Quantum expectation values on black hole space-times

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Quantum fields on curved space-time

QFT on curved space-time

QFT on curved space-time

- Classical background space-time
- Quantum field on this background

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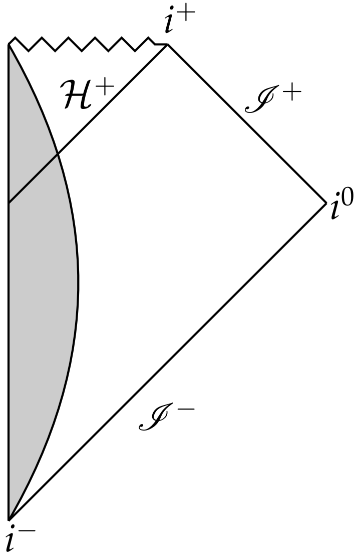
Hawking radiation

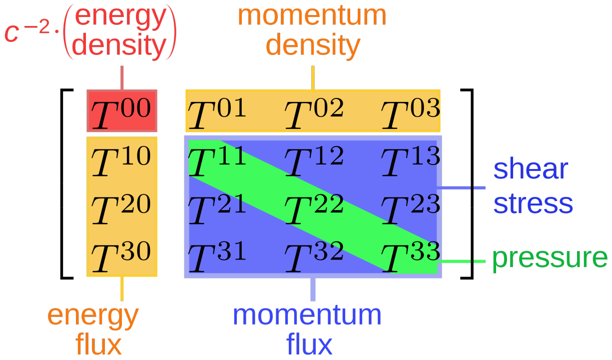
- Black hole formed by gravitational collapse
- Thermal flux at $\mathscr{I}^+$
$$T_H = \frac{\kappa}{2\pi}$$



Stress-energy tensor expectation value

Semi-classical Einstein equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi \langle \hat{T}_{\mu\nu} \rangle$$



Massless, conformally coupled scalar field Φ

Klein-Gordon equation

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Classical stress-energy tensor

$$T_{\mu\nu} = \frac{2}{3}\Phi_{;\mu}\Phi_{;\nu} - \frac{1}{6}g_{\mu\nu}\Phi^{;\alpha}\Phi_{;\alpha} - \frac{1}{3}\Phi\Phi_{;\mu\nu} + \frac{1}{3}g_{\mu\nu}\Phi\Box\Phi + \frac{1}{6}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\Phi^2$$

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Expectation values of field operators

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$$\langle \hat{T}_{\mu\nu}(x) \rangle = \lim_{x' \to x} \left[\mathcal{T}_{\mu\nu} \left(-\mathrm{i} G_{\mathrm{F}}(x, x') \right) \right]$$

$\mathcal{T}_{\mu\nu}$ second order differential operator

Feynman Green's function

Static, spherically symmetric black hole

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Renormalization

DeWitt *Phys. Rept.* **19** 295 (1975)
Christensen *PRD* **14** 2490 (1976)
Wald *CMP* **54** 1 (1977)
Christensen *PRD* **17** 946 (1978)
Decanini & Folacci *PRD* **78** 044025 (2008)

Overall strategy

Stress-energy tensor operator $\hat{T}_{\mu\nu}$

- Involves products of field operators at the same space-time point
- Expectation values are divergent

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Regularization by point-splitting

- $\mathcal{T}_{\mu\nu}\left(-\mathrm{i}G_{\mathrm{F}}(x,x')\right)$ finite for $x' \neq x$
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Renormalized expectation value

- Subtract off appropriate divergent terms $G_{\mathrm{S}}(x,x')$

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Hadamard renormalization

[Decanini & Folacci *PRD* **78** 044025 (2008)]

Hadamard renormalization

Hadamard parametrix

$$-\mathrm{i}G_{\mathrm{S}}(x,x') = \frac{U(x,x')}{\sigma(x,x')} + V(x,x')\,\log\left(\frac{\sigma(x,x')}{\ell^2}\right)$$

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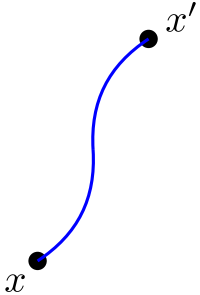

