Centerless BMS_4 charge algebra and (A)dS uplift

Geoffrey Compère
Université Libre de Bruxelles (ULB)

Spanish-Portuguese Relativity Meeting (EREP2021)
Sept 13-16th, 2021

References

- G.C., David Nichols,
  Classical and Quantized General-Relativistic Angular Momentum
  2103.17103

- G.C., Adrien Fiorucci and Romain Ruzziconi,
  The Λ-BMS_4 Charge Algebra
  2004.10769

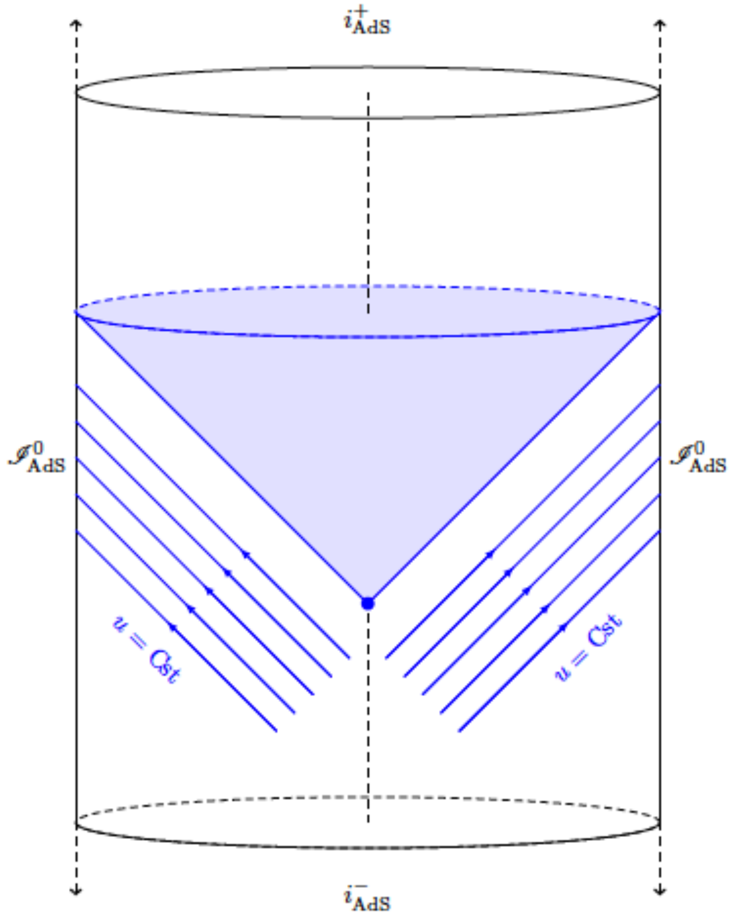
- G.C., Roberto Oliveri, Ali Seraj,
  The Poincaré and BMS flux-balance laws with application to binary systems
  1912.03164

- G.C., Adrien Fiorucci and Romain Ruzziconi,
  The Λ-BMS_4 group of dS_4 and new boundary conditions for AdS_4.
  1905.00971

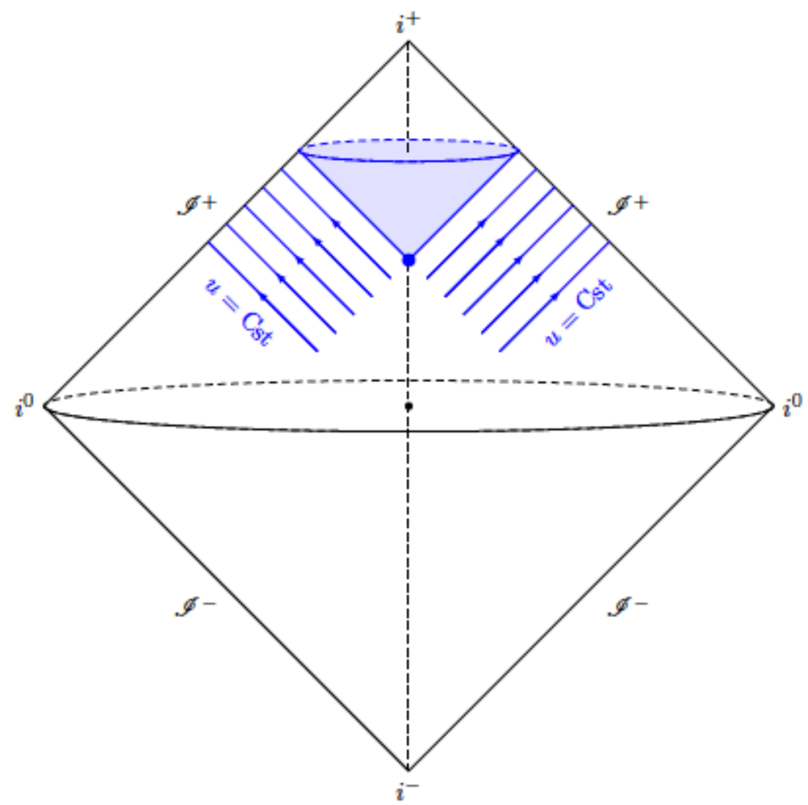
- G.C., Adrien Fiorucci and Romain Ruzziconi,
  Superboost transitions, refraction memory and super-Lorentz charge algebra
  1810.00377

1. The centerless BMS_4 charge algebra

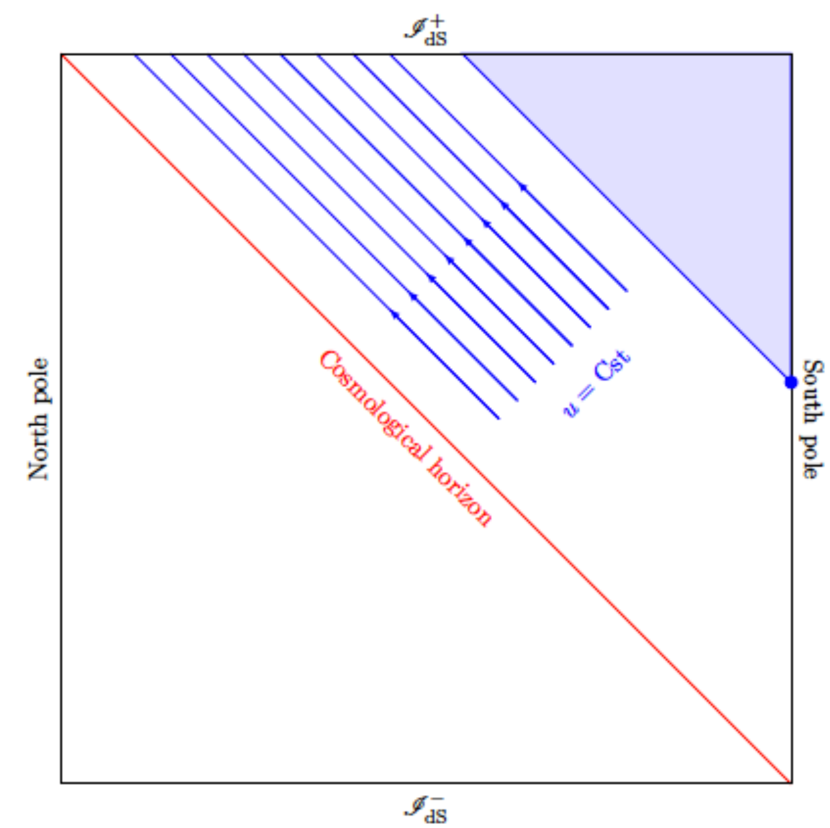
Infrared structure of gravity



AdS case $\Lambda < 0$.



