Covariant formulation of Entropic Forces: Cosmic Acceleration from First Principles

with Llorenç Espinosa, arXiv:2106.16012 and 16014

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Forces in Physics

- **Fundamental Forces**

  Gravitation, Strong, Weak, E.M.

- **Residual Forces**

  Molecular, Nuclear, Surface Tension

- **Collective Forces**

  Brownian motion, $\boxed{\text{Entropic Forces}}$

Our proposal

- Entropic Forces are responsible for present cosmic acceleration and many other LSS phenomena.
- Need a covariant formalism of out-of-equilibrium phenomena in GR.
- Just Quantum Mechanics (QFT), (Non eq.) Thermodynamics and GR.

Motivation

- General Relativity is a time-reversible ($t \to -t$) theory.
- Most of the history of the universe is adiabatic (actually in local th. eq.), although there is an arrow of time.
- Irreversible phenomena are **not** included in GR in a systematic way.

Cosmology

Homogeneous and isotropic universe

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - k\, r^2} + r^2 d\Omega_2^2 \right)$$

Filled with a perfect (ideal) fluid

$$T_{\mu\nu} = p\, g_{\mu\nu} + (\rho + p)\, u_\mu u_\nu$$

Friedmann (Einstein) equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\rho \qquad \text{(F1)}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\,(\rho + 3p) \qquad \text{(F2)}$$

Cosmology

Covariant Energy-Momentum Tensor Conservation

$$D_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

Also derived from Second Law Thermodynamics (in eq.)

$$T\frac{dS}{dt} = \frac{d}{dt}\left(\rho\, a^3\right) + p\frac{d}{dt}\left(a^3\right) = 0$$

On a few occasions (e.g. Big Bang)

$$T\frac{dS}{dt} \geq 0$$

entropy production

Cosmology

Beyond adiabatic cosmology

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = \frac{T\dot{S}}{a^3}$$ (continuity eq.)

Together with Friedmann equation (F1)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

entropic force

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p - \frac{T\dot{S}}{a^2\dot{a}}\right)$$

Entropic forces in mechanics

Variational formulation of non-equilibrium thermodynamics
(Gay-Balmaz & Yoshimura, 2017)

Laws Thermodynamics + Stationary action principle

Constraints on dynamics

Defines physical
Variational problem

Entropic forces in mechanics

General mechanical system with canonical coord. q

$$\mathcal{S} = \int dt\, L(q,\, \dot{q},\, S) \quad \Rightarrow \quad \delta\mathcal{S} = \int dt \left(\left(\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} \right) \delta q + \frac{\partial L}{\partial S}\,\delta S \right)$$

+ thermodynamic constraint: $\dfrac{\partial L}{\partial S}\,\delta S = f(q,\, \dot{q})\,\delta q$

Constrained equations of motion

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = f(q,\, \dot{q}) \quad \longleftarrow \quad \text{Friction or entropic force}$$

Entropic forces in mechanics

What is the physics of $f(q,\, \dot{q})$?

Has to be obtained from the thermodynamic constraint

$$\frac{\partial L}{\partial S}\, \dot{S} = f(q,\, \dot{q})\, \dot{q}$$

Usually $\dfrac{\partial L}{\partial S} = -T \;\Rightarrow\; f(q,\, \dot{q})\, \dot{q} \leq 0$ entropy production

Dynamics is modified + Symmetry under time-reversal is broken

Entropic forces in GR

$$\mathcal{S} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + \int d^4x\mathcal{L}_m(g_{\mu\nu}, S)$$

straightforward generalization

$$\delta\mathcal{S} = \int d^4x \left(\frac{1}{2\kappa}\frac{\delta(\sqrt{-g}R)}{\delta g^{\mu\nu}} + \frac{\delta\mathcal{L}_m}{\delta g^{\mu\nu}}\right)\delta g^{\mu\nu} + \int d^4x\frac{\partial\mathcal{L}_m}{\partial S}\delta S = 0$$