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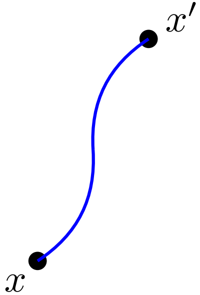

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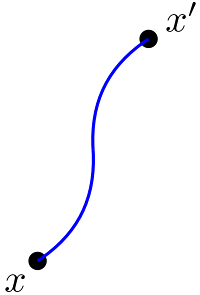
- ℓ renormalization length scale



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Renormalized expectation values

$$\langle \hat{T}_{\mu\nu}(x) \rangle_{\rm ren} = \lim_{x' \to x} \left[\mathcal{T}_{\mu\nu} \left\{ -{\rm i} \left[G_{\rm F}(x,x') - G_{\rm S}(x,x') \right] \right\} \right]$$

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Challenge

How can we subtract $G_{\rm S}(x,x')$ from $G_{\rm F}(x,x')$ so that limit can be taken and answer computed numerically?

WKB-based method

Candelas & Howard *PRD* **29** 1618 (1984)
Howard & Candelas *PRL* **53** 403 (1984)
Howard *PRD* **30** 2532 (1984)
Anderson, Hiscock & Samuel *PRD* **51** 4337 (1995)
EW & Young *PRD* **77** 024008 (2008)
Flachi & Tanaka *PRD* **78** 064011 (2008)
Breen & Ottewill *PRD* **82** 084019 (2010)
Breen & Ottewill *PRD* **85** 084029 (2012)

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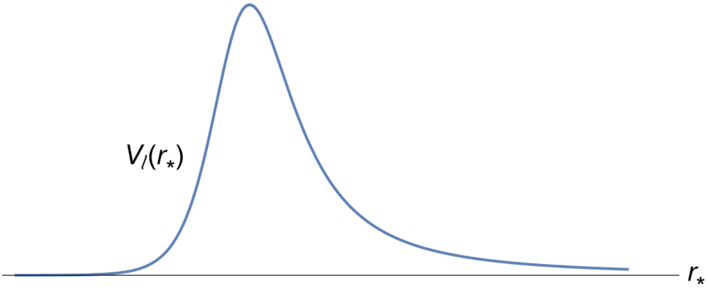

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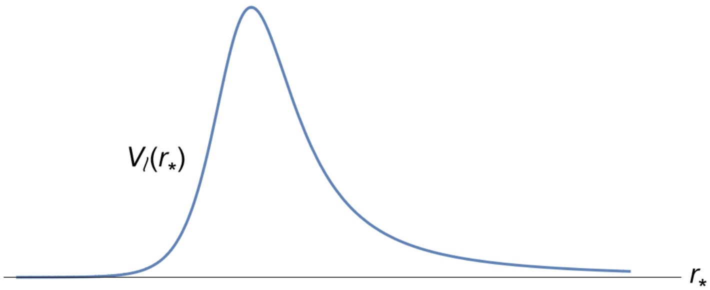

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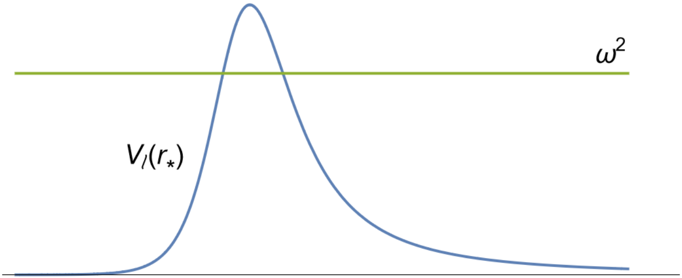

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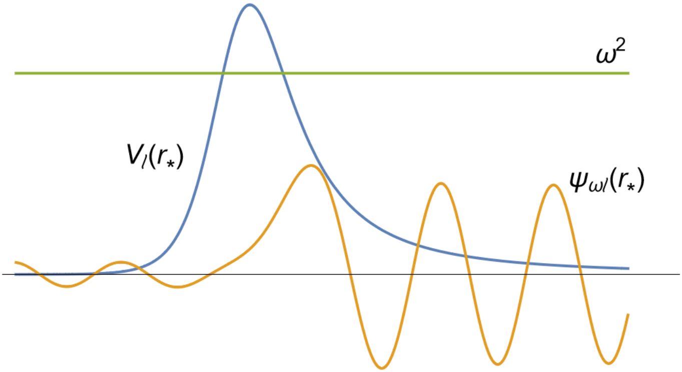

