless Scalar Fields

$$\omega^2 \neq \omega^2(\phi)$$

$$\left[\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\phi^2} + \omega^2(\alpha, K, \Lambda)\right]\psi = 0$$

→ Separable

$$\left[\frac{\partial^2}{\partial\alpha^2} + \omega^2(\alpha, K, \Lambda)\right]\psi = -E_\phi\psi$$

$$-\frac{\partial^2\psi}{\partial\phi^2} = E_\phi\psi$$

$$\left[\frac{\partial^2}{\partial\alpha^2} + \omega^2(\alpha, K, \Lambda, E_\phi)\right]\psi = 0$$

Time-Dependent Harmonic Oscillator-like Equation

2. Third Quantization

- 80's
  - 1984 Caderni & Martellini (!)
  - 1988 McGuigan
  - 1989 Giddings & Strominger
  - 1989 Hosoya & Morikawa

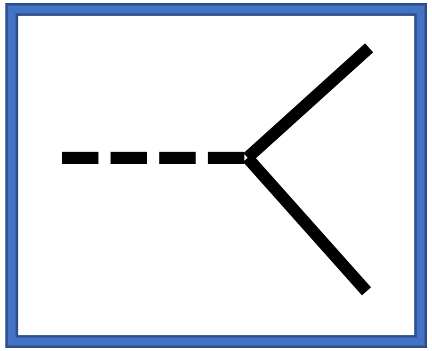
… and others

- Quantum Field Theory of Universes

$$\psi \rightarrow \hat{\psi}$$

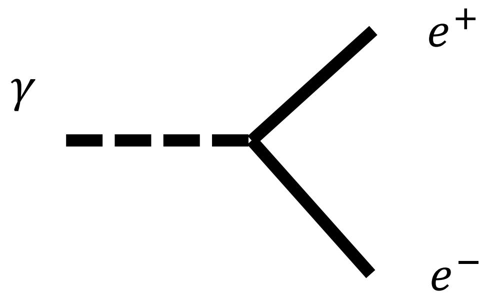
<u>The Universe is an Excitation of a Field of Universes</u>

Our Interest:



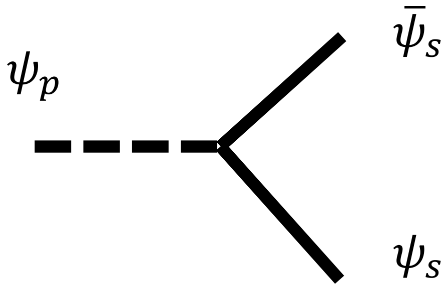
2. Third Quantization

- Creation of Two Universes $\equiv$ Pair Creation

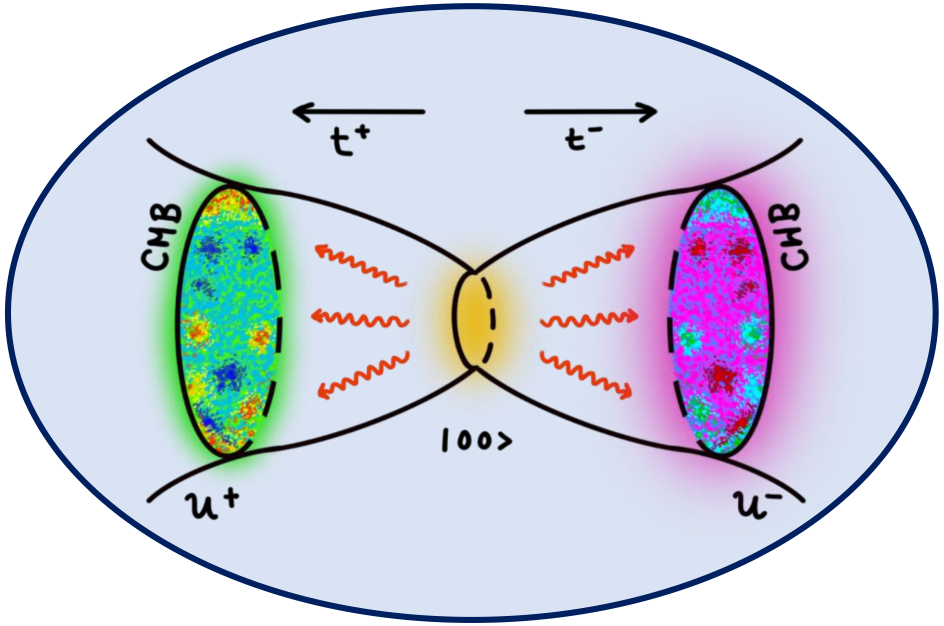


QED

$\equiv$



Third Quantization



$$\left[\frac{\partial^2}{\partial \alpha^2} + \omega^2(\alpha, K, \Lambda, E_\phi)\right]\psi = 0$$

It Always Admits Two Conjugated Solutions !!

ψ_s and $\bar{\psi}_s$

2. Third Quantization

- Entanglement
  - $\rho = |00\rangle\langle 00|$ — $|00\rangle$ is the initial state
  - Von Neumann Entropy: $S_{ent} = -\mathrm{Tr}[\rho_r \ln \rho_r]$

- The Problem of the Particle Creation

The Vacuum State is not Well-Defined for Different Observers

- Solution: The Invariant Vacuum

3. The Invariant Vacuum

- The Invariant Representation
  for Time-Dependent Harmonic Oscillator !!

1969 Lewis & Riesenfeld
1996 Kim, Lee, Ji & Kim

⟹ We can Do It for Our Wheeler-DeWitt Equation

$$\left[\frac{\partial^2}{\partial\alpha^2} + \omega^2(\alpha, K, \Lambda, E_\phi)\right]\psi = 0$$

3. The Invariant Vacuum

- The Invariant Vacuum

$$|00\rangle_i = \frac{1}{|\alpha_B|}\sum_{n=0}^{\infty}\left(\frac{|\beta_B|}{|\alpha_B|}\right)^n |n_- n_+\rangle_d$$

"n labels the mode of excitation of the universe"

Note!
The Entanglement Entropy is into the interval:
$[0, \dim(\mathcal{H})] = [0, \infty)$ $\Longrightarrow$ The Entanglement Entropy can Be Infinite!

4. Entanglement Entropy

- We Found:

About the Image of the function $S_{ent}(a, \phi)$

- $S_{ent}(a, \phi) \in \mathbb{R}^+(\cup \{\infty\})$ $\Leftrightarrow$ Classically Allowed Region
- $S_{ent}(a, \phi) \in \mathbb{C}_\infty \setminus \mathbb{R}$ $\Leftrightarrow$ Classically Forbidden Region

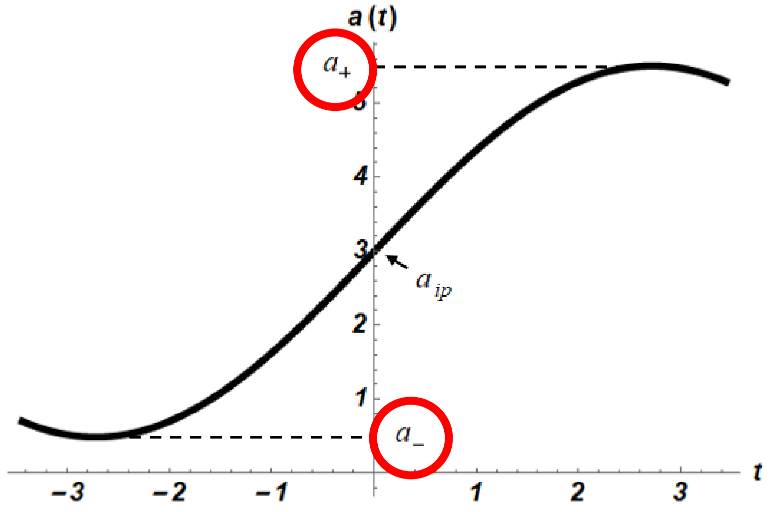
About $S_{ent}(a, \phi)$ at the Initial Singularity