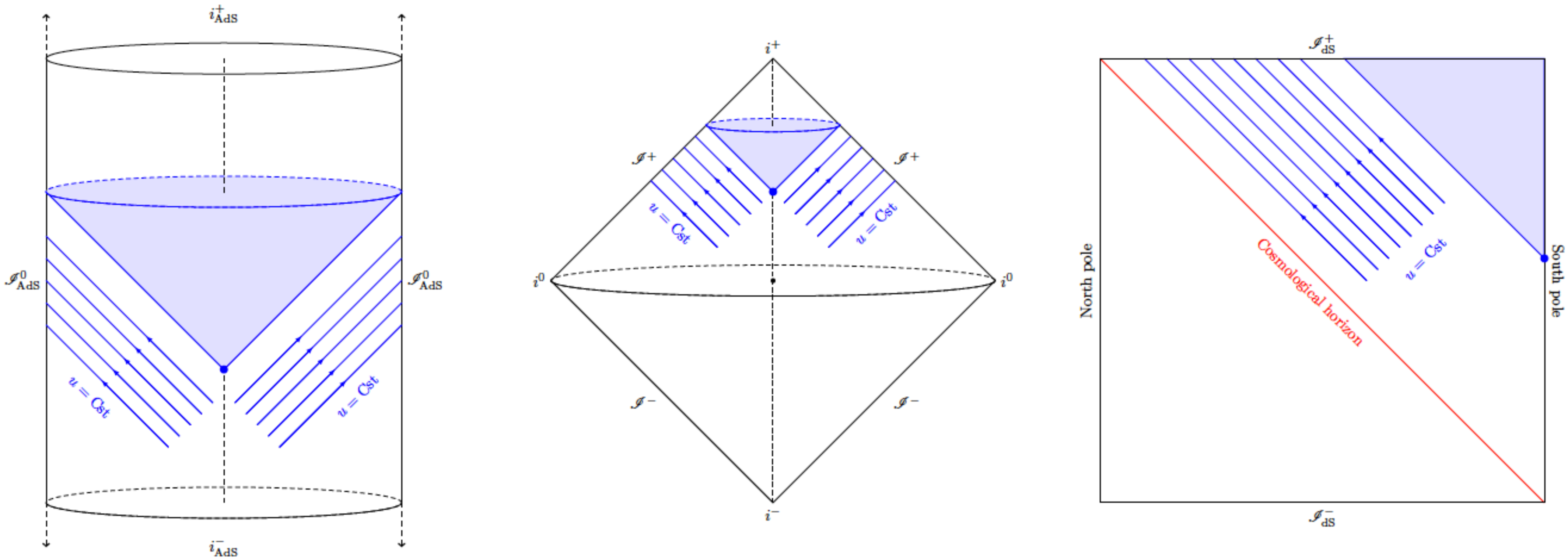
Flat case $\Lambda = 0$.



dS case $\Lambda > 0$.

"The Hamiltonian in General Relativity is a surface term. Therefore, gravity is holographic."

Infrared structure of gravity



AdS case $\Lambda < 0$. | Flat case $\Lambda = 0$. | dS case $\Lambda > 0$.

80's

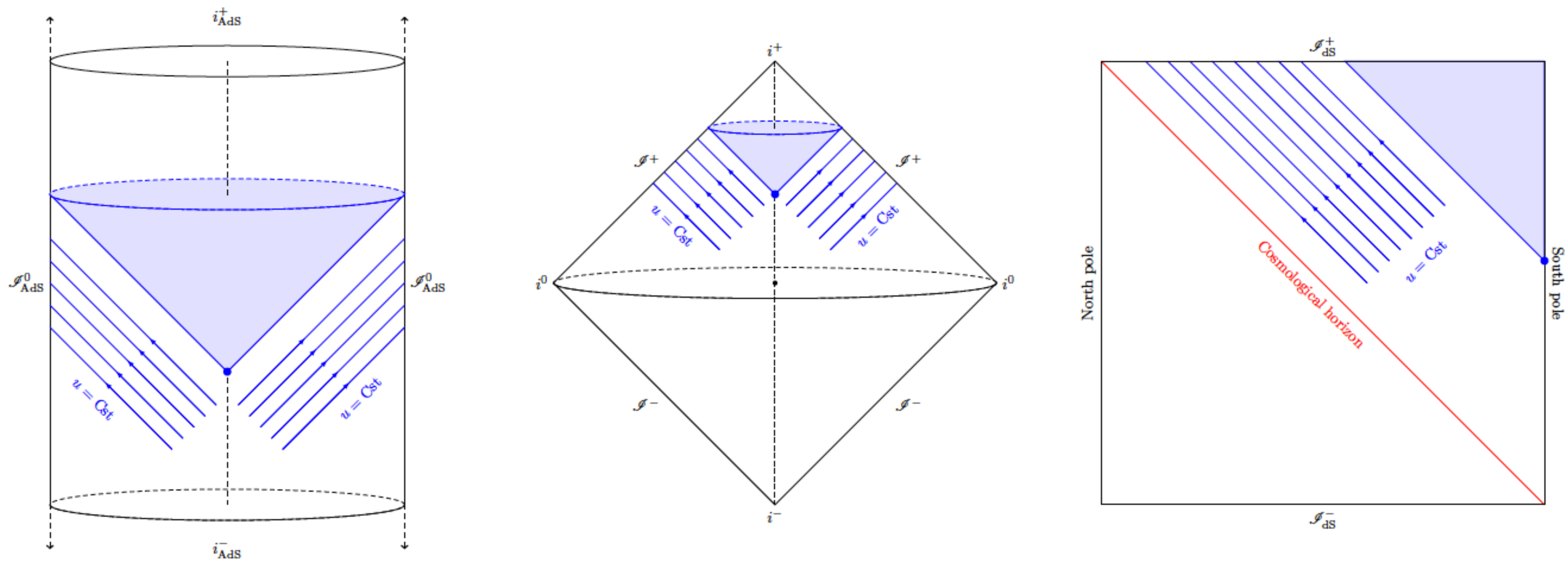
[Ashtekar,Brown,Bunster,Henneaux]
[Maldacena,Witten]

60's [ADM ; BMS]

2010-2021 [Barnich-Troessaert ; Strominger et al]

2000's

Boundary conditions -> Global symmetries



AdS case $\Lambda < 0$.

Flat case $\Lambda = 0$.

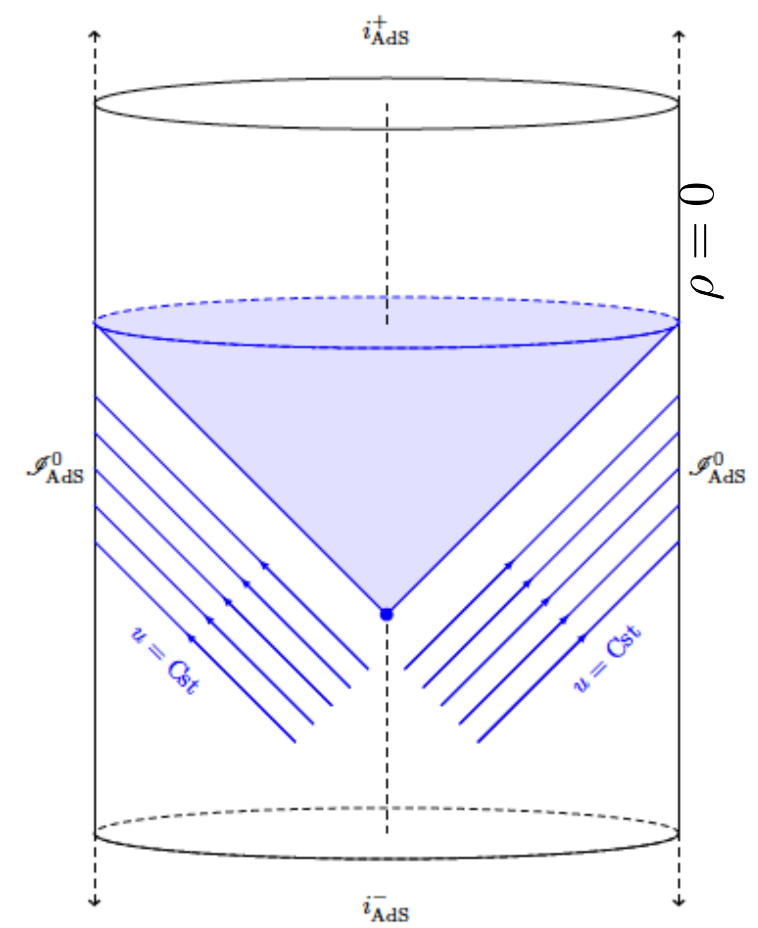
dS case $\Lambda > 0$.

Conformal group SO(d-1,2)
OR None
OR Λ-BMS
...

Poincaré
BMS Supertranslations
BMS Super-Lorentz
...

Group SO(d,1)
OR None
OR Λ-BMS
...

Dirichlet Anti-de Sitter



AdS case $\Lambda < 0$.