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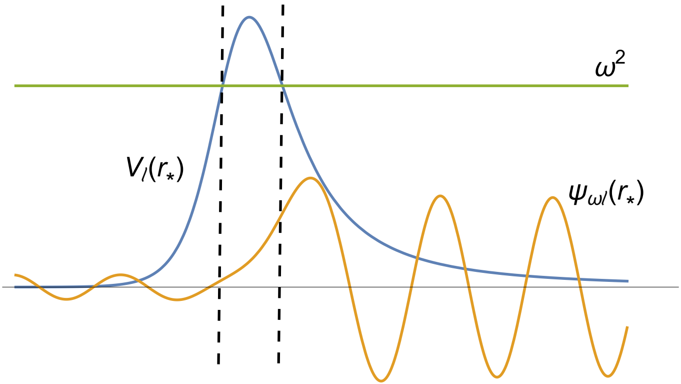

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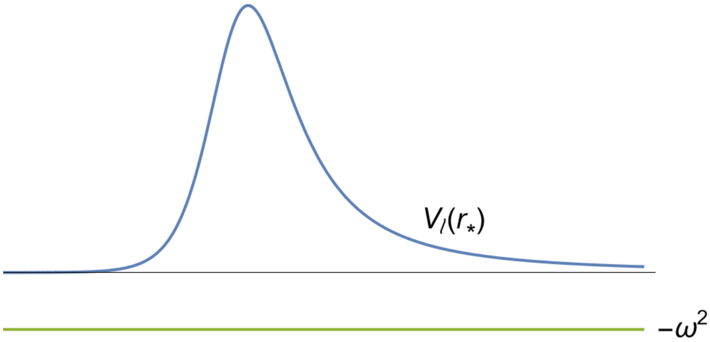

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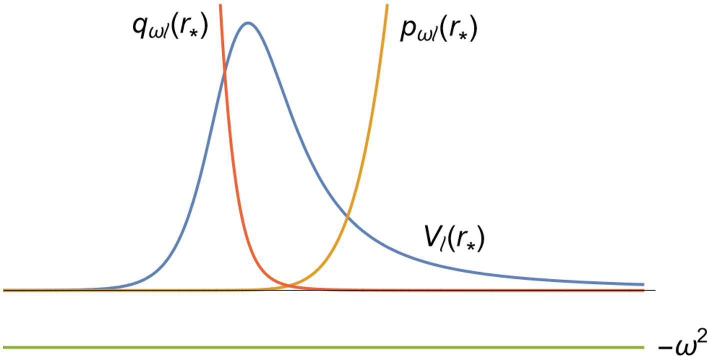
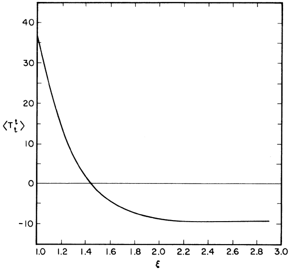
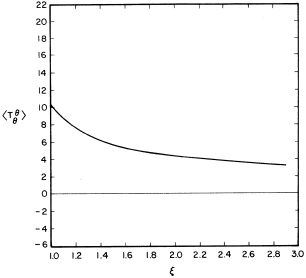


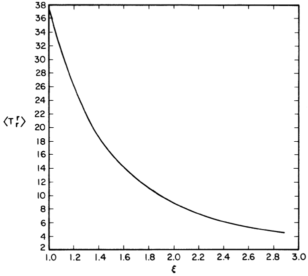
RSET on Schwarzschild



$$ds^2 = f(r)\,d\tau^2 + f(r)^{-1}\,dr^2 + r^2\,d\Omega^2$$

$$f(r) = 1 - \frac{2M}{r}$$

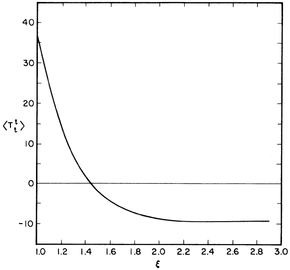


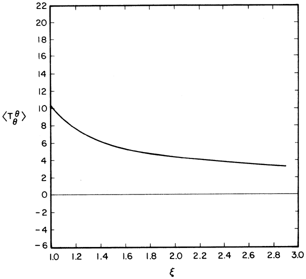


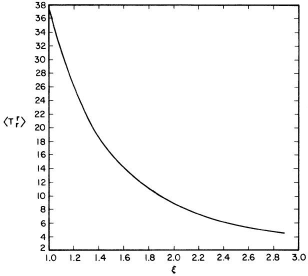
[Howard & Candelas *PRL* **53** 403 (1984)]

