Gravitational wave Poynting vector and gravitoelectromagnetism

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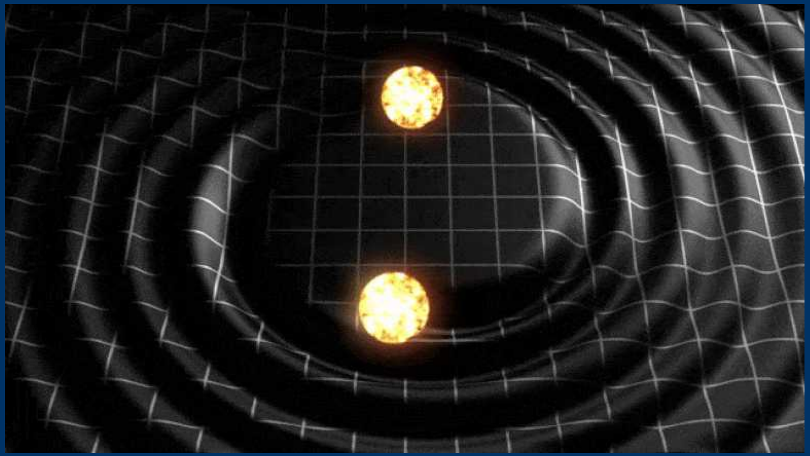
EREP 2021, Virtual – September 14th

Outline of this talk

- Gravitational waves
- Gravitoelectromagnetism
- Poynting vector in GR and in GEM
- An illustration with GW detectors
- Final remarks

Gravitational waves



Phenomena predicted by general relativity

Weak fields: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $|h_{\mu\nu}| \ll 1$

$$[\eta_{\mu\nu}] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Low speeds: $v \ll c$

Linear approximation: only keeps $O(h_{\mu\upsilon})$ terms

Einstein equations + gauge TT → wave equation for $h_{\mu\upsilon}$.

https://scitechdaily.com/images/Gravitational-Waves.gif

Gravitational waves

TT gauge → [$h_{\mu\upsilon}$] has 5 independent components

These components allow the determination of all the wave's parameters:
direction (q, f), polarization (h+, hx) and amplitude.

$$\boldsymbol{h}^{\mathrm{TT}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

https://scitechdaily.com/images/Gravitational-Waves.gif

Gravitoelectromagnetism (GEM)

Similarities between electromagnetism (EM) and gravitation are known for a long time.

1918: Thirring pointed out an analogy between GR (in first approximation) and EM.
He and Lense calculated a “magnetic” effect (Lense-Thirring precession)
that was tested with satellites.

LAGEOS satellite

https://upload.wikimedia.org/wikipedia/commons/7/78/LAGEOS-NASA.jpg

Gravitoelectromagnetism (GEM)

1961: Forward presented a direct analogy between the equations of EM and GR to help experimentalists deal with GR experiments.

He recognized that this was an approximation to the tensorial formalism.

TABLE I

	EM Symbol	Gravitational Symbol	Value or Definition
Force Vector	$-\boldsymbol{E}$ →	$\boldsymbol{G}$	$= -\nabla\chi - \frac{\partial \boldsymbol{K}}{\partial t}$
Solenoidal Force Vector	$-\boldsymbol{B}$ →	$\boldsymbol{P}$	$= \nabla \times \boldsymbol{K}$
Scalar Potential	$-\phi$ →	χ	$\approx -\frac{1}{4\pi\gamma}\int_V \frac{\mu}{r}\, dV$
Vector Potential	$-\boldsymbol{A}$ →	$\boldsymbol{K}$	$\approx -\frac{\eta}{4\pi}\int_V \frac{\mu \boldsymbol{v}}{r}\, dV$
Source Density	ρ →	μ	$= \frac{dM}{dV}$
Source Quantity	Q →	M	$= \int_V \mu\, dV$

Gravitoelectromagnetism (GEM)

1977: Braginski, Caves & Thorne presented a similar analogy, based on the PPN expansion.

Over the years, various applications of this analogy – the gravitoelectromagnetism – have resulted.
The subject is present in textbooks on gravity, like those by Ohanian & Ruffini, Rindler and Moore.