1. SOLUTION

$$ds^2 = -\frac{3}{\Lambda}\frac{d\rho^2}{\rho^2} + \gamma_{ab}(\rho, x^c)dx^a dx^b.$$

[Fefferman-Graham theorem]

$$\gamma_{ab} = \frac{1}{\rho^2}\, g^{(0)}_{ab} + \frac{1}{\rho}\, g^{(1)}_{ab} + g^{(2)}_{ab} + \rho\, g^{(3)}_{ab} + \mathcal{O}(\rho^2)$$

$$T_{ab} = \frac{\sqrt{3|\Lambda|}}{16\pi G} g^{(3)}_{ab}$$

$$D^{(0)}_a T^{ab} = 0, \quad g^{(0)}_{ab} T^{ab} = 0$$

Two holographic fields: the boundary metric $g^{(0)}_{ab}$ and the stress-tensor T^{ab}

2. SYMMETRY

Fixing the boundary metric to be the flat cylinder, there are SO(2,3) symmetries.

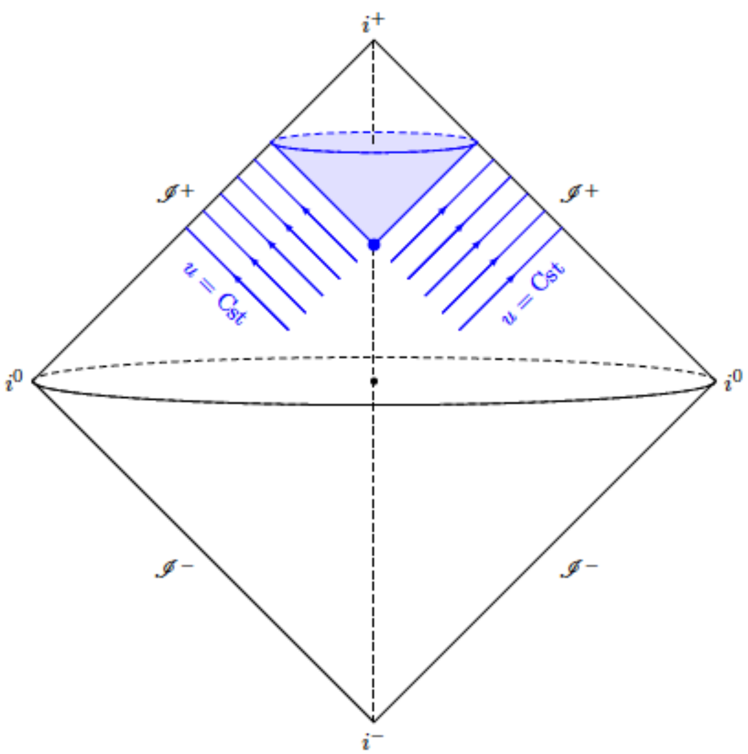
$$\mathcal{L}_{\xi^{(0)}} g^{(0)}_{ab} \sim g^{(0)}_{ab} \qquad \mathcal{L}_\xi g_{\mu\nu} = g_{\mu\nu}(T_{ab} + \delta_\xi T_{ab}) - g_{\mu\nu}(T_{ab}).$$

3. CHARGES

The associated charges are conserved and represent the group SO(2,3) under the Peierls bracket

$$Q_\xi = \int_S d^2\Omega T_{ab} \xi^a_{(0)} n^b, \qquad \{Q_\xi, Q_\eta\} = Q_{[\xi,\eta]}$$

Asymptotically Flat Spacetimes: Null infinity



Flat case $\Lambda = 0$.

Gauge fixing :

Bondi / Newman-Unti coordinates

$$g_{ur} = -1, \quad g_{uu} = 0, \quad g_{uA} = 0, \qquad x^A = \{\theta, \phi\}$$

1. SOLUTION

$$ds^2 = e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}dudr + g_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

$$g_{AB} = r^2 q_{AB} + r\,C_{AB} + D_{AB} + \frac{1}{r}E_{AB} + \frac{1}{r^2}F_{AB} + \mathcal{O}(r^{-3})$$

Infinite number of holographic fields:

the boundary metric q_{AB}, the shear $C_{AB}(u, x^C)$,
mass $m(u, x^C)$ and angular momentum aspects $N_A(u, x^C)$,
subleading fields $E_{AB}(u, x^C)$, $F_{AB}(u, x^C)$

Flux-balance laws:

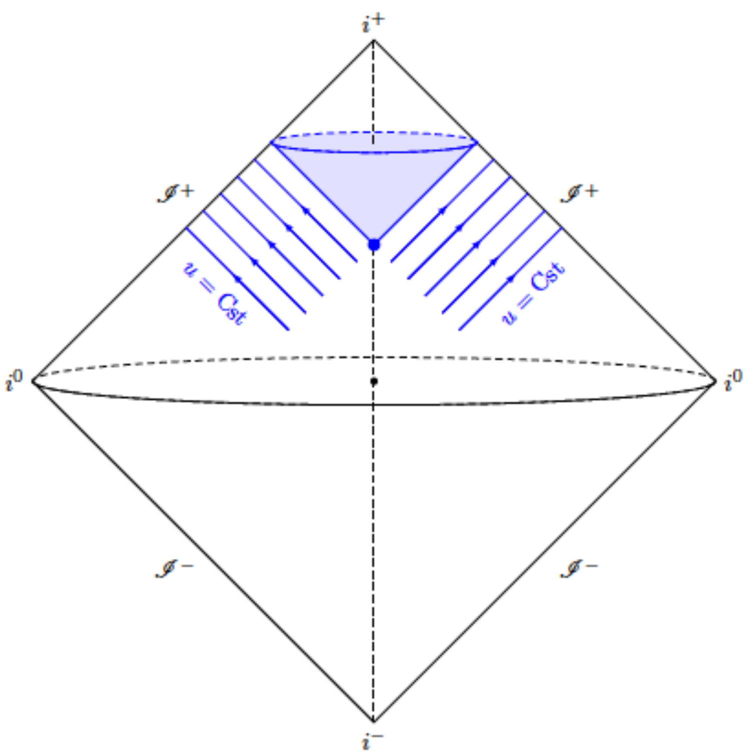
$$\partial_u m + D_A(\cdots) = \text{HARD TERMS}(N_{AB}),$$
$$\partial_u N_A + D_B(\cdots) = \text{HARD TERMS}(N_{AB}),$$

I. Supertranslations

$$T(\theta,\phi)\partial_u + \frac{1}{2}\nabla^2 T\partial_r - \frac{1}{r}(\partial_\theta T\partial_\theta + \frac{1}{\sin^2\theta}\partial_\phi T\partial_\phi) + \dots$$

- The associated Noether charge is the Bondi mass aspect $m(u,x^A)$ integrated over the celestial sphere
- The 4 lowest harmonics are the translations associated with Momenta.
- Supertranslations transitions are associated with displacement memory and are caused by any null radiation exiting null infinity

2. SYMMETRY



Flat case $\Lambda = 0$.

II. Super-Lorentz transformations

$$\frac{1}{2}uD_AR^A\partial_u + (-\frac{1}{2}(r+u)D_AR^A + \mathcal{O}(\frac{1}{r}))\partial_r + (R^A - \frac{u}{2r}D^AD_BR^B + \mathcal{O}(\frac{1}{r^2}))\partial_A$$

- Associated Noether charge: $N_A(u,x^A)$ integrated over the celestial sphere (after renormalization of radial divergences)
- The 6 lowest harmonics are associated with the Lorentz charges: angular momentum and center-of-mass charge (orbital angular momentum).
- Lorentz transformations are asymptotic symmetries. Super-Lorentz transformations are asymptotic symmetries after renormalization.
- Superrotations and superboosts