RSET on Schwarzschild

- Euclidean time coordinate periodic $\tau \rightarrow \tau + 2\pi/\kappa$







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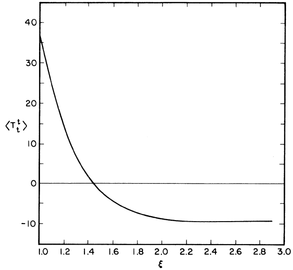
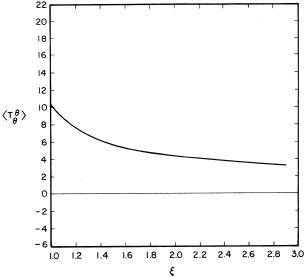
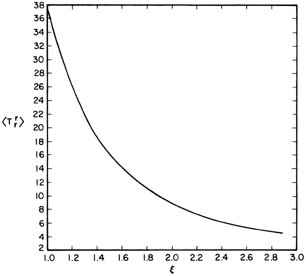




- Euclidean time coordinate periodic $\tau \to \tau + 2\pi/\kappa$
- Thermal state at temperature $\kappa/2\pi$



[Howard & Candelas *PRL* **53** 403 (1984)]

Extended coordinates method

Taylor & Breen *PRD* **94** 125024 (2016)
Taylor & Breen *PRD* **96** 105020 (2017)
Morley, Taylor & EW *CQG* **35** 235010 (2018)
Breen & Taylor *PRD* **98** 105006 (2018)
Morley, Taylor & EW *PRD* **103** 045007 (2021)

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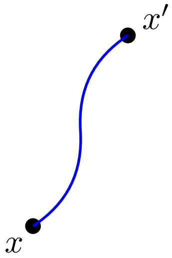

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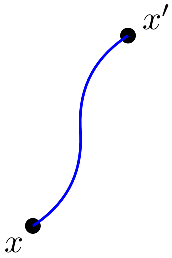

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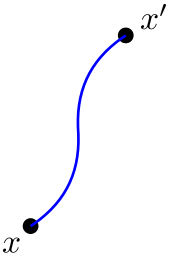

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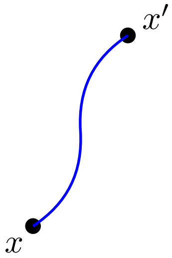

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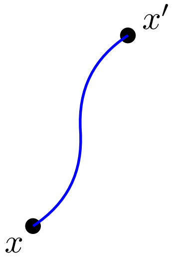

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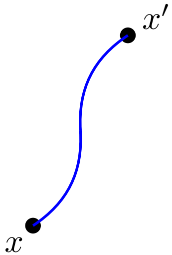

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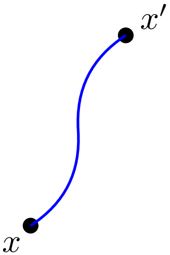

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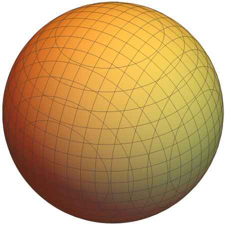
Topological black holes

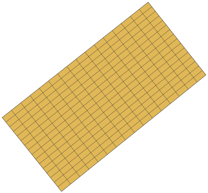
Schwarzschild-anti-de Sitter space-time

$$ds^2 = -\left(k - \frac{2M}{r} + \frac{r^2}{L^2}\right) dt^2 + \left(k - \frac{2M}{r} + \frac{r^2}{L^2}\right)^{-1} dr^2 + r^2\, d\Omega_k^2$$

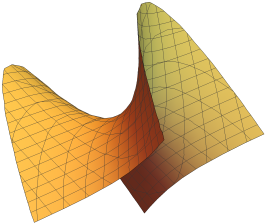
$k = 1$



$k = 0$

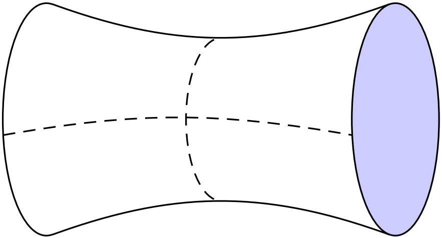


$k = -1$



$d\Omega_1^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$ $\quad d\Omega_0^2 = d\theta^2 + \theta^2\, d\varphi^2$ $\quad d\Omega_{-1}^2 = d\theta^2 + \sinh^2\theta\, d\varphi^2$

Geometry of anti-de Sitter space-time



- Time-like boundary
- Boundary conditions required

[Avis, Isham & Storey *PRD* **18** 3565 (1978)]

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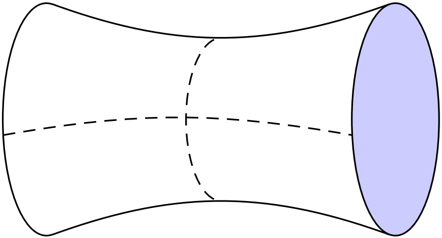

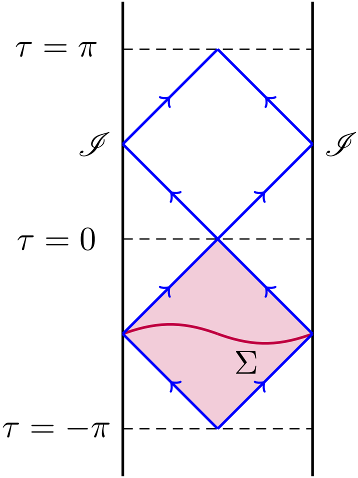

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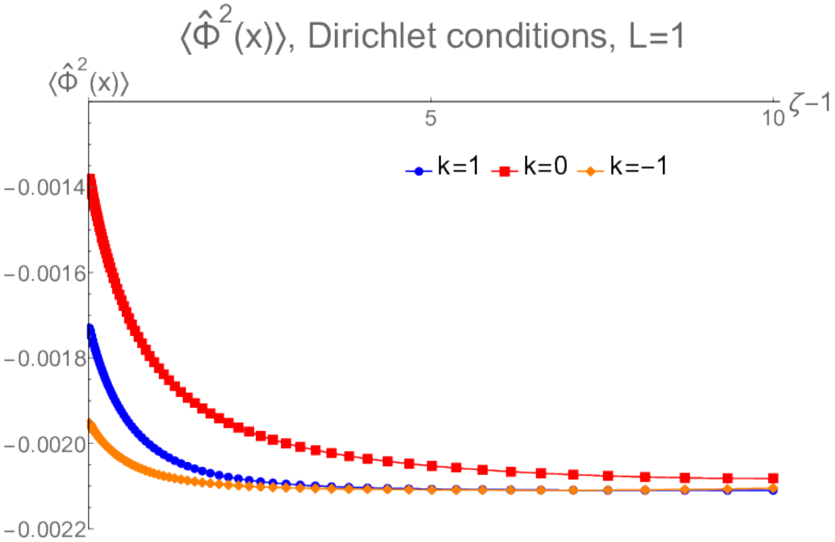
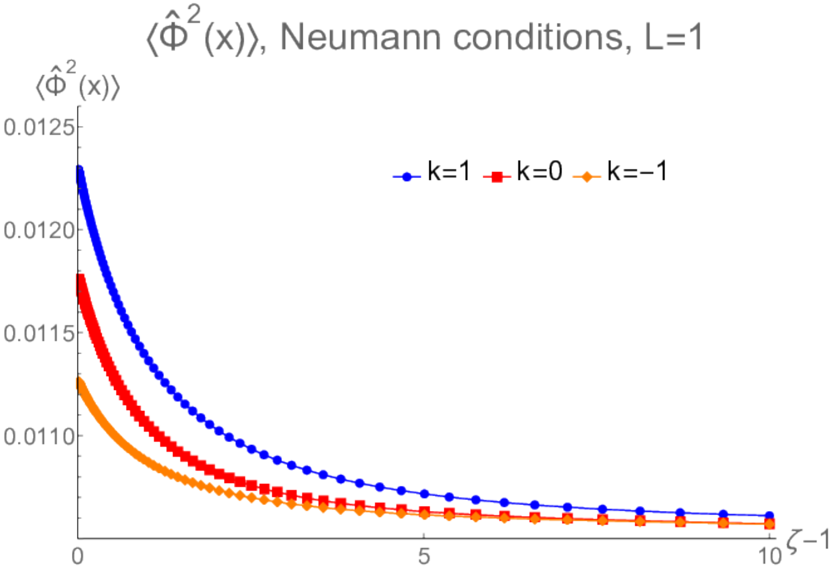