Variational constraint: 2nd law thermodynamics

$$\frac{\partial\mathcal{L}_m}{\partial s}\delta s = \frac{1}{2}\sqrt{-g}f_{\mu\nu}\delta g^{\mu\nu}$$

Non-equilibrium Einstein field equations

$$\boxed{R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa\left(T_{\mu\nu} - f_{\mu\nu}\right)} \Rightarrow D^\mu T_{\mu\nu} = D^\mu f_{\mu\nu}$$

entropic force

Bianchi identities

ADM Formalism

(3+1)-splitting of space-time

$$ds^2 = -(N dt)^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

Lie derivative $\pounds_n$ along the normal vector n
as a generalization of the time derivative

$$n_\alpha = (-N, 0, 0, 0)$$
$$h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$$

$$K_{ij} = \frac{1}{2}\pounds_n h_{ij} = \frac{1}{2N}\left(\partial_0 h_{ij} - \nabla_i N_j - \nabla_j N_i\right)$$

$$\frac{\partial \mathcal{L}}{\partial s}\pounds_n s = \frac{1}{2} N \sqrt{h} \tilde{f}_{ij} \pounds_n h^{ij} \qquad \tilde{f}_{ij} = h_i^\mu h_j^\nu f_{\mu\nu}$$

Hamilton equations

$$\frac{\delta \mathcal{H}_G}{\delta \Pi^{ij}} = \dot{h}_{ij}$$
$$\frac{\delta \mathcal{H}_G}{\delta h_{ij}} = -\dot{\Pi}^{ij} - 2\kappa \frac{\delta \mathcal{L}_m}{\delta h_{ij}} - \kappa N \sqrt{h} \tilde{f}^{ij}$$

Hamiltonian and momentum constraints

$$\frac{\delta \mathcal{H}_G}{\delta N} = \mathcal{H} = 2\kappa \frac{\partial \mathcal{L}_m}{\partial N}$$
$$\frac{\delta \mathcal{H}_G}{\delta N_i} = \mathcal{H}^i = 2\kappa \frac{\partial \mathcal{L}_m}{\partial N_i}$$

Raychaudhuri Eq.

Congruence of worldlines

$$\Theta_{\mu\nu} = D_\nu n_\mu = \frac{1}{3}\Theta h_{\mu\nu} + \sigma_{\mu\nu} + \omega_{\mu\nu} - a_\mu n_\nu$$

$$\pounds_n \Theta = -\frac{1}{3}\Theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}n^\mu n^\nu + D_\mu a^\mu$$

$\sigma_{\mu\nu}\sigma^{\mu\nu} > 0$ and $\Theta^2 > 0$. vorticity vanishes $\omega_{\mu\nu} = 0$.

$$R_{\mu\nu}n^\mu n^\nu = 8\pi G\left(T_{\mu\nu}n^\mu n^\nu + \frac{1}{2}T - f_{\mu\nu}n^\mu n^\nu - \frac{1}{2}f\right)$$

If the strong energy condition is satisfied, then: $T_{\mu\nu}n^\mu n^\nu \geq -\frac{1}{2}T$ and, in the absence of intrinsic acceleration, $a_\mu = 0$, we can establish the bound:

$$\boxed{\pounds_n \Theta + \frac{1}{3}\Theta^2 \leq 8\pi G\left(f_{\mu\nu}n^\mu n^\nu + \frac{1}{2}f\right)}$$

A positive & suff. large entropic contribution can avoid recollapse

Entropic forces in GR

Temperature and Entropy from the matter content

- Mechanical system

$$L(q,\,\dot{q},\,S) = K(q,\,\dot{q}) - U(q,\,S) \quad\Rightarrow\quad T = -\frac{\partial L}{\partial S} = \frac{\partial U}{\partial S}$$

- Hydrodynamical matter

$$\mathcal{L} = -\sqrt{-g}\,\rho(g_{\mu\nu},\,s) \quad\Rightarrow\quad T = -\frac{1}{\sqrt{-g}}\frac{\partial \mathcal{L}}{\partial s} = -\frac{\partial \rho}{\partial s}$$