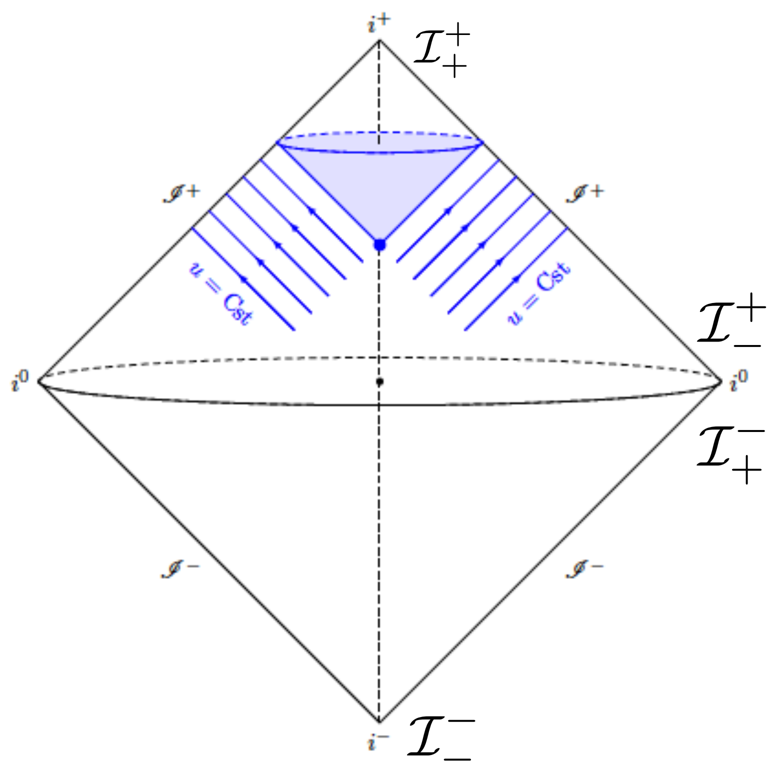
$$\text{Symmetry group} = \mathrm{Vect}(S^2) \ltimes \mathrm{Diff}(S^2)$$

Generalized BMS4 group

3. CHARGES

$$Q_T(u) = \int_S d^2S\, \overline{m}(u, x^C) T(x^C)$$

$$Q_R(u) = \int_S d^2S\, \overline{N}_A(u, x^C) R^A(x^C)$$



Flat case $\Lambda = 0$.

Junction condition between past and future null infinity: Antipodal map at spatial infinity

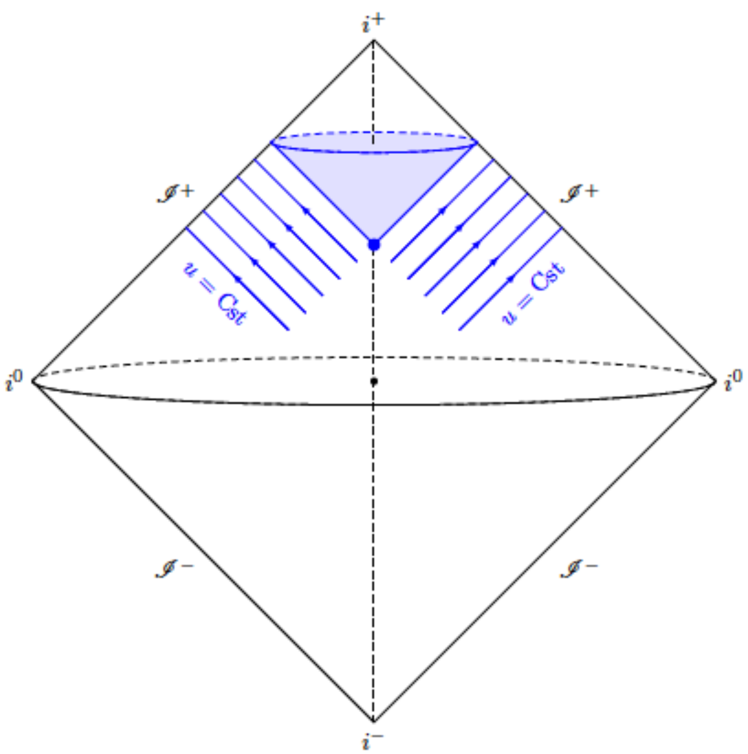
Scattering around Minkowski obeys

$$F_{T,R} = \int_{-\infty}^{\infty} du \partial_u Q^+_{T,R}(u) = \int_{-\infty}^{\infty} dv \partial_v Q^-_{T,R}(v)$$

This is the Ward identity of BMS symmetry. It is equivalent to the leading and subleading soft theorems.

BMS$_4$ flux asymptotic symmetry algebra

3. CHARGES



Flat case $\Lambda = 0$.

Given a prescription

$$\overline{m} = m + f(q_{AB}, C_{AB}, N_{AB}),$$
$$\overline{N}_A = N_A + f_A(q_{AB}, C_{AB}, N_{AB})$$

and boundary conditions at past/future times.

The BMS$_4$ algebra can be represented under the Peierls bracket without central extension

$$\{F_{T_1}, F_{T_2}\} = 0,$$

$$\{F_{R_1}, F_{T_2}\} = F_{R_1(T_2)}, \qquad R_1(T_2) \equiv (R_1^A \partial_A - \frac{1}{2} D_A R_1^A) T_2$$

$$\{F_{R_1}, F_{R_2}\} = F_{[R_1, R_2]}$$

[Campiglia, Peraza, 2020]
[G.C., Fiorucci, Ruzziconi, 2020]

Prescription for the experts:

$$\bar{M} = M + \frac{1}{8} C_{AB} N^{AB}_{\text{vac}},$$
$$\bar{N}_A = N_A^{[B.T.]} - u \partial_A \bar{M} + \frac{1}{4} C_{AB} D_C C^{BC} + \frac{3}{32} \partial_A (C_{BC} C^{BC}).$$

Its quantization leads to the soft graviton theorems.

2. The angular momentum in GR

Three ambiguities to define J

1. Center-of-mass frame

$$SO(2) \subset SO(3) \subset SO(3,1)$$

- Pauli-Lubanski spin pseudo-vector $W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}J^{\nu\rho}P^\sigma$

- Local rotation vector :

$$R_i'^A = \gamma R_i^A + (1-\gamma)\frac{v_i(\vec{v}\cdot\vec{R}^A)}{v^2} + \gamma\epsilon_{ijk}v_j K_k^A, \qquad v_i \equiv \frac{\mathcal{P}_i}{\mathcal{P}_0}, \qquad \gamma = (1-\frac{v^2}{c^2})^{-\frac{1}{2}}, \quad v = \sqrt{\vec{v}\cdot\vec{v}}.$$

In GR, the 4-momentum evolves according to the mass loss formula:

$$\dot{\mathcal{P}}^\mu = -\frac{c^2}{8G}\oint_S \dot{C}_{AB}\dot{C}^{AB}k^\mu \qquad k^\mu = (1, n_i)$$

Three ambiguities to define J

2. Supertranslation frame

$$SO(2) \subset SO(3) \subset SO(3,1) \subset SO(3,1) \ltimes Vect(S^2)$$

- Boundary condition on the shear

$$C_{AB}|_{u=\pm\infty} = -2D_A D_B C^{\pm} + \gamma_{AB} D^C D_C C^{\pm} + O(u^{-1}).$$

$$\delta C^{\pm} = T(\theta, \phi)$$

- Fix supertranslation frame at $r \to \infty, u \to -\infty \; (\mathcal{I}^{+}_{-})$

$$C^{-} = 0$$

- The displacement memory effect is generally present

Three ambiguities to define J

3. α-ambiguity

$$\mathcal{J}_i^{(\alpha)} \equiv -\frac{1}{2} \oint_S \epsilon^{AD} \partial_D n_i \left(\bar{N}_A - \frac{\alpha c^3}{4G} C_{AB} D_C C^{BC} \right)$$

For all α

(i) Vanishing for Minkowski
(ii) Standard J of Kerr
(iii) Locally constructed from tensors
(iv) Obey the BMS algebra
(v) Satisfy the BMS flux-balance laws

Three ambiguities to define J

3. α-ambiguity

$$\mathcal{J}_i^{(\alpha)} \equiv -\frac{1}{2}\oint_S \epsilon^{AD}\partial_D n_i \left(\bar{N}_A - \frac{\alpha c^3}{4G} C_{AB} D_C C^{BC}\right)$$

Definitions used in the literature:

$\alpha = 1$ $\quad \mathcal{J}_i = \frac{1}{16\pi G}\oint_S D^\mu \xi_i^\nu \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} dx^\alpha \wedge dx^\beta$ [Komar][Iyer,Wald,1992] [Wald,Zoupas,1999]

$$\dot{\mathcal{J}}_i = -\frac{G}{c^5}\left(\frac{2}{5}\epsilon_{ijk} I_{jl}^{(2)} I_{kl}^{(3)}\right) - \frac{G}{c^7}\left(\frac{1}{63}\epsilon_{ijk} I_{jlm}^{(3)} I_{klm}^{(4)} + \frac{32}{45}\epsilon_{ijk} J_{jl}^{(2)} J_{kl}^{(3)}\right) + O\left(c^{-9}\right),$$
[Thorne, 1980]

$$\dot{\mathcal{J}}_i = \frac{c^3}{32\pi G}\oint_S d^2\Omega \epsilon_{ijk}(x^i \dot{f}_{ab}\partial_j f_{ab} - 2 f_{ia}\dot{f}_{ja})$$
[Landau-Lifshitz]

$$\dot{\mathcal{J}}_i = \frac{1}{16\pi G}\oint_S d^2\Omega(\mathcal{L}_{\xi_i} D_c - D_c \mathcal{L}_{\xi_i}) l_d q^{ac} q^{bd}$$
[Dray-Streubel,84] [Ashtekar, Streubel,81]

(vi) No background structure required

(vii) Axisymmetry implies J=0

Three ambiguities to define J

3. α-ambiguity

$$\mathcal{J}_i^{(\alpha)} \equiv -\frac{1}{2}\oint_S \epsilon^{AD}\partial_D n_i \left(\bar{N}_A - \frac{\alpha c^3}{4G} C_{AB} D_C C^{BC}\right)$$

Definitions used in the literature:

$\alpha = 0$ [Strominger, Zhiboedov, 2014]
[Pasterski, Strominger, Zhiboedov, 2015]
[G.C., Fiorucci, Ruzziconi, 2020]

The change of definition leads to a numerically 0.01%-0.1% effect for binary coalescences [Elhashash, Nichols, 2021]

(vi) Background structure required (radial foliation)

(vii) Generalized BMS group represented (including super-Lorentz)

$$\{F_{T_1}, F_{T_2}\} = 0, \qquad \{F_{R_1}, F_{T_2}\} = F_{R_1(T_2)}, \qquad \{F_{R_1}, F_{R_2}\} = F_{[R_1,R_2]}$$

$$R_1(T_2) \equiv (R_1^A \partial_A - \frac{1}{2} D_A R_1^A) T_2$$

3. Extension of the BMS group to (A)dS

Question

- Three-dimensional case :

Asymptotically AdS_3 :
$\mathrm{Diff}(S^1) \times \mathrm{Diff}(S^1)$
[Brown-Henneaux '86]

$\Longrightarrow$
$\Lambda \to 0$

Asymptotically flat :
$BMS_3 = \mathrm{Diff}(S^1) \ltimes \mathrm{Vect}(S^1)$
[Ashtekar-Bicak-Schmidt '96]
[Barnich-Compère '07]

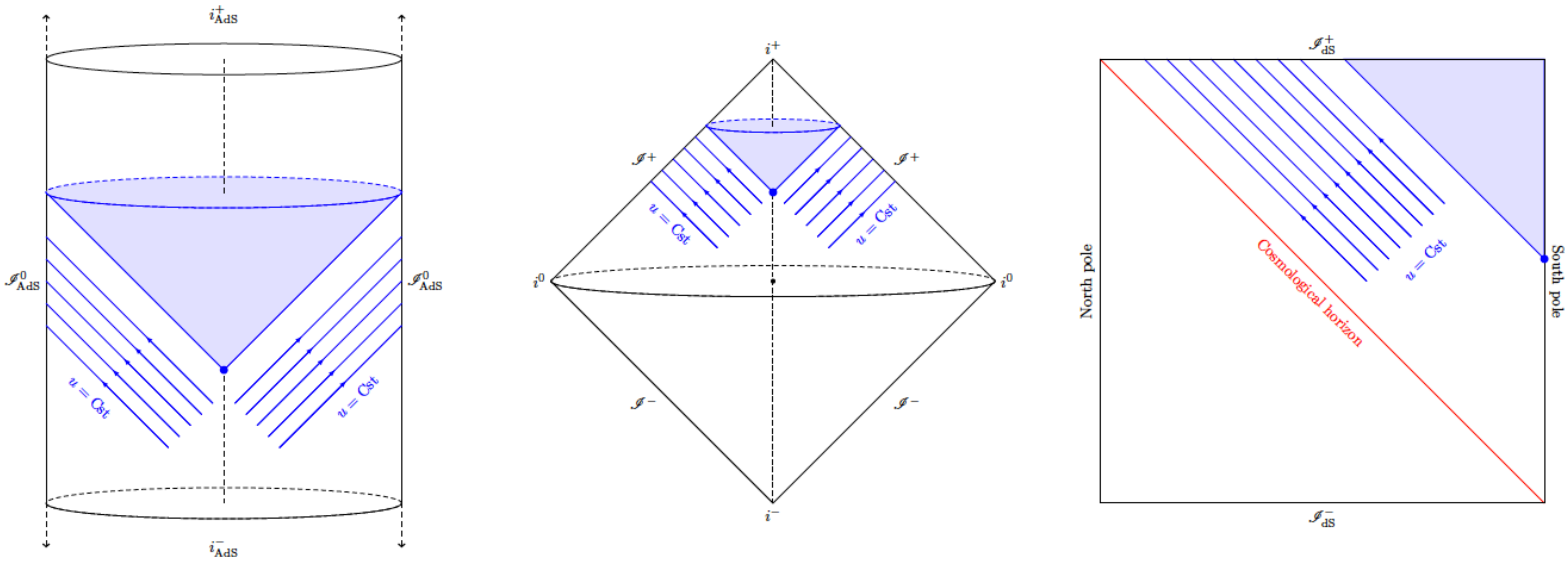
- Four-dimensional case :

Asymptotically AdS_4 :
? ? ?

$\Longrightarrow$
$\Lambda \to 0$

Asymptotically flat :
$BMS_4 = \mathrm{Diff}(S^2) \ltimes \mathcal{T}$
[Bondi-van der Burg-Metzner '62]
[Sachs '62]
[Barnich-Troessaert, '11][Campiglia-Laddha, '15]

Universal BMS structure (keeping all dynamics)



AdS case $\Lambda < 0$.

Flat case $\Lambda = 0$.

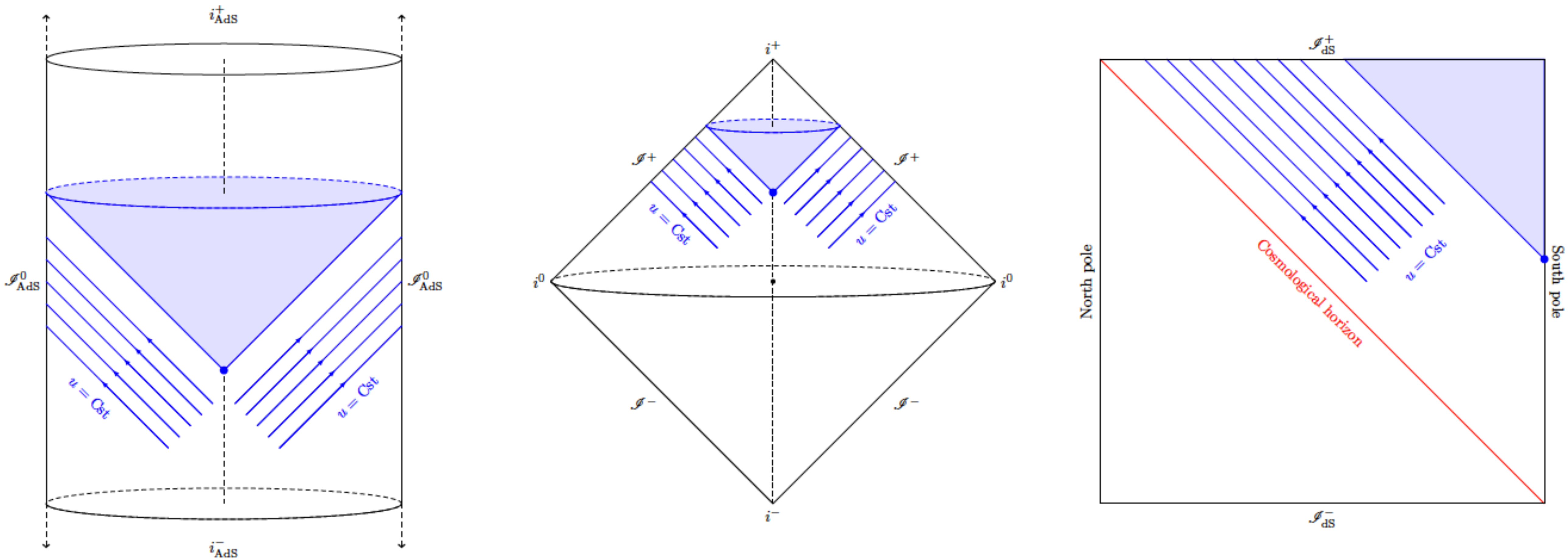
dS case $\Lambda > 0$.