[Flachi & Tanaka *PRD* **78** 064011 (2008), Morley, Taylor & EW *CQG* **35** 235010 (2018)]



[Morley, Taylor & EW *PRD* **103** 045007 (2021)]

Pragmatic mode sum method

Levi & Ori *PRD* **91** 104028 (2015)
Levi & Ori *PRD* **94** 044054 (2016)
Levi & Ori *PRL* **117** 231101 (2016)
Levi, Eilon, Ori & van de Meent *PRL* **118** 141102 (2017)
Levi *PRD* **95** 025007 (2017)
Lanir, Levi, Ori & Sela *PRD* **97** 024033 (2018)
Lanir, Levi & Ori *PRD* **98** 084017 (2018)
Lanir, Ori, Zilberman, Sela, Maline & Levi *PRD* **99** 061502 (2019)
Zilberman, Levi & Ori *PRL* **124** 171302 (2020)
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Lorentzian black hole space-times

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$$-\mathrm{i}G_{\mathrm{F}}(x,x') = \int_{\omega=0}^{\infty} d\omega \sum_{\ell=0}^{\infty} (2\ell+1) \frac{\mathrm{e}^{\mathrm{i}\omega\epsilon}}{r^2} \left| \psi_{\omega\ell}(r) \right|^2$$

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Generalized integrals

[Levi & Ori *PRD* **91** 104028 (2015)]

Generalized integrals

$$\mathcal{H}(\omega) = \int_{\varpi=0}^{\omega} d\varpi \, \mathrm{e}^{\mathrm{i}\varpi\epsilon} \mathcal{G}_{\varpi}(r)$$

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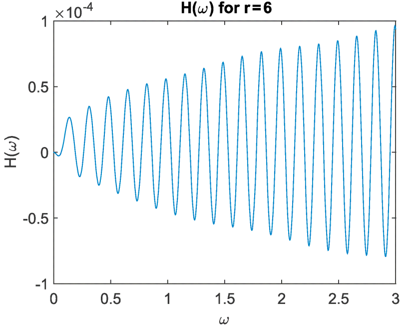

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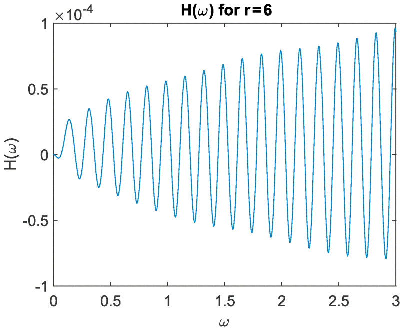

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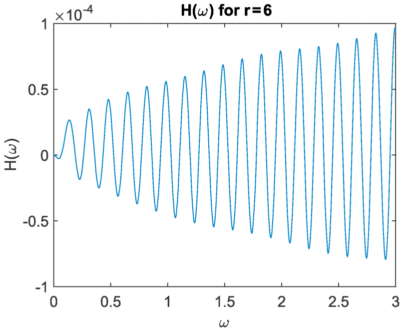

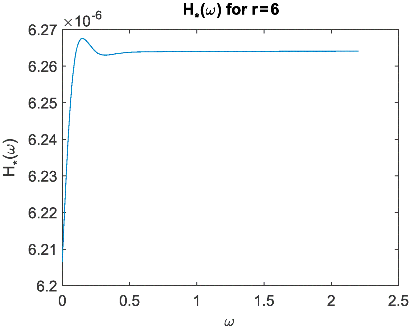

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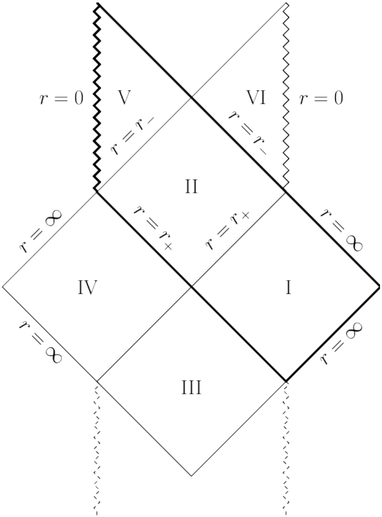

[Figure: Paston & Sheykin *SIGMA* **10** 003 (2014)]

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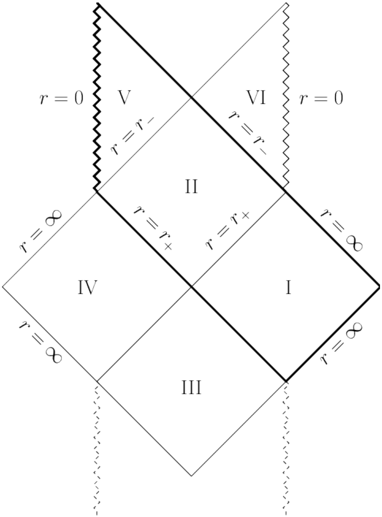
Cosmic censorship

[Penrose (1974)]

- Cauchy horizon $r = r_-$ unphysical
- Classically unstable



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RSET at the Cauchy horizon in RN

[Zilberman, Levi & Ori *PRL* **124** 171302 (2020)
Zilberman & Ori *PRD* **104** 024066 (2021)]

RSET at the Cauchy horizon in RN

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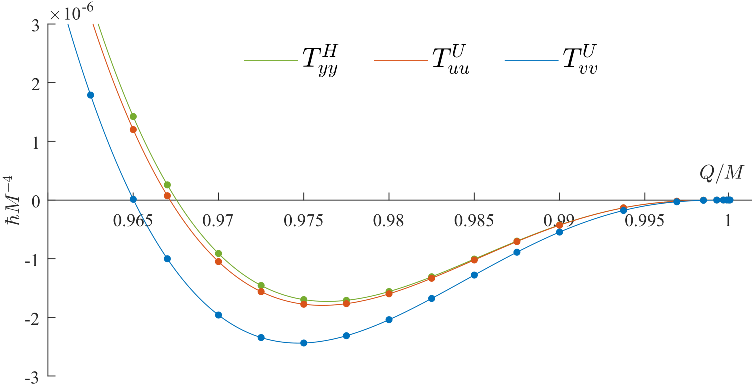

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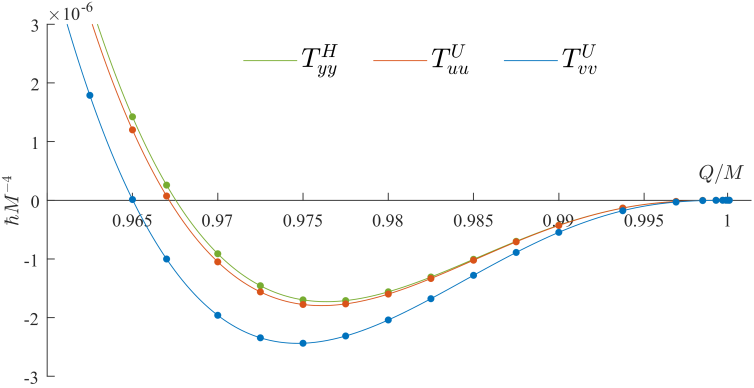

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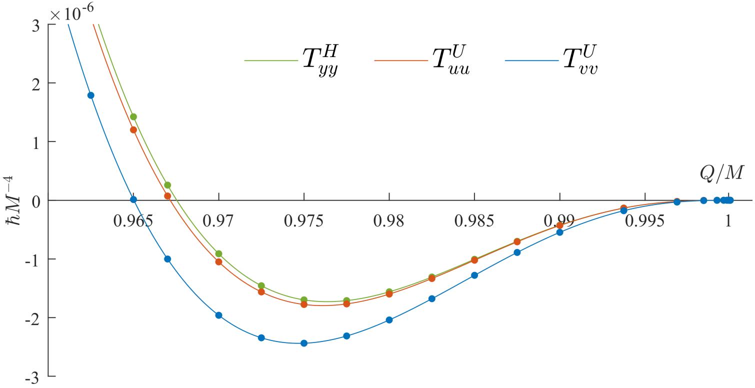

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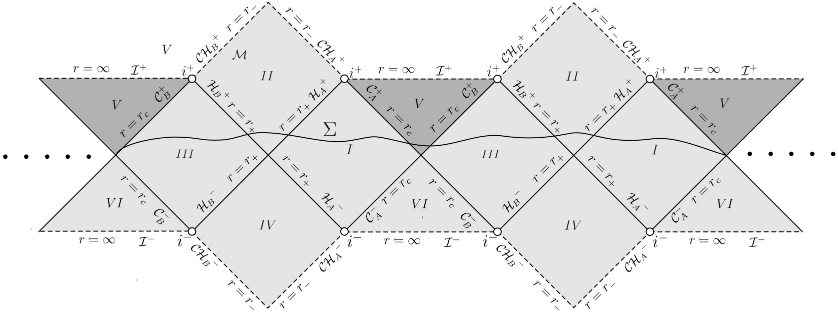

[Figure: Costa, Natario & Oliveira *Ann. H. Poincaré* **20** 3059 (2019)]

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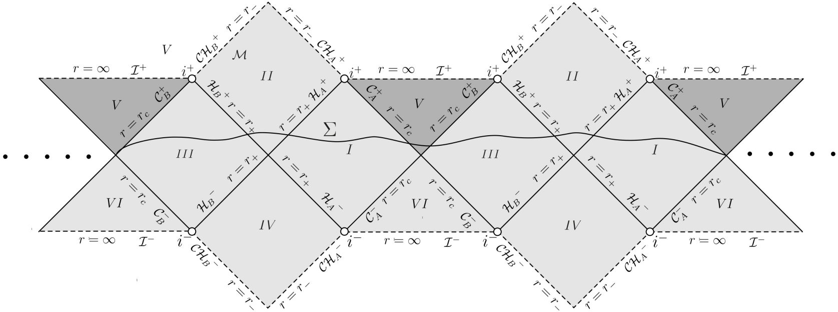

[Figure: Costa, Natario & Oliveira *Ann. H. Poincaré* **20** 3059 (2019)]

Strong cosmic censorship fails classically for some RNdS black holes
[Cardoso et al *PRL* **120** 031103 (2018)]

RSET at the Cauchy horizon in RNdS

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$$\langle \hat{T}_{VV} \rangle \sim \kappa_-^2 \, \widetilde{C} \, |V|^{-2} + \ldots \qquad \text{as } V \to 0$$

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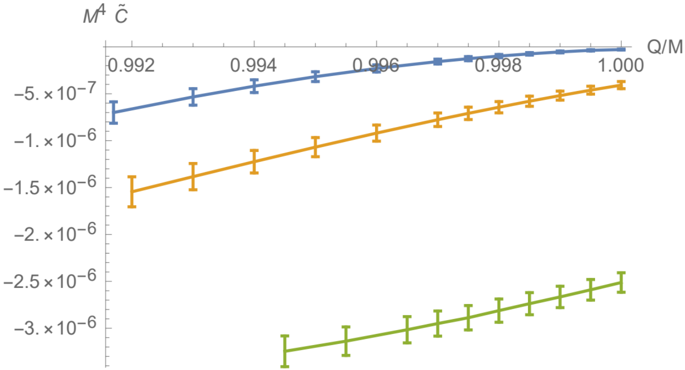

[Hollands, Klein & Zahn *PRD* **102** 085004 (2020)]

Renormalized vacuum polarization and SET

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Computing renormalized expectation values

- WKB-based method
- Extended coordinates method
- Pragmatic mode sum method

Renormalized vacuum polarization and SET

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Outlook

- Cosmic censorship

Renormalized vacuum polarization and SET

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- WKB-based method
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Outlook

- Cosmic censorship
- Back-reaction problem

Semi-classical Einstein equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi \langle \hat{T}_{\mu\nu} \rangle$$