Entropic forces in GR

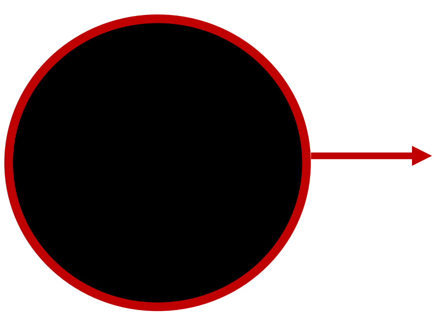
Temperature and Entropy from the gravity sector

- Horizon $\mathcal{H}$ with induced metric h

$$\mathcal{S}_{\mathrm{GHY}} = \frac{1}{8\pi G}\int_H d^3y\,\sqrt{h}\,K = \frac{1}{8\pi G}\int_H dt\,\sin\theta d\theta\, d\phi\,\sqrt{h}\,K$$

- Schwarzschild black hole

$$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2\,.$$



$$n = -\sqrt{1-\frac{2GM}{r}}\,\partial_r$$

normal vector to S_2 of radius r

Entropic forces in GR

$$\mathcal{S}_{\mathrm{GHY}} = \frac{1}{8\pi G}\int_H d^3y\,\sqrt{h}\,K = \frac{1}{8\pi G}\int_H dt\,\sin\theta d\theta\,d\phi\,\sqrt{h}\,K$$

$\sqrt{h}K = (3GM - 2r)\sin\theta$ @ event horizon $r = 2GM$

$$\mathcal{S}_{\mathrm{GHY}} = -\frac{1}{2}\int dt\,Mc^2 = -\int dt\,T_{\mathrm{BH}}S_{\mathrm{BH}}$$

Classical (emergent)

$$T_{\mathrm{BH}} = \frac{\hbar c^3}{8\pi GM}$$

quantum origin

$$S_{\mathrm{BH}} = \frac{A\,c^3}{4G\hbar} = \frac{4\pi GM^2}{\hbar c}$$

Entropic forces in GR

- Contribution to bulk entropy of the **inevitable** Schwarzschild black hole component of Dark Matter

Assuming BH total comoving number is conserved, their Total energy density and entropy density $(\hbar = c = 1)$ is

$$\rho_{BH} = n_{BH}\, M \,, \qquad s_{BH} = n_{BH}\, 4\pi G M^2$$

Therefore $$a^3 \frac{d}{dt}(\rho_{BH} a^3) = T_{BH} \frac{d}{dt}(s_{BH} a^3) = 0$$

No contribution to entropic force of the universe unless multiple **black hole mergers** or significant **mass accretion,** Which may change **mass or number density** of black holes.

Entropic forces in FLRW

Non-equilibrium thermodynamics in expanding universe

$$ds^2 = -N(t)^2 dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right)$$

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + p g^{\mu\nu}$$

$$D^\mu T_{\mu\nu} = D^\mu f_{\mu\nu}$$

2nd law thermodynamics

$$T dS = d(\rho a^3) + p\, d(a^3)$$

$$\dot\rho + 3H(\rho + p) = \frac{T\dot S}{a^3}$$

Hamiltonian constraint $\dot a^2 + k = \frac{8\pi G}{3}\rho\, a^2$

Friedmann/Raychaudhuri equation

$$\boxed{\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{4\pi G}{3}\frac{T\dot S}{a^3 H}}$$

Entropic forces in FLRW

- Apparent Cosmological Horizon H

$$n = g^{rr}\partial_r = a^{-1}\sqrt{1-kr^2}\,\partial_r$$

unit normal vector to comoving sphere of radius r

$$\sqrt{h}K = 2N(t)\,r\,a\sqrt{1-kr^2}\sin\theta$$

Trace extrinsic curvature

$$r_{AH} = 1/\sqrt{H^2 - k^2/a^2}$$

Apparent horizon distance

$$\mathcal{S}_{\mathrm{GHY}} = -\frac{1}{2G}\int dt\,N(t)\,H\,r_{AH}^2 = -\int dt\,N(t)\,T_{AH}S_{AH}$$

$$T_{AH} = \frac{\hbar c\,H}{2\pi}\,, \qquad S_{AH} = \frac{c^3}{\hbar}\frac{\pi\,r_{AH}^2}{G}$$