Boundary structure: codimension 1 codimension 2

"Boundary gauge fixing": Fixing a foliation and a measure

$$ds^2 = sign(\Lambda)du^2 + q_{AB}dx^A dx^B \qquad \sqrt{q}$$

A dictionary exists between distinct bulk gauges



AdS case $\Lambda < 0$. Flat case $\Lambda = 0$. dS case $\Lambda > 0$.

Fefferman-Graham

$$g^{(0)}_{ab}, T^{ab}$$

Bondi

$$C_{AB}, M, N_A,$$
$$D_{AB}, E_{AB}, \ldots$$

Starobinsky

$$g^{(0)}_{ab}, T^{ab}$$

Definitions

Starobinsky/
Fefferman-Graham
(SFG) gauge

$$(\rho, x^a)$$

$$g_{\rho a} = 0,$$

$$g_{\rho\rho} = -\frac{3}{\Lambda}\frac{1}{\rho^2}$$

Bondi gauge

$$(u, r, x^A)$$

$$g_{rr} = 0, \quad g_{rA} = 0$$

$$\partial_r \left(\frac{\det(g_{AB})}{r^4} \right) = 0$$

The dictionary between Bondi and Starobinsky/Fefferman-Graham gauge has been worked out

- One can solve the large radius expansion of Einstein's equations in both gauges
- A diffeomorphism exists between the two gauges when $\Lambda \neq 0$
- The (2-covariant) map between the free fields in each gauge can be formulated

[Poole, Skenderis, Taylor, 2018]
[G.C., Fiorucci, Ruzziconi, 2019]

Solution space (AL(A)dS$_4$)

SFG gauge

$$ds^2 = -\frac{3}{\Lambda}\frac{d\rho^2}{\rho^2} + \gamma_{ab}(\rho, x^c)dx^a dx^b.$$

$$\gamma_{ab} = \frac{1}{\rho^2}\, g^{(0)}_{ab} + \frac{1}{\rho}\, g^{(1)}_{ab} + g^{(2)}_{ab} + \rho\, g^{(3)}_{ab} + \mathcal{O}(\rho^2)$$

[FG theorem]

$$T_{ab} = \frac{\sqrt{3|\Lambda|}}{16\pi G} g^{(3)}_{ab}$$

Bondi gauge

$$ds^2 = e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}du dr + g_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

$$g_{AB} = r^2 q_{AB} + r\, C_{AB} + D_{AB} + \frac{1}{r} E_{AB} + \frac{1}{r^2} F_{AB} + \mathcal{O}(r^{-3})$$

$$l = l^A_A = \frac{1}{2} q^{AB}\partial_u q_{AB} = \partial_u \ln\sqrt{q}.$$

[Blanchet-Damour]

$$U^A = U^A_0(u, x^B) + \overset{(1)}{U^A}(u, x^B)\frac{1}{r} + \overset{(2)}{U^A}(u, x^B)\frac{1}{r^2} + \overset{(3)}{U^A}(u, x^B)\frac{1}{r^3} + \overset{(L3)}{U^A}(u, x^B)\frac{\ln r}{r^3} + o(r^{-3})$$

$$\beta(u, r, x^A) = \beta_0(u, x^A) + \frac{1}{r^2}\left[-\frac{1}{32}C^{AB}C_{AB}\right] + \frac{1}{r^3}\left[-\frac{1}{12}C^{AB}\mathcal{D}_{AB}\right] + \frac{1}{r^4}\left[-\frac{3}{32}C^{AB}\mathcal{E}_{AB} - \frac{1}{16}\mathcal{D}^{AB}\mathcal{D}_{AB} + \frac{1}{128}(C^{AB}C_{AB})^2\right] + \mathcal{O}(r^{-5}).$$

$$\frac{V}{r} = \frac{\Lambda}{3}e^{2\beta_0}r^2 - r(l + D_A U^A_0) - e^{2\beta_0}\left[\frac{1}{2}\left(R[q] + \frac{\Lambda}{8}C_{AB}C^{AB}\right) + 2D_A\partial^A\beta_0 + 4\partial_A\beta_0\partial^A\beta_0\right] - \frac{2M}{r} + o(r^{-1})$$

$$\frac{\Lambda}{3}C_{AB} = e^{-2\beta_0}\left[(\partial_u - l)q_{AB} + 2D_{(A}U^0_{B)} - D^C U^0_C q_{AB}\right].$$

Holographic fields $\Lambda \neq 0$

SFG gauge | **Bondi gauge (2+1 boundary split)**

$$g^{(0)}_{ab}dx^a dx^b \quad \Longleftrightarrow \quad \frac{\Lambda}{3}e^{4\beta_0}du^2 + q_{AB}(dx^A - U_0^A du)(dx^B - U_0^B du)$$

T^{ab}

$$g^{(0)}_{ab}T^{ab} = 0.$$

$$T_{ab} = \frac{\sqrt{3|\Lambda|}}{16\pi G}\begin{bmatrix} -\frac{4}{3}M^{(\Lambda)} & -\frac{2}{3}N_B^{(\Lambda)} \\ -\frac{2}{3}N_A^{(\Lambda)} & J_{AB} + \frac{2}{\Lambda}M^{(\Lambda)}q_{AB} \end{bmatrix}$$

$$M^{(\Lambda)} = M + \frac{1}{16}(\partial_u + l)(C_{CD}C^{CD}),$$

$$N_A^{(\Lambda)} = N_A - \frac{3}{2\Lambda}D^B(N_{AB} - \frac{1}{2}lC_{AB}) - \frac{3}{4}\partial_A(\frac{1}{\Lambda}R[q] - \frac{3}{8}C_{CD}C^{CD}),$$

$$\begin{aligned} J_{AB} = &- \mathcal{E}_{AB} - \frac{3}{\Lambda^2}\Big[\partial_u(N_{AB} - \frac{1}{2}lC_{AB}) - \frac{\Lambda}{2}q_{AB}C^{CD}(N_{CD} - \frac{1}{2}lC_{CD})\Big] \\ &+ \frac{3}{\Lambda^2}(D_A D_B l - \frac{1}{2}q_{AB}D_C D^C l) \\ &- \frac{1}{\Lambda}(D_{(A}D^C C_{B)C} - \frac{1}{2}q_{AB}D^C D^D C_{CD}) \\ &+ C_{AB}\Big[\frac{5}{16}C_{CD}C^{CD} + \frac{1}{2\Lambda}R[q]\Big]. \end{aligned}$$

3 boundary ODEs
=
3 flux-balance laws

$$D^{(0)}_a T^{ab} = 0 \quad \Longleftrightarrow$$

$$(\partial_u + \frac{3}{2}l)M^{(\Lambda)} + \frac{\Lambda}{6}D^A N_A^{(\Lambda)} + \frac{\Lambda^2}{24}C_{AB}J^{AB} = 0.$$

$$(\partial_u + l)N_A^{(\Lambda)} - \partial_A M^{(\Lambda)} - \frac{\Lambda}{2}D^B J_{AB} = 0.$$