- $S_{ent}(0, \phi)$ is finite $\Leftrightarrow$ The Scalar Field is Quantum
- $S_{ent}(0, \phi)$ is infinite $\Leftrightarrow$ The Scalar Field is Classical

(Perfect Fluids: $\phi \longrightarrow \omega_\phi$)

4. Entanglement Entropy

- We Found:



$S_{ent}(a, \phi)$ at the Critical Points (a_+, a_-)
of the Classical Evolution

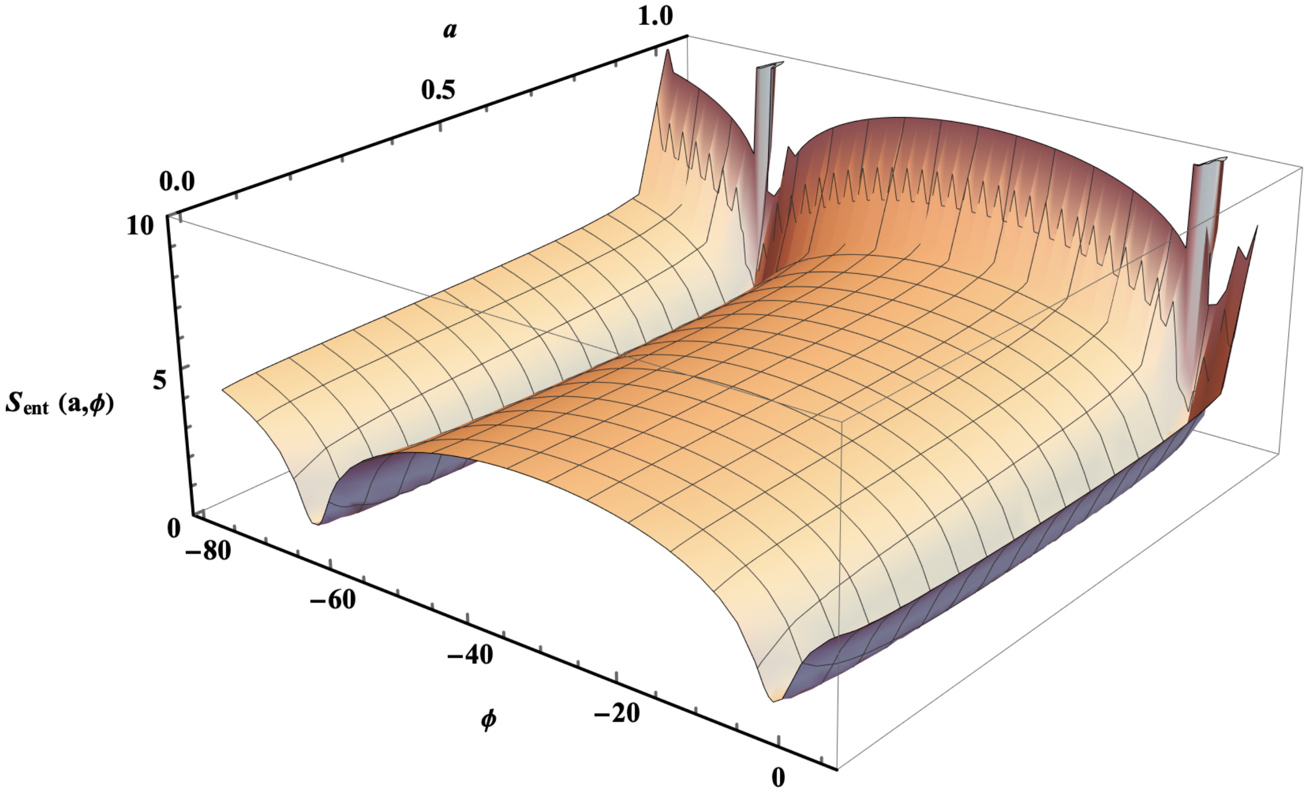
$$\left(0 = H := \frac{\dot{a}}{a}\right)$$