Emergent

Not enough to contribute to accelerated expansion universe

Entropic forces in FLRW

- Causal Cosmological Horizon $\mathcal{H}$

$$\sqrt{h}K = 2N(t)\, r\, a\sqrt{1-kr^2}\,\sin\theta$$ Trace extrinsic curvature

$$d_H \;=\; a\,\eta$$ Causal horizon distance

$$r \;=\; \sinh(\eta\sqrt{-k})/\sqrt{-k}.$$ Conformal time η

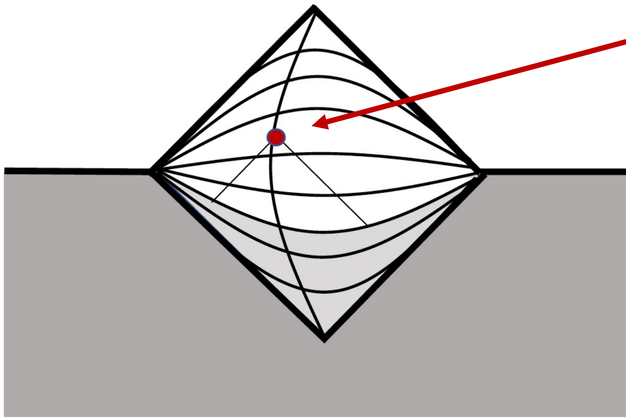
$$S_{GHY} \;=\; -\frac{1}{2G}\int dt\, N(t)\,\frac{a}{\sqrt{-k}}\sinh(2\eta\sqrt{-k})$$

$$= \;-\int dt\, N(t)\, T_H S_H = -\int dt\, N\, a^3 \rho_H$$

$$T_H = \frac{\hbar c}{2\pi}\;\frac{\sinh(2\eta\sqrt{-k})}{a\eta^2\sqrt{-k}}\,, \qquad S_H = \frac{\pi c^3}{\hbar}\frac{a^2\eta^2}{G}$$

Emergent

Cosmic Acceleration



Observer's causal horizon

$$\rho_H\, a^2 = \frac{T_H S_H}{a} = \frac{x_0}{2G} \sinh(2a_0 H_0 \eta)\,,$$

$$x_0 \equiv \frac{1-\Omega_0}{\Omega_0} = e^{-2N} \left(\frac{T_{\rm rh}}{T_{\rm eq}}\right)^2 (1+z_{\rm eq})\,.$$