Mashoon (2008) wrote a brief review about it.
Costa & Herdeiro (2008) proposed a new approach to a physical analogy between GR and EM, based on tidal tensors of both theories and wrote a covariant form for the gravitational analogues of the Maxwell equations.

Research using GEM and on GEM is ongoing.

GEM applied to GW detected by LIGO

After decades of technological improvement (since 1960's), the announcement of the first direct detection of GW occurred in 2015 using the interferometric detectors LIGO.

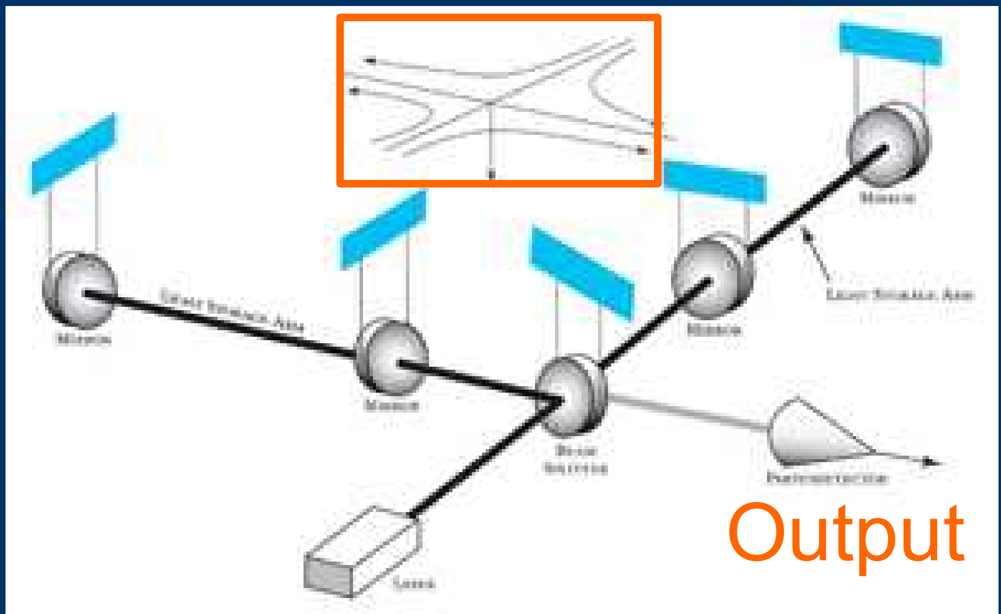
https://www.ligo.caltech.edu/system/media_files/binaries/180/large/LIGOs_Dual_Detectors.jpg?1431726608

GEM applied to GW detected by LIGO

A detector has a response function that depends on the input signal

Iorio & Crosta (2011) showed that the total response fuction of the LIGO had relevant contribution at higher frequencies (e.g. 8kHz) from the (gravito)magnetic component.

Input: GW



https://www.aps.org/publications/apsnews/200306/images/ligo_detector.jpg

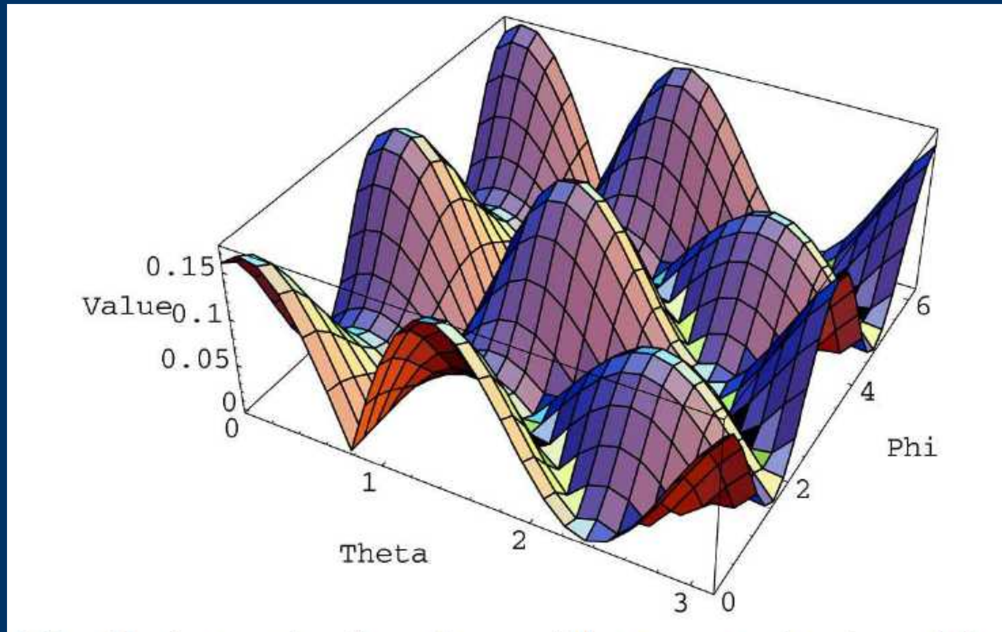


Fig. (3). the angular dependence of the response function of the LIGO interferometer to the magnetic component of the + polarization for $f = 8000$ Hz.

Poynting vector in GEM

From Mashoon (2008), the GEM field equations: $O(h_{\mu\upsilon})$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{B}\right), \quad \nabla \cdot \left(\frac{1}{2}\mathbf{B}\right) = 0.$$

$$\nabla \cdot \mathbf{E} = 4\pi G\rho, \quad \nabla \times \left(\frac{1}{2}\mathbf{B}\right) = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E} + \frac{4\pi G}{c}\mathbf{j}.$$

The GEM fields defined by: $\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{A}\right), \quad \mathbf{B} = \nabla \times \mathbf{A},$

The GEM potentials defined by: $\bar{h}_{00} = 4\Phi/c^2$ $\bar{h}_{0i} = -2A_i/c^2$

Trace-reversed amplitude: $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h,$ where $h = \eta^{\mu\nu}h_{\mu\nu}$

Poynting vector in GEM

From Mashoon (2008), the Landau-Lifshitz pseudotensor ($t_{\mu\upsilon}$) is employed to determine the local stress-energy content of the GEM fields. For a stationary configuration:

$$\partial\Phi/\partial t = 0$$
$$\partial\mathbf{A}/\partial t = 0$$

$$4\pi G t_{0i} = 2(\mathbf{E}\times\mathbf{B})_i$$

The GEM Poynting vector defined by:

$$\mathcal{S} = -\frac{c}{2\pi G}\mathbf{E}\times\mathbf{B}.$$

$$\bar{h}_{00} = 4\Phi/c^2$$
$$\bar{h}_{0i} = -2A_i/c^2$$

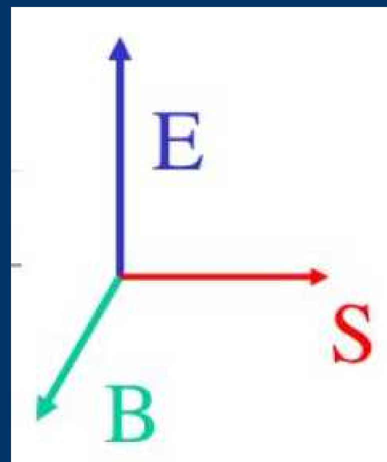
$O(h_{\mu\upsilon})$

Electromagnetic Poynting vector

From classical electromagnetism, the electromagnectic flux is characterized by the Poynting vector:

$$\mathbf{S} = \frac{c}{4\pi} \mathbf{E} \times \mathbf{H}$$

It represents the EM energy that flows per unit time (power) through a unit area in a direction perpendicular to the electric field E and the magnetic field H.



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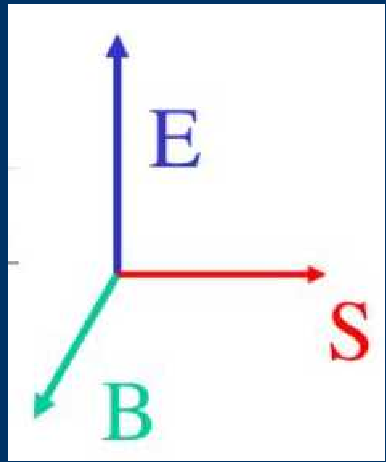

The GEM Poynting vector:

$$\mathcal{S} = -\frac{c}{2\pi G} \mathbf{E} \times \mathbf{B}$$

Poynting vector in EM and in GEM

EM: $$\mathbf{S} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{H}$$

GEM: $$\mathcal{S} = -\frac{c}{2\pi G}\mathbf{E}\times\mathbf{B}$$

It represents the energy that flows per unit time (power) through a unit area in a direction perpendicular to the electric field and the magnetic field.

GEM: $$4\pi G t_{0i} = 2(\mathbf{E}\times\mathbf{B})_i$$

The component of the Landau-Lifshitz pseudotensor ($t_{\mu\upsilon}$) represents this energy flux.

Energy flux in general relativity

In GR the energy flux of a system is given by the stress-energy tensor ($T_{\mu\upsilon}$), specifically its T_{0k} component.

For a gravitational wave this component is given by

MTW 1973 § 35.7. c=G=1.

< > denotes an average over several wavelengths.

$$T_{0k} = 1/(32\pi) \langle h^{TT}_{ij,0}\, h^{TT}_{ik,k} \rangle \; .$$

Calculating this average using t_{0k} yields the same as above.

There is no Poynting vector in this context.

Energy flux in general relativity: an illustration

Magalhaes et al. (1997)*, studying the detection of GW by spherical resonant-mass detectors, analysed the influence of a GW on a spherical distribution of free masses.

*Magalhaes et al. ApJ 475, 462 (1997)

Energy flux in general relativity: an illustration

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A brief comment on spherical geometry for the detection of GW

*Magalhaes et al. ApJ 475, 462 (1997)

Spherical detectors of GW

The last generation of resonant-mass GW detector was under study in the early 1990's, when the LIGO detectors got funds to be constructed.

The US projects for resonant-mass detector were shut down. A Brazilian and a Dutch prototypes with spherical geometry were developed.

The Brazilian prototype evolved to the Schenberg dector and is still under study.

Spherical detectors of GW: SCHENBERG

One only, well-tuned spherical GW detector should be capable of yielding all 5 independent components of $h_{\mu\upsilon}$.

The Schenberg dector was designed to detect GW around 3200 Hz, with narrow bandwidth and limited by quantum noise.

It is presently disassembled.

Picture Credit: Odylio D. Aguiar

Optimization studies are ongoing aiming at coincidence operation with interferometers.

Bortoli, Frajuca, Magalhes et al. BPJ 50, 541 (2020)

Spherical detectors of GW:
The Laser Gravitational Compass

The laser gravitational compass:
A spheroidal interferometric gravitational observatory

It is shown that a minimum of four non-coplanar mass probes are necessary for a Michelson-Morley interferometer to fully detect gravitational waves within the context of GR.

With fewer probes, some alternative theories of gravitation could also explain the observations.

The conversion of the existing gravitational wave detectors to four probes is also suggested.

Ferreira, Magalhes et al. IJMPA 35, 2040020 (2020).

Back to

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Gravitational wave in GR

The wave of space-time (GW) distorts the background space-time (Minkowskian) where the particles are located.

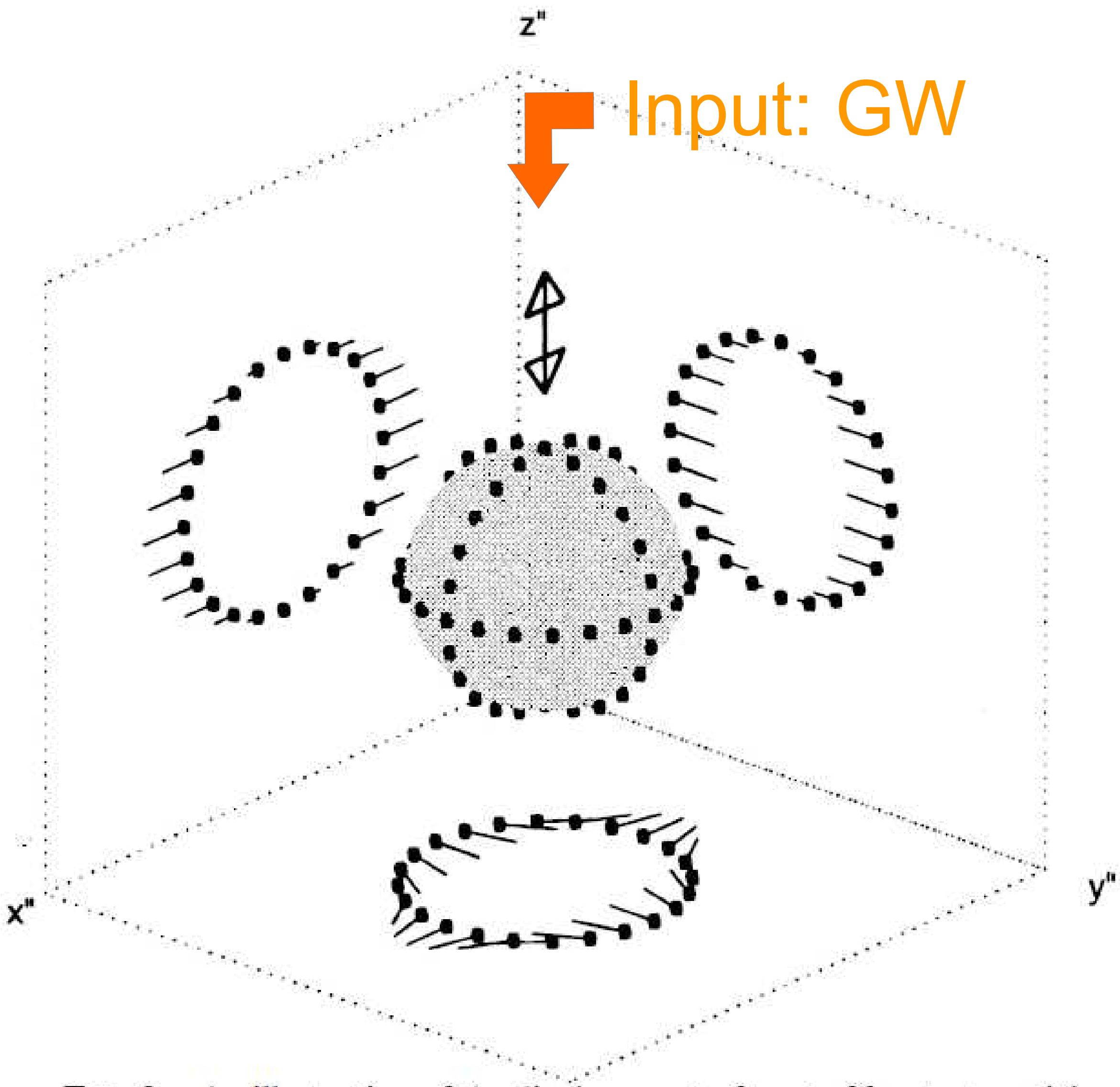


FIG. 2.—An illustration of the displacement of a set of free test particles by a gravitational wave traveling in the z'' direction. The particles are initially arranged in three orthogonal concentric rings. Each undeformed ring of particles is projected onto a plane. Their subsequent gravitational displacements are shown by the lines, which all lie in a plane normal to the direction of propagation.

Magalhaes et al. ApJ 475, 462 (1997)

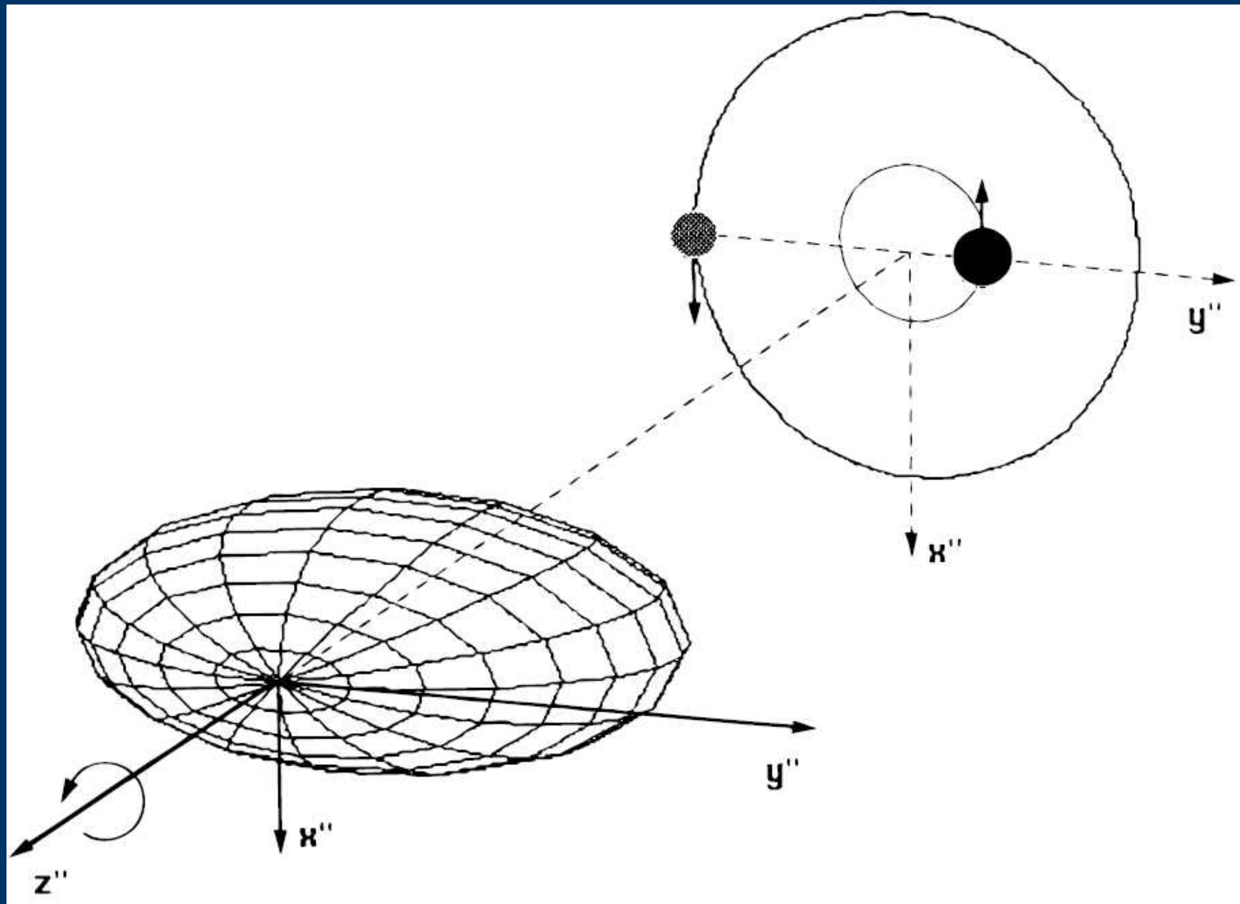
Gravitational wave in GR

The wave of space-time distorts the background space-time while it travels.

The volume of 3-space depends on g = Det(**g**).

In the TT gauge

$$g = 1 - h_t^2$$



$$\boldsymbol{h}'' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_t & 0 & 0 \\ 0 & 0 & -h_t & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

In the diagonal frame of the ellipsoid

Magalhaes et al. ApJ 475, 462 (1997)

Gravitational wave flux in GR

In the TT gauge $g = 1 - h_t^2$

This shows that the particles move relative to the center of the sphere.

We can show that the energy flux, $T_{0k} = 1/(32\pi) \langle h^{TT}_{ij,0}\, h^{TT}_{ik,k} \rangle$, is not zero.

Consider the simple example of a monochromatic wave:

$$h_{t''} = \mathscr{A} \cos \omega(t'' - z''/c)$$

Gravitational wave flux in GEM

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GEM:

$$4\pi G t_{0i} = 2(\mathbf{E} \times \mathbf{B})_i$$

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The GEM Poynting vector is null for the GW.

Final remarks

Gravitoelectromagnetism provides useful insights on the Poyinting vector of statationary space-times.

But it proves unfit to analyse gravitational wave using this vector.

Recently a paper published by our collaborators* extended GEM to higher orders and explained the precession of the orbit of Mercury with the formalism.

We intend to apply this beyond-GEM formalism to the study of gravitational waves.

* Rocha et al. Int. J. Mod. Phys. D (2021) 2150073

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Thank you

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