$$S_{ent}(a_{cp}, \phi) \longrightarrow \infty$$

4. Entanglement Entropy

- Examples

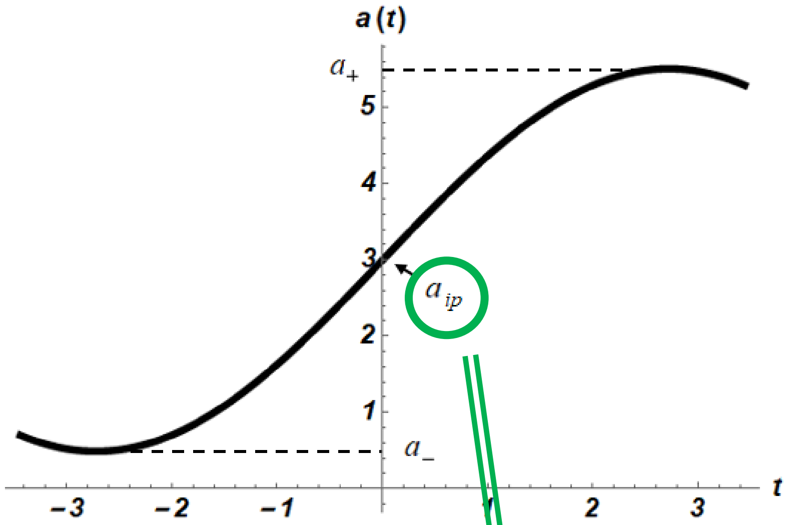
Closed Universe with a Quantum Massless Scalar Field

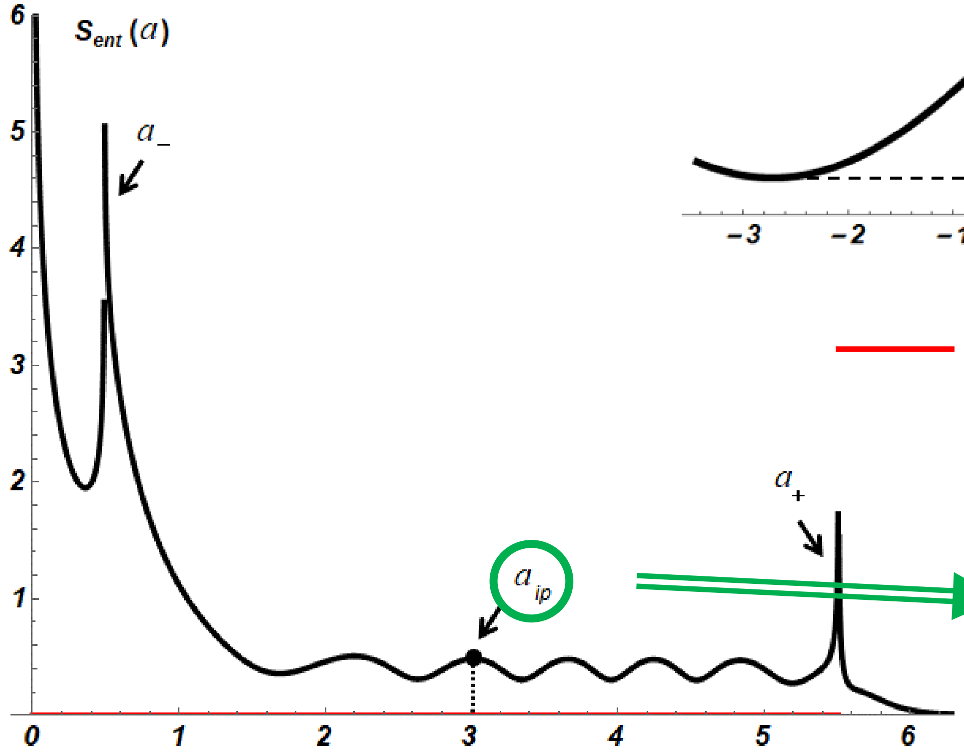


4. Entanglement Entropy

- Examples

Sinusoidal Universe
(includes Classical Scalar Fields)