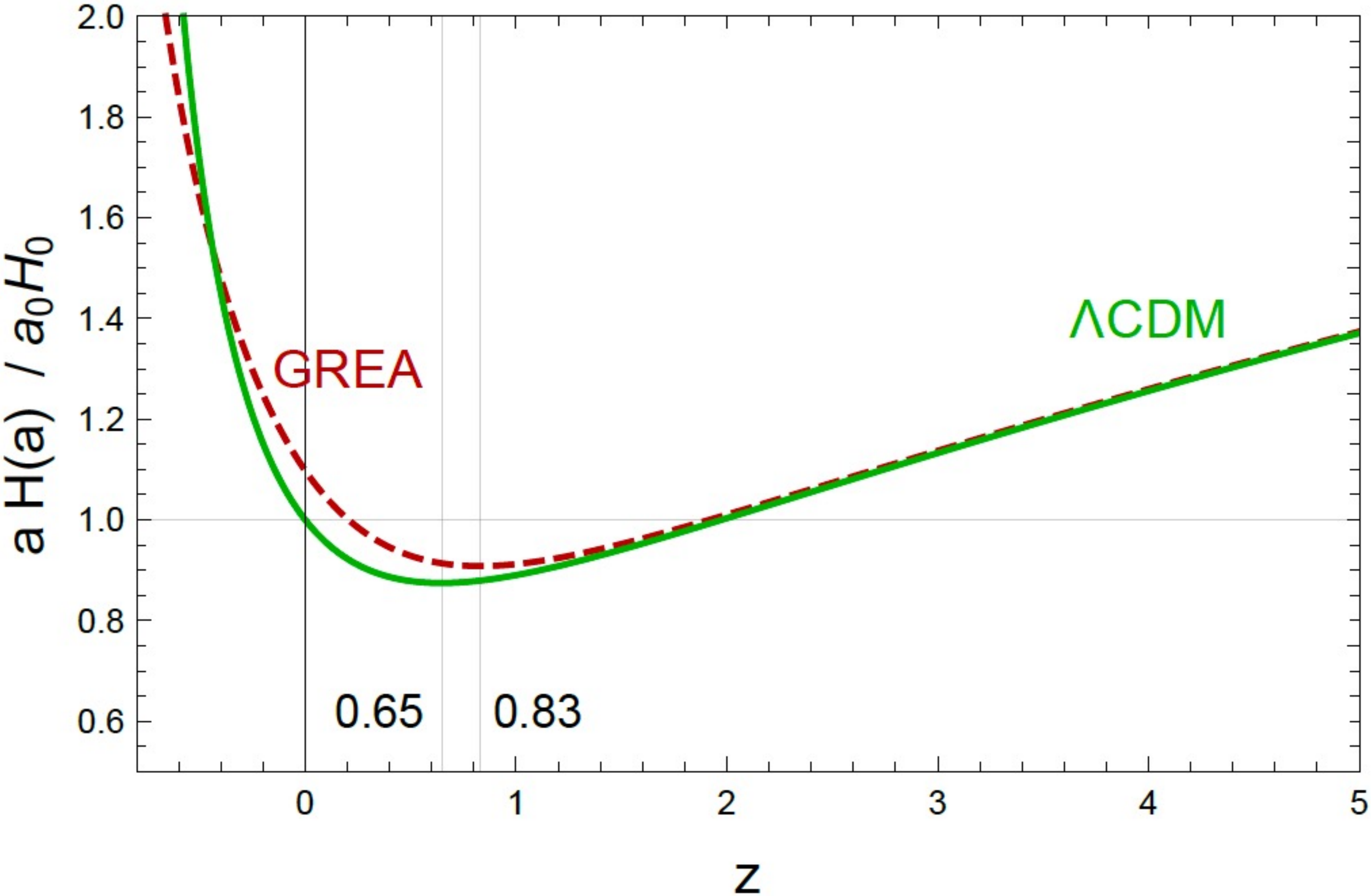
Hamiltonian constraint in conformal time
(primes denote derivatives w.r.t. $\tau = a_0 H_0 \eta$)

$$\left(\frac{a'}{a_0}\right)^2 = \Omega_{\rm M} \left(\frac{a}{a_0}\right) + \Omega_{\rm K} \left(\frac{a}{a_0}\right)^2 + \frac{4\pi}{3}\Omega_{\rm K} \left(\frac{a}{a_0}\right)^2 \sinh(2\tau)$$