More holographic fields for $\Lambda = 0$

SFG gauge

infinite number of boundary ODEs / flux-balance laws

Bondi gauge

$$\frac{\Lambda}{3} e^{4\beta_0} du^2 + q_{AB}(dx^A - U_0^A du)(dx^B - U_0^B du)$$

$$M, N_A, C_{AB}, D_{AB}, E_{AB}, F_{AB}, \cdots$$

Constraints

$$\partial_u q_{AB} = l q_{AB} + 2D_{(A} U^0_{B)} - D^C U^0_C q_{AB}$$

$$\partial_u M = -\frac{1}{8} N_{AB} N^{AB} + \frac{1}{4} D_A D_B N^{AB} + \frac{1}{8} D_A D^A \mathring{R},$$

$$\begin{aligned}\partial_u N_A = D_A M &+ \frac{1}{16} D_A (N_{BC} C^{BC}) - \frac{1}{4} N^{BC} D_A C_{BC} \\ &- \frac{1}{4} D_B (C^{BC} N_{AC} - N^{BC} C_{AC}) - \frac{1}{4} D_B D^B D^C C_{AC} \\ &+ \frac{1}{4} D_B D_A D_C C^{BC} + \frac{1}{4} C_{AB} D^B \mathring{R}.\end{aligned}$$

$$\partial_u D_{AB} = 0,$$

$$\partial_u E_{AB} = \cdots$$

$$\partial_u F_{AB} = \cdots$$

see [Barnich, Troessart, 2012]

Boundary gauge condition: Definition of Λ-BMS

$$g^{(0)}_{tt} = \frac{\Lambda}{3}, \qquad g^{(0)}_{tA} = 0, \qquad \det(g_{(0)}) = \frac{\Lambda}{3}\bar{q}.$$

$$ds^2_{(0)} = \frac{\Lambda}{3}dt^2 + q_{AB}dx^A dx^B$$

- Can always be reached
- Does not constraint the Cauchy problem

The residual diffeomorphisms in a given bulk gauge and in this boundary gauge form the Λ-BMS group.

Symmetry generators

Preserving SFG gauge: Diff(S³)xWeyl

$$\xi^u = f,$$
$$\xi^A = Y^A + I^A, \quad I^A = -\partial_B f \int_r^\infty dr' (e^{2\beta} g^{AB}),$$
$$\xi^r = -\frac{r}{2}(\mathcal{D}_A Y^A - 2\omega + \mathcal{D}_A I^A - \partial_B f U^B + \frac{1}{2} f g^{-1} \partial_u g),$$

$$g = \det(g_{AB})$$

$$\partial_r f = 0 = \partial_r Y^A$$

Preserving further boundary gauge:

$\delta_\xi \sqrt{q} = 0$ → $\omega = 0.$

$\delta_\xi \beta_0 = 0$ → $\left(\partial_u - \frac{1}{2} l\right) f = \frac{1}{2} D_A Y^A,$

$$l = l^A_A = \frac{1}{2} q^{AB} \partial_u q_{AB} = \partial_u \ln \sqrt{q}.$$

$\delta_\xi U_0^A = 0$ → $\partial_u Y^A = -\frac{\Lambda}{3} \partial^A f.$

Flat spacetime limit: $Y^A = V^A(x^B), \; f = T(x^A) + \frac{u}{2} D_A V^A$

(Soft) Algebra / Algebroid

$\bar{\xi} = f\partial_u + Y^A\partial_A$

$[\bar{\xi}_1, \bar{\xi}_2] = \hat{\bar{\xi}},$

$\hat{\bar{\xi}} = \hat{f}\partial_u + \hat{Y}^A\partial_A$

$$\hat{f} = Y_1^A\partial_A f_2 + \frac{1}{2}f_1 D_A Y_2^A - (1 \leftrightarrow 2),$$
$$\hat{Y}^A = Y_1^B\partial_B Y_2^A - \frac{\Lambda}{3}f_1 q^{AB}\partial_B f_2 - (1 \leftrightarrow 2).$$

The structure constants of the Λ-BMS algebra are field-dependent.

In the flat limit, the structure constants are field-independent and reproduce the generalized BMS algebra.

Λ-BMS: Surface charges

$$\mathrm{d}\boldsymbol{k}_{\xi,\mathrm{ren}}[\delta\phi;\phi] = \boldsymbol{\omega}_{\mathrm{ren}}[\delta_\xi\phi,\delta\phi;\phi],$$

$$\boxed{\delta\!\!\!/ H_\xi[\phi] = \int_{S^2_\infty} 2(\mathrm{d}^2x)_{\rho t}\left[\delta\left(\sqrt{|g^{(0)}|}T^t_{(\mathrm{tot})b}\right)\xi^b_{(0)} - \frac{1}{2}\sqrt{|g^{(0)}|}\,\xi^t_{(0)}\,T^{bc}_{(\mathrm{tot})}\delta g^{(0)}_{bc}\right].}$$

- Charges are finite thanks to renormalization
- Charges are neither conserved or integrable $\delta\!\!\!/ H_\xi[\phi] = \delta H_\xi[\phi] + \Xi_\xi[\delta\phi;\phi],$
- Charges associated with Weyl are zero
- They obey the surface charge algebra

$$\boxed{\{H_\xi[\phi], H_\chi[\phi]\}_\star = H_{[\xi,\chi]_\star}[\phi].}$$

where the Barnich-Troessaert brackets are

$$[\xi,\chi]_\star = [\xi,\chi] - \delta_\xi\chi + \delta_\chi\xi.$$

$$\{H_\xi[\phi], H_\chi[\phi]\}_\star \equiv \delta_\chi H_\xi[\phi] + \Xi_\chi[\delta_\xi\phi,\phi].$$

Completeness of Λ-BMS

Number of flux-balance laws: d-1

$$(\partial_u + \frac{3}{2}l)M^{(\Lambda)} + \frac{\Lambda}{6}D^A N_A^{(\Lambda)} + \frac{\Lambda^2}{24}C_{AB}J^{AB} = 0.$$

$$(\partial_u + l)N_A^{(\Lambda)} - \partial_A M^{(\Lambda)} - \frac{\Lambda}{2}D^B J_{AB} = 0.$$

Number of generators: d-1 : f , Y^A

Number of charges: d-1 : $M^{(\Lambda)},\ N_A^{(\Lambda)}$

In comparison with Diff(S^3) studied in [Anninos,Ng,Strominger,2011] the Λ-BMS groupoid is the subset of Diff(S^3) associated with non-trivial flux-balance laws / Ward identities

Symplectic structure and (A)dS equivalent of Bondi shear/news

Action with holographic counterterms

$$S = \frac{1}{16\pi G}\int_{\mathscr{M}} d^4x\sqrt{-g}\,(R[g]-2\Lambda) + \frac{1}{16\pi G}\int_{\mathscr{I}} d^3x\sqrt{|\gamma|}\,(2K+\frac{4}{\ell}-\ell R[\gamma]).$$

[Balasubramanian, Kraus]

Symplectic structure gets a contribution from the counterterm

$$\delta L = \frac{\delta L}{\delta g_{\mu\nu}}\delta g_{\mu\nu} + d\Theta, \qquad \omega = \delta\Theta$$

$$\omega = \omega_{EH}[\delta g,\delta g; g] - d\omega_{EH}[\delta\gamma,\delta\gamma;\gamma]$$

[Skenderis,Papadimitriou,2005]

[G.C.,Marolf,2008]

At the boundary, the orthogonal component of the symplectic structure is

$$\omega^\rho = \frac{1}{2\ell^2}\int_{\mathscr{I}} d^3x\,\delta\Big(\sqrt{|g_{(0)}|}T^{ab}\Big)\wedge\delta g^{(0)}_{ab}.$$

$$g^{(0)}_{tt} = \frac{\Lambda}{3}, \qquad g^{(0)}_{tA} = 0, \qquad \det(g_{(0)}) = \frac{\Lambda}{3}\bar{q}.$$

$$T_{ab} = \frac{\sqrt{3|\Lambda|}}{16\pi G}\begin{bmatrix} -\frac{4}{3}M^{(\Lambda)} & -\frac{2}{3}N^{(\Lambda)}_B \\ -\frac{2}{3}N^{(\Lambda)}_A & J_{AB}+\frac{2}{\Lambda}M^{(\Lambda)}q_{AB}\end{bmatrix}$$