Black: Real Part
Red: Imaginary Part

Nothing special at the inflection point

4. Entanglement Entropy

- Exotic Singularities

TABLE I. Classification of basic singularities in Friedmann cosmology. Here t_s is the time when a singularity appears, $w = p/\rho$ is the barotropic index, T is Tipler's definition and K is the Królak definition. In this paper we mainly concentrate on types 0, I_l, II_a, III_a and IV.

Type	Name	t	$a(t_s)$	$\varrho(t_s)$	$p(t_s)$	$\dot{p}(t_s)$	$w(t_s)$	T	K
0	big bang (BB)	0	0	∞	∞	∞	finite	strong	strong
I	big rip (BR)	t_s	∞	∞	∞	∞	finite	strong	strong
I_l	little rip (LR)	∞	∞	∞	∞	∞	finite	strong	strong
II	sudden future (SFS)	t_s	a_s	ϱ_s	∞	∞	finite	weak	weak
II_a	big brake (BBr)	t_s	a_s	0	∞	∞	finite	weak	weak
III	finite scale factor (FSF)	t_s	a_s	∞	∞	∞	finite	weak	strong
III_a	big freeze (BF)	t_s	0	∞	∞	∞	finite	weak	strong
IV	big separation (BS)	t_s	a_s	0	0	∞	∞	weak	weak
V	w-singularity (w)	t_s	a_s	0	0	0	∞	weak	weak

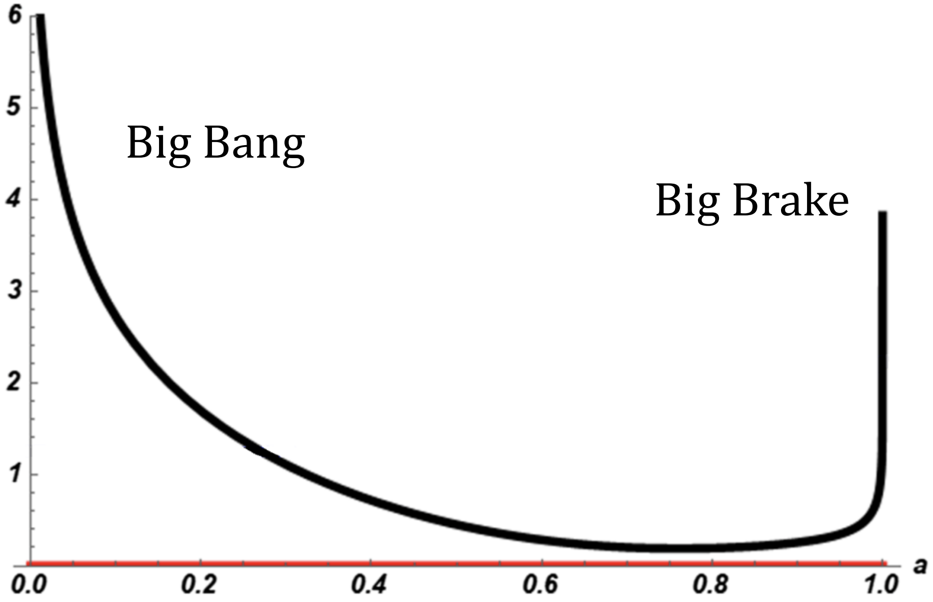
4. Entanglement Entropy

- Big Brake

$K, \Lambda = 0$

$p \sim \frac{1}{\rho^{\beta}}, \quad \beta > 0, \quad p(t_s) \to \infty$

$\rho(t_s) \to 0$

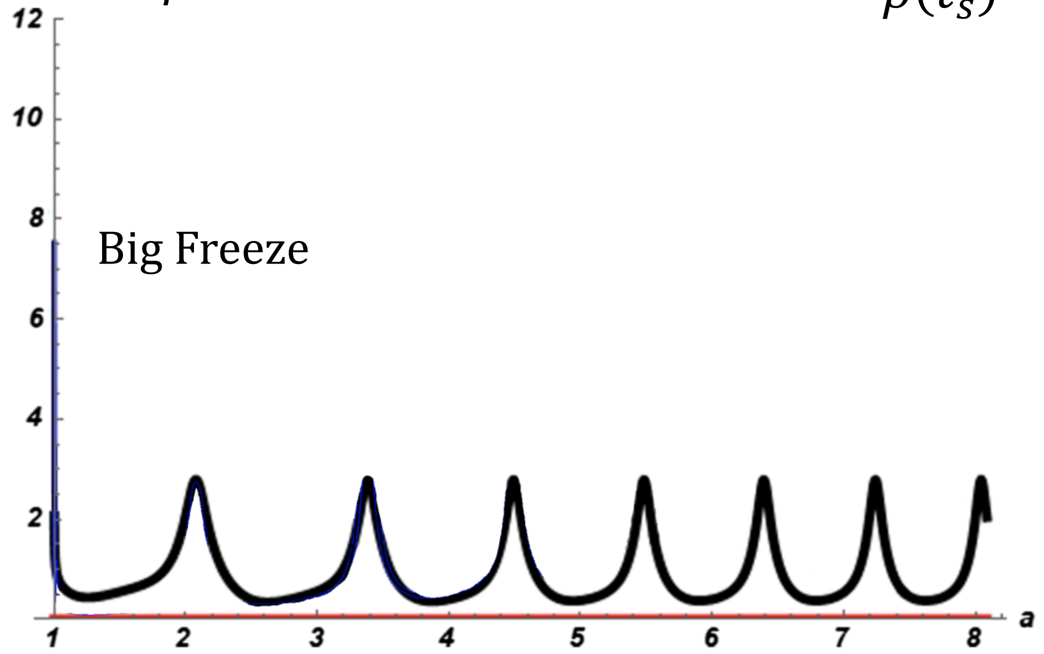


- Big Freeze

$K, \Lambda = 0$

$p \sim \frac{1}{\rho^{\beta}}, \quad \beta < -1, \quad p(t_s) \to \infty$

$\rho(t_s) \to \infty$



4. Entanglement Entropy

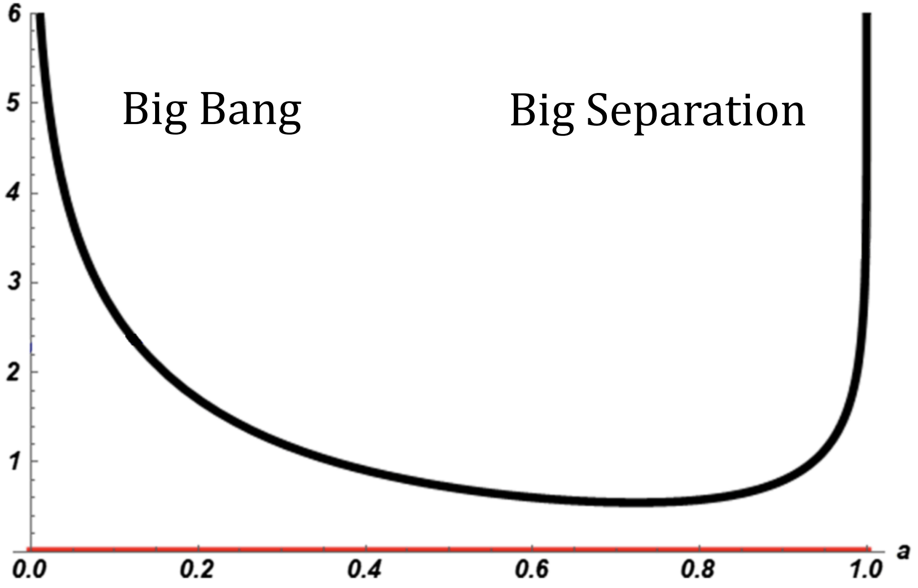
$K, \Lambda = 0$

- Big Separation

$p \sim \frac{1}{\rho^{\beta}}, \quad \beta \in (-1/2, 0),$

$p(t_s) \to 0$

$\rho(t_s) \to 0$

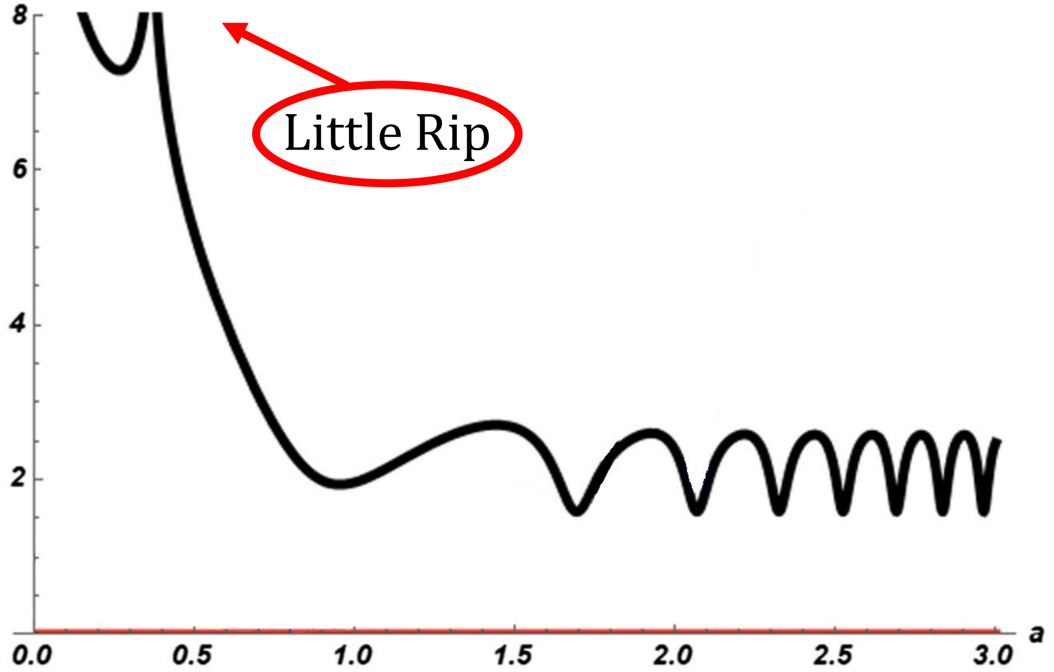


$K, \Lambda = 0$

- Little Rip

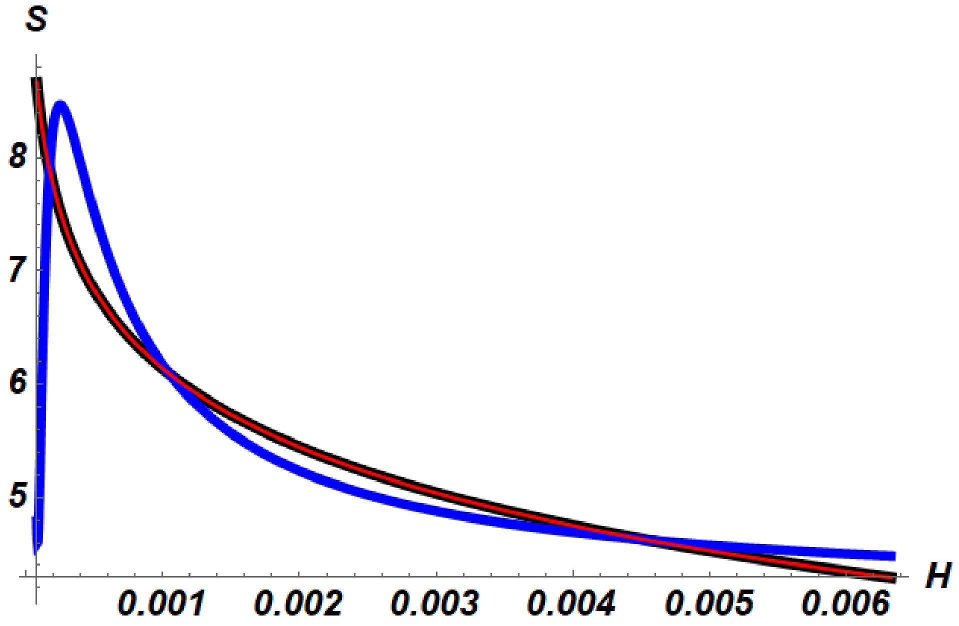
$p = -\rho - A\sqrt{\rho}, \quad A > 0, \quad \rho(t_{special}) \to 0$

Big Bang



4. Entanglement Entropy

- Relation to the Hubble Parameter Around the CP's?



Fits

Blue: Inverse Polynomials

$$S_{ent} \approx c_o + \frac{c_1}{H} + \frac{c_2}{H^2} + \frac{c_3}{H^3}$$

It does not work!

Red: Logarithmic

$$S_{ent} \approx c_o + c_1 \ln H$$

$c_1 \cong -1$!! $\Longrightarrow$ *It works!*

Shannon Information

$$S_{ent} \sim I(H)$$

5. Conclusions

- We Studied the Entanglement Entropy of an Entangled Pair of Universes
- The Entanglement Entropy is Finite/Infinite when the Scalar Field is Treated as Quantum/Classical
- It Diverges at the Critical Points ($H = \frac{\dot{a}}{a} = 0$) of the Classical Evolution $\Longrightarrow$ Strongly Entangled!
- The Asymptotical Behavior around H=0 goes like the Shannon Information

5. Conclusions

- Other Critical Points like Exotic Singularities are Also Strongly Entangled:

  Big Brake, Big Freeze, Big Separation and Little Rip

- Prospectives
  - Look for Observational Imprints of our Twin Universe
  - Improve the 3rd Quantization of CQG


Thanks

What: Quantum or Classical?

$$H = \frac{1}{2}\left[-\frac{p_a^2}{a} + \frac{p_\phi^2}{a^3} - aK + a^3\left(\frac{\Lambda}{3} + 2V(\phi)\right)\right]$$

Quantum

$$\left[\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\phi^2} - \mathrm{e}^{4\alpha}K + \mathrm{e}^{6\alpha}\left(\frac{\Lambda}{3} + 2V(\phi)\right)\right]\Psi_Q(\alpha,\phi) = 0$$

Classical

$$p_\phi = a^3\dot{\phi} \qquad \rho_\phi = \dot{\phi}^2/2 + V(\phi)$$

$$p_\phi = \omega\rho_\phi \qquad \rho_\phi(\alpha,\omega) = \rho_o \mathrm{e}^{-3\alpha(1+\omega)}$$

$$\left[\frac{\partial^2}{\partial\alpha^2} - \mathrm{e}^{4\alpha}K + \mathrm{e}^{6\alpha}\left(\frac{\Lambda}{3} + 2\rho_\phi(\alpha,\omega)\right)\right]\Psi_C(\alpha) = 0$$