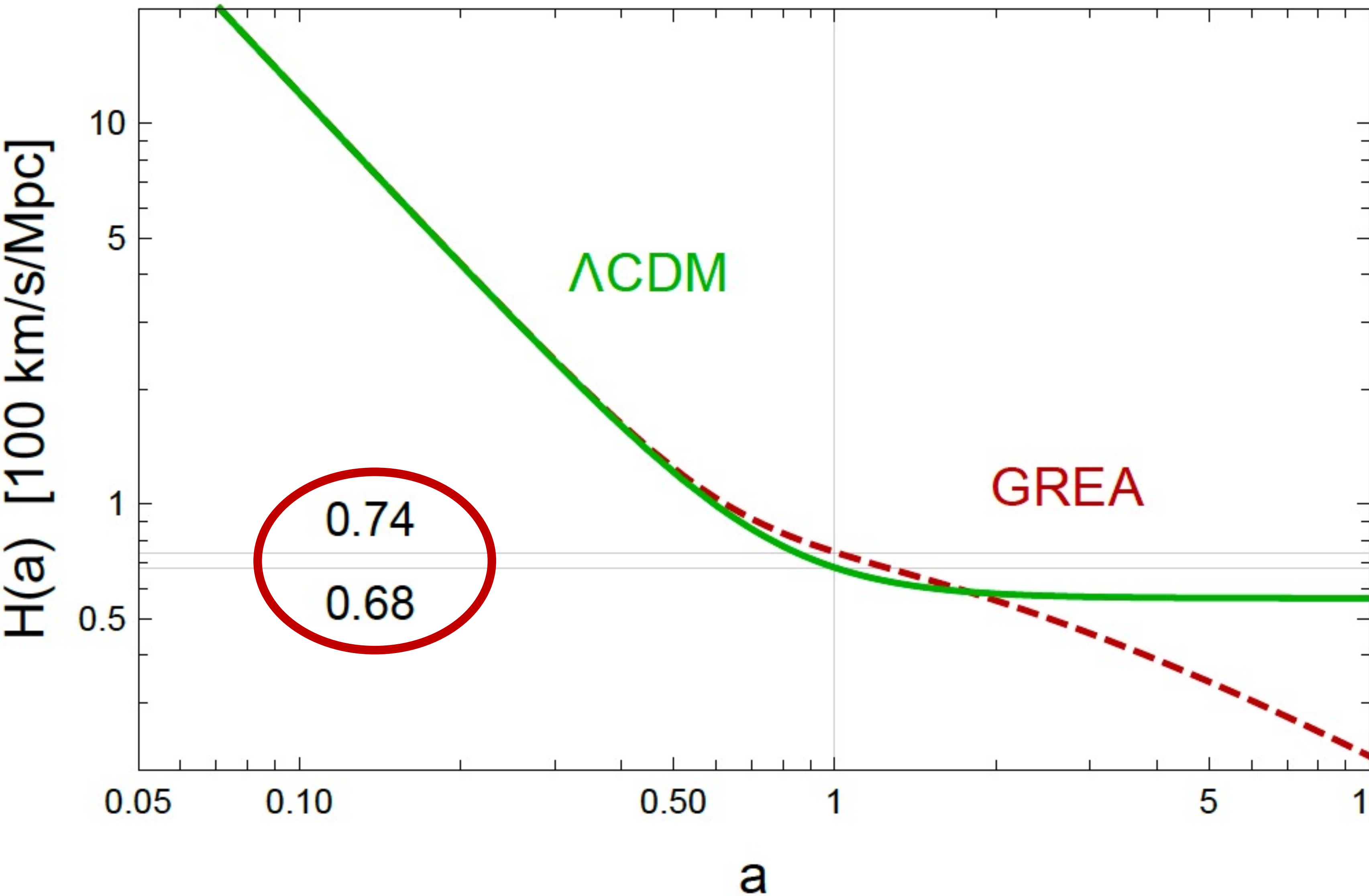
Entropic force term

Note: $\Lambda = 0$

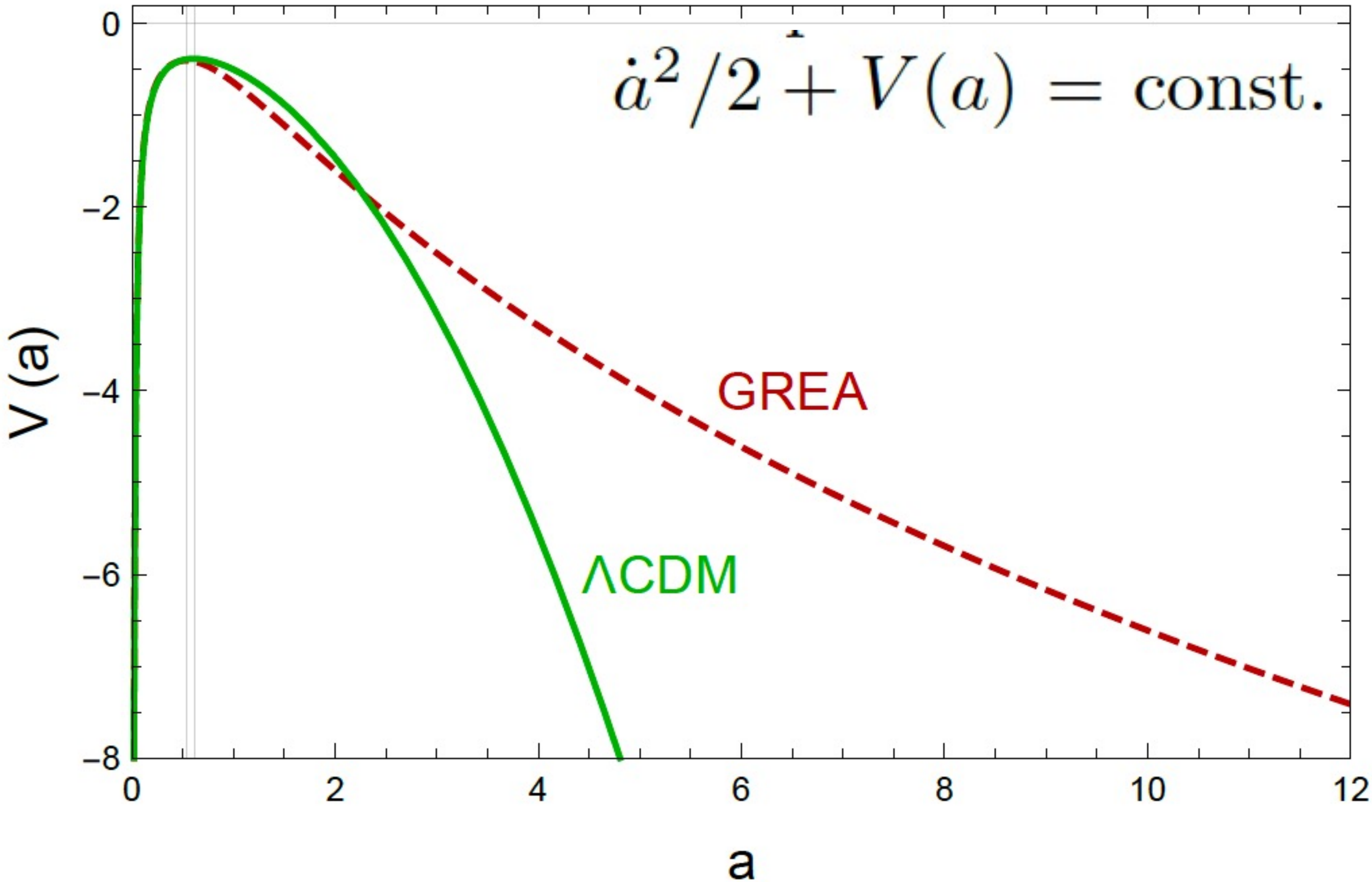
Cosmic Acceleration



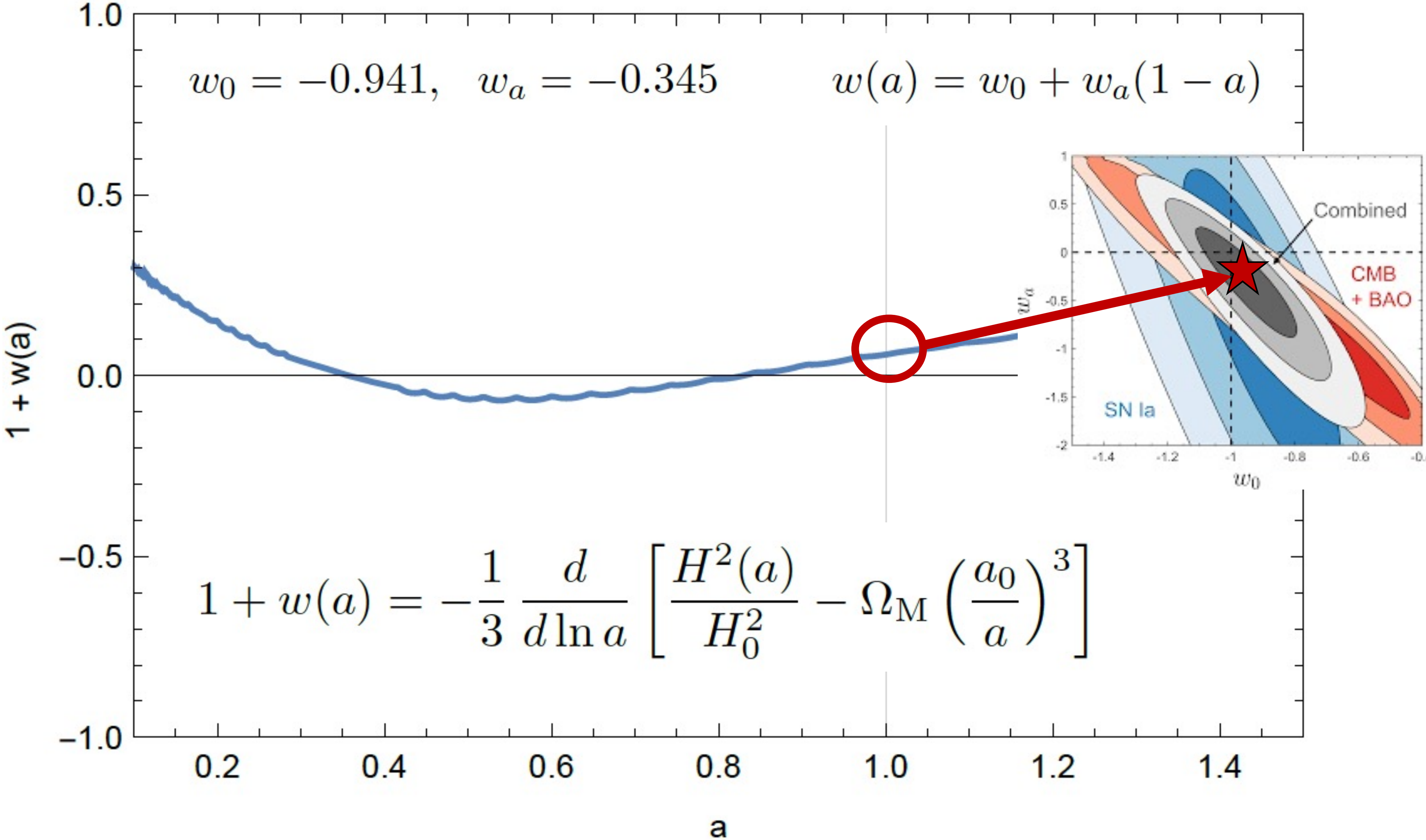
Cosmic Acceleration



Cosmic Acceleration



Cosmic Acceleration



Conclusions

- Non-equilibrium phenomena in GR: entropic forces
- ADM (3+1) slicing: Raychaudhuri eq. grav. collapse
- Cosmic acceleration from first principles
- No need for a Cosmological Constant
- Just QFT, GR and Non eq. Thermodynamics
- Multiple consequences for Large Scale Structure
- Possible solution of the H_0 , S_8 tensions
- <u>Future</u>: Preheating after inflation (Big Bang)
- <u>Future</u>: Connection w/ Verlinde's emergent gravity
- <u>Future</u>: Connection w/ Buchert's backreaction prob.