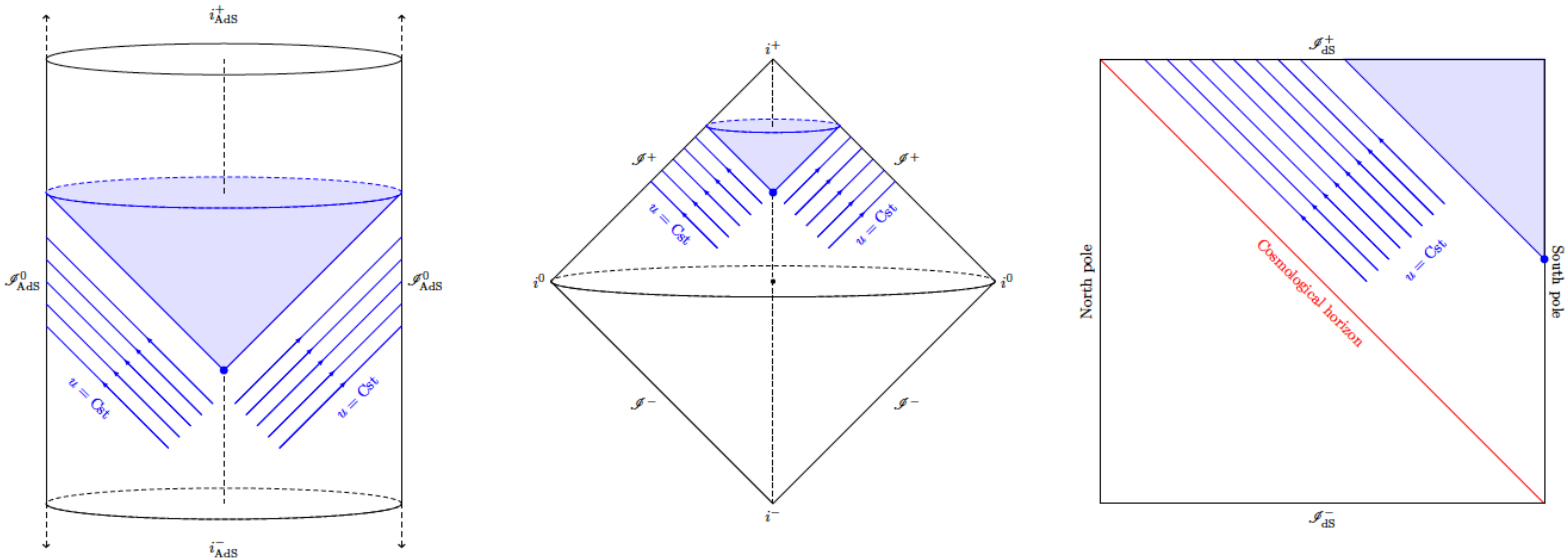
$\longrightarrow$

$$\omega^\rho = \frac{3}{32\pi G\ell^4}\int_{\mathscr{I}} d^3x\,\sqrt{\bar{q}}\,\delta J^{AB}\wedge\delta q_{AB}.$$

Therefore, energy is transferred due to changes of both J^{AB} and q_{AB}. They are the analogue in dS/AdS of the Bondi shear/news.

[G.C., Fiorucci, Ruzziconi, 2019]

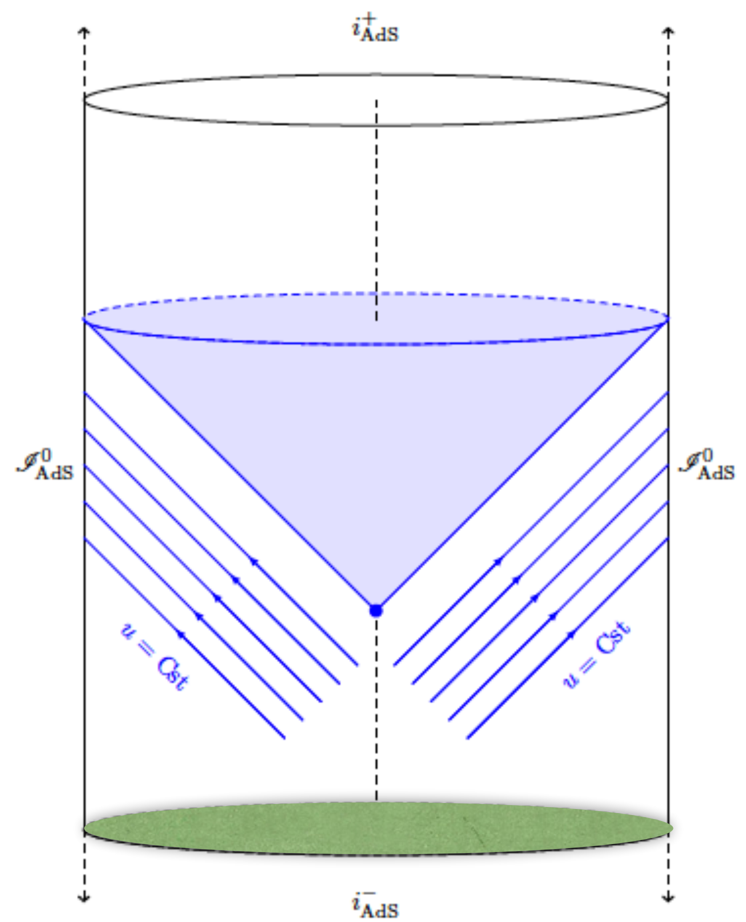
Symplectic flux and the Cauchy problem



AdS case $\Lambda < 0$. Flat case $\Lambda = 0$. dS case $\Lambda > 0$.

In AdS, the Cauchy problem requires an additional boundary condition (standard or "leaky")

Example of "leaky" boundary condition

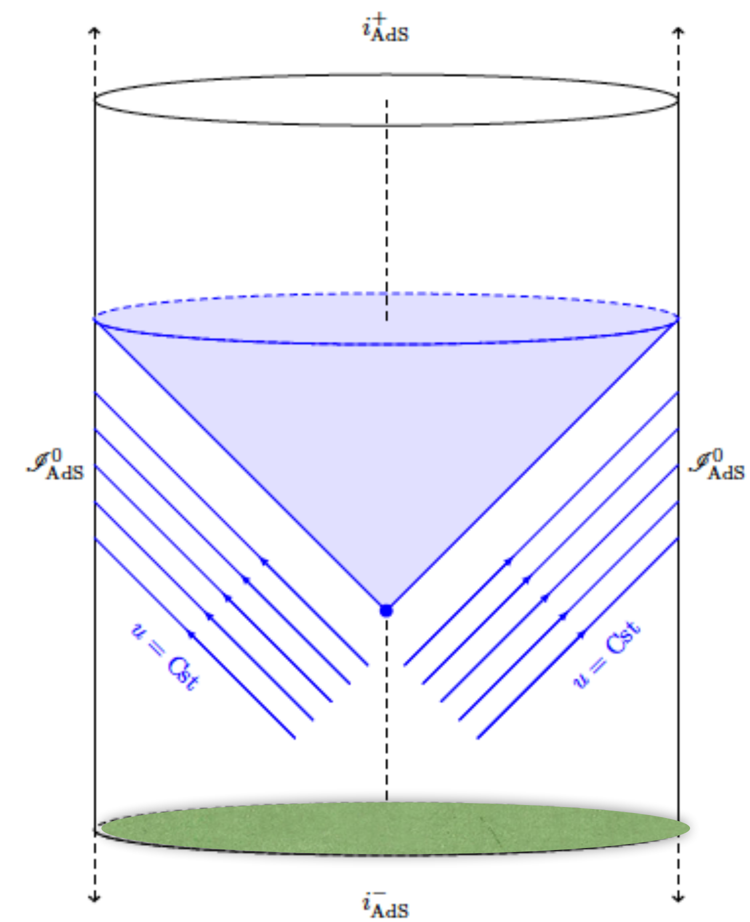


AdS case $\Lambda < 0$.

Glued at the boundary ⟷

(identification of codimension 2 boundary metrics)

$q_{AB} = q'_{AB}$



AdS case $\Lambda < 0$.

Fix the initial data of both AdS's.

Then the Cauchy problem of the first AdS is well-defined.

Related example: [Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini, 2019]

Conclusion

- The extended BMS charge algebra (supertranslations and Diff(S^2) super-Lorentz transformations) is realized without center at the past and future of null infinity (at spatial and timelike infinity). The asymptotic symmetry group at spatial infinity therefore includes the extended BMS group.
- The extended BMS asymptotic symmetry algebra leads to a preferred definition of the quantized angular momentum, which differs from many existing classical prescriptions.
- The extended BMS charge algebra admits a natural extension to (A)dS: the Λ-BMS algebroid. It is the asymptotic symmetry group of Al(A)dS spacetimes with "leaky boundary conditions": without intrinsic boundary conditions (except a boundary gauge fixing condition that does not constraint the Cauchy problem) but with external boundary